

Thinking Inductively

COS 326

David Walker

Princeton University

Administration

Assignment 1 due at 11:59pm tonight!

Program style guide:

- <http://www.cs.princeton.edu/~cos326/style.php>

if style:

```
if b then
  exp1
else
  exp2
```

Read notes:

- functional basics, type-checking, typed programming
- thinking inductively (today)
- Real World OCaml Chapter 2, 3 (optional)

For Windows users:

- I pinned some install instructions to the top of the Piazza feed
- They look relatively easy to follow and have the side effect of installing bash, which will be generally useful for you in the future

Options

A value v has type t **option** if it is either:

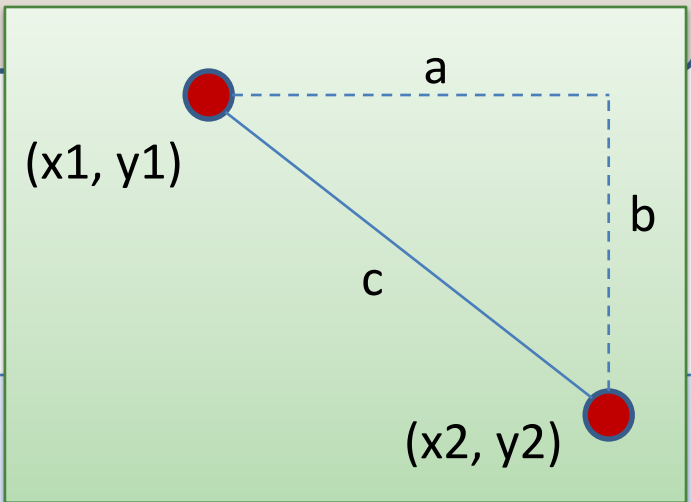
- the value **None**, or
- a value **Some v'** , and v' has type t

Options can signal there is no useful result to the computation

Example: we look up a value in a hash table using a key.

- **If the key is present**, return **Some v** where v is the associated value
- **If the key is not present**, we return **None**

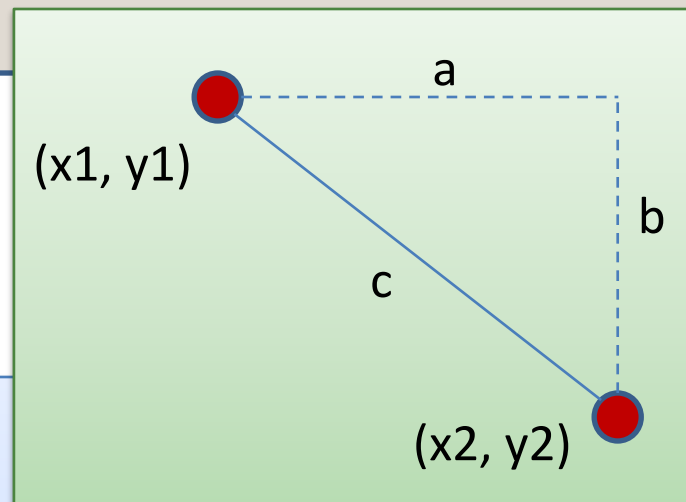
Slope between two points



```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float =
```


Slope between two points



```
type point = float * float
```

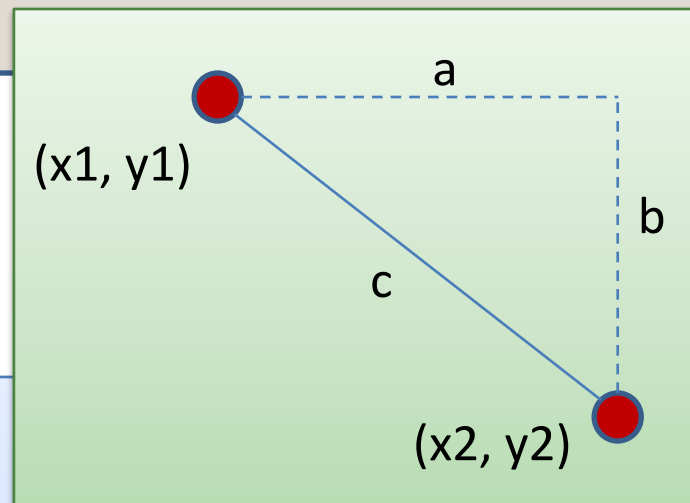
```
let slope (p1:point) (p2:point) : float =
```

```
  let (x1,y1) = p1 in
```

```
  let (x2,y2) = p2 in
```

deconstruct tuple

Slope between two points



```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float =
```

```
  let (x1,y1) = p1 in
```

```
  let (x2,y2) = p2 in
```

```
  let xd = x2 -. x1 in
```

```
  if xd != 0.0 then
```

```
    (y2 -. y1) /. xd
```

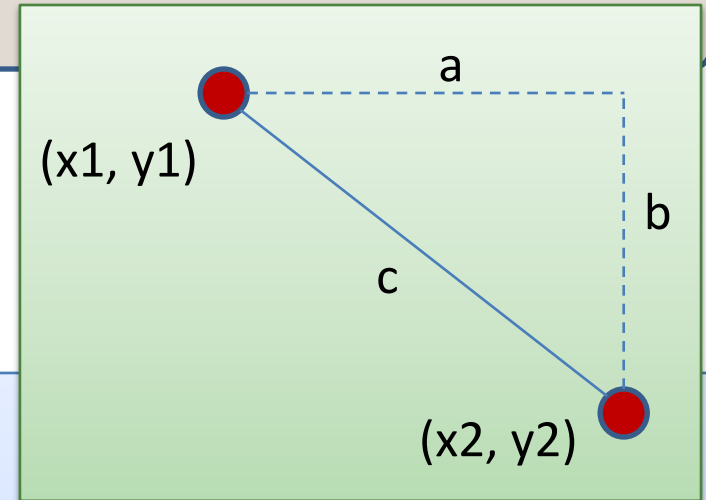
```
  else
```

```
    ???
```

avoid divide by zero

what can we return?

Slope between two points

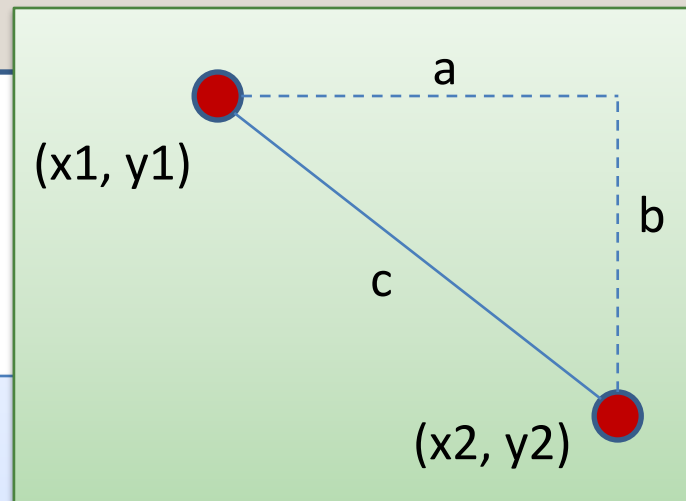


```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =  
  let (x1,y1) = p1 in  
  let (x2,y2) = p2 in  
  let xd = x2 -. x1 in  
  if xd != 0.0 then  
    ???  
  else  
    ???
```

we need an option
type as the result type

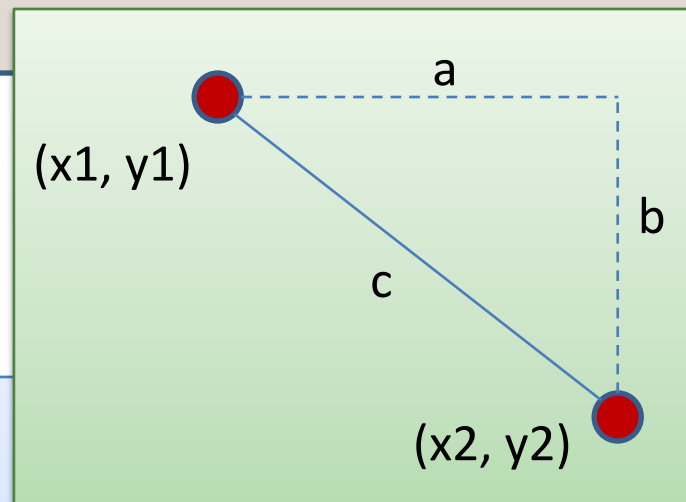
Slope between two points



```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =  
  let (x1,y1) = p1 in  
  let (x2,y2) = p2 in  
  let xd = x2 -. x1 in  
  if xd != 0.0 then  
    Some ((y2 -. y1) /. xd)  
  else  
    None
```

Slope between two points



```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =
```

```
  let (x1,y1) = p1 in
```

```
  let (x2,y2) = p2 in
```

```
  let xd = x2 -. x1 in
```

```
  if xd != 0.0 then
```

```
    (y2 -. y1) /. xd
```

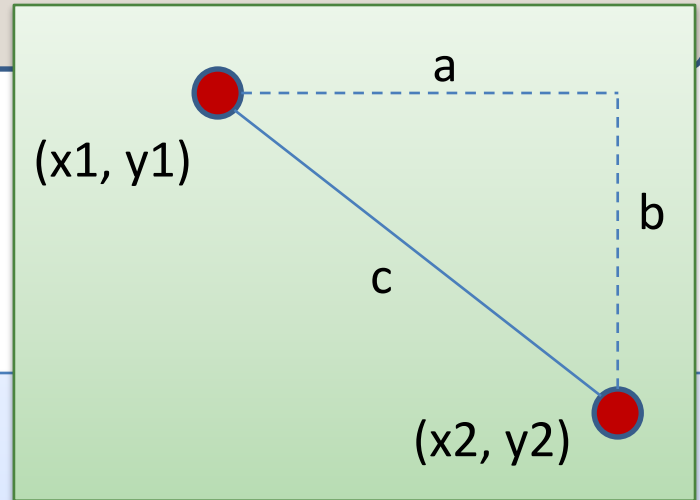
```
  else
```

```
    None
```

Has type **float**

Can have type **float option**

Slope between two points



```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =
```

```
  let (x1,y1) = p1 in
```

```
  let (x2,y2) = p2 in
```

```
  let xd = x2 -. x1 in
```

```
  if xd != 0.0 then
```

```
    (y2 -. y1) /. xd
```

```
  else
```

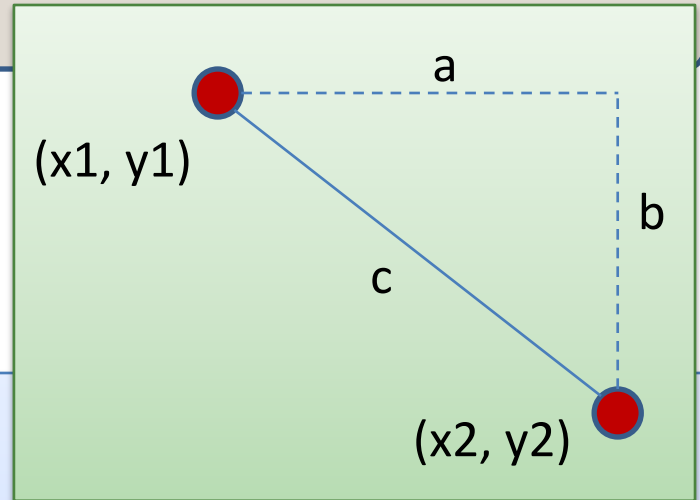
```
    None
```

Has type **float**

Can have type **float option**

WRONG: Type mismatch

Slope between two points



```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =
```

```
  let (x1,y1) = p1 in
```

```
  let (x2,y2) = p2 in
```

```
  let xd = x2 -. x1 in
```

```
  if xd != 0.0 then
```

```
    (y2 -. y1) /. xd
```

```
  else
```

```
    None
```

Has type **float**

doubly WRONG:
result does not
match declared result

Remember the typing rule for if

```
if e1 : bool
and e2 : t and e3 : t (for some type t)
then if e1 then e2 else e3 : t
```

Returning an optional value from an if statement:

```
if ... then
  None           : t option
else
  Some ( ... )   : t option
```


How do we use an option?

```
slope : point -> point -> float option
```

returns a float option



How do we use an option?

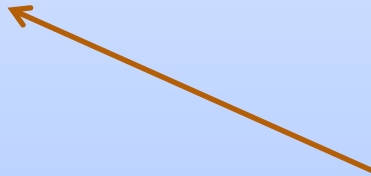
```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =
```

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
    slope p1 p2
```



returns a float option;
to print we must discover if it is
None or Some

How do we use an option?

```
slope : point -> point -> float option
```

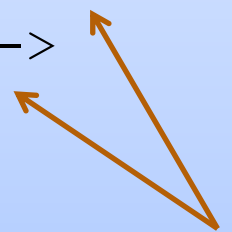
```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with
```

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with  
  | Some s ->  
  | None ->
```

There are two possibilities



Vertical bar separates possibilities



How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with  
  | Some s ->  
  | None ->
```

The "Some s" pattern includes the variable s



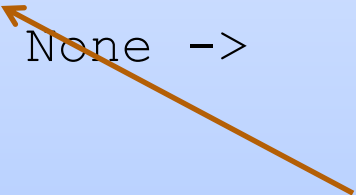
The object between | and -> is called a pattern



How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with  
  | Some s ->  
  | None ->
```



You can put a “|” on the first line if you want. It is generally considered better style to do so. When I learned OCaml, that wasn’t an option so I forget to do it a lot...

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with  
  | Some s ->  
    print_string ("Slope: " ^ string_of_float s)  
  | None ->  
    print_string "Vertical line.\n"
```


Writing Functions Over Typed Data

- Steps to writing functions over typed data:
 1. Write down the function and argument names
 2. Write down argument and result types
 3. Write down some examples (in a comment)
 4. **Deconstruct** input data structures
 5. **Build** new output values
 6. Clean up by identifying repeated patterns
- For option types:

when the **input** has type **t option**,
deconstruct with:

```
match ... with
| None -> ...
| Some s -> ...
```

when the **output** has type **t option**,
construct with:

Some (...)

None

MORE PATTERN MATCHING

Recall the Distance Function


```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```



(x_2, y_2) is an example of a pattern – a pattern for tuples.


So let declarations can contain patterns just like match statements

The difference is that a match allows you to consider multiple different data shapes

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match p1 with
  | (x1,y1) ->
    let (x2,y2) = p2 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```




There is only 1 possibility when matching a pair

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match p1 with
  | (x1,y1) ->
    match p2 with
    | (x2,y2) ->
      sqrt (square (x2 -. x1) +. square (y2 -. y1))
```



We can nest one match expression inside another.

(We can nest any expression inside any other, if the expressions have the right types)

Better Style: Complex Patterns

we built a pair of pairs

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match (p1, p2) with
  | ((x1, y1), (x2, y2)) ->
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Pattern for a pair of pairs: **((variable, variable), (variable, variable))**

All the variable names in the pattern must be different.

Better Style: Complex Patterns

we built a pair of pairs

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match (p1, p2) with
  | (p3, p4) ->
    let (x1, y1) = p3 in
    let (x2, y2) = p4 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

A pattern must be **consistent with** the type of the expression
in between **match ... with**
We use (p3, p4) here instead of ((x1, y1), (x2, y2))

Pattern-matching in function parameters

```
type point = float * float

let distance ((x1,y1):point) ((x2,y2):point) : float =
  let square x = x *. x in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Function parameters are patterns too!

What's the best style?

```
let distance (p1:point) (p2:point) : float =  
  let square x = x *. x in  
  let (x1,y1) = p1 in  
  let (x2,y2) = p2 in  
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

```
let distance ((x1,y1):point) ((x2,y2):point) : float =  
  let square x = x *. x in  
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Either of these is reasonably clear and compact.

Code with unnecessary nested matches/lets is particularly ugly to read.

You'll be judged on code style in this class.

What's the best style?

```
let distance (x1,y1) (x2,y2) =  
  let square x = x *. x in  
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

This is how I'd do it ... the types for tuples + the tuple patterns are a little ugly/verbose ... but for now in class, use the explicit type annotations. We will loosen things up later in the semester.

Combining patterns

```
type point = float * float
```

```
(* returns a nearby point in the graph if one exists *)  
nearby : graph -> point -> point option
```

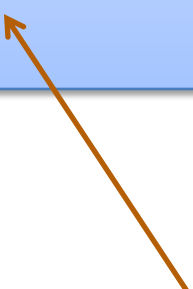
```
let printer (g:graph) (p:point) : unit =  
  match nearby g p with  
  | None -> print_string "could not find one\n"  
  | Some (x,y) ->  
    print_float x;  
    print_string ", ";  
    print_float y;  
    print_newline();
```

Other Patterns

Constant values can be used as patterns

```
let small_prime (n:int) : bool =  
  match n with  
  | 2 -> true  
  | 3 -> true  
  | 5 -> true  
  | _ -> false
```

```
let iffy (b:bool) : int =  
  match b with  
  | true -> 0  
  | false -> 1
```



the underscore pattern
matches anything
it is the "don't care" pattern

INDUCTIVE THINKING

Inductive Programming and Proving

An *inductive data type* T is a data type defined by:

- a collection of base cases
 - that don't refer to T
- a collection of inductive cases that build new data of type T from pre-existing data of type T
 - the pre-existing data is guaranteed to be *smaller* than the new values

Programming principle:

- solve programming problem for base cases
- solve programming problem for inductive cases *by calling the function recursively on smaller data and assuming your function already works correctly on those smaller data values*

Proving principle:

- prove program satisfies property P for base cases
- prove inductive cases satisfy property P *by assuming inductive calls on smaller data values satisfy property P*

LISTS: AN INDUCTIVE DATA TYPE

Lists are Inductive Data

In OCaml, a list value is:

- `[]` (the empty list)
- `v :: vs` (a value `v` followed by a shorter list of values `vs`)

Inductive
Case

Base Case

Lists are Inductive Data

In OCaml, a list value is:

- `[]` (the empty list)
- `v :: vs` (a value `v` followed by a shorter list of values `vs`)

An example:

- `2 :: 3 :: 5 :: []` has type `int list`
- is the same as: `2 :: (3 :: (5 :: []))`
- `::` is called "cons"

An alternative syntax ("syntactic sugar" for lists):

- `[2; 3; 5]`
- But this is just a shorthand for `2 :: 3 :: 5 :: []`. If you ever get confused fall back on the 2 basic *constructors*: `::` and `[]`

Typing Lists

Typing rules for lists:

- (1) `[]` may have any list type `t list`
- (2) if `e1 : t` and `e2 : t list`
then `(e1 :: e2) : t list`

Typing Lists

Typing rules for lists:

- (1) $[]$ may have any list type t list
- (2) if $e1 : t$ and $e2 : t$ list
then $(e1 :: e2) : t$ list

More examples:

$(1 + 2) :: (3 + 4) :: []$: ??

$(2 :: []) :: (5 :: 6 :: []) :: []$: ??

$[[2]; [5; 6]]$: ??

Typing Lists

Typing rules for lists:

- (1) $[]$ may have any list type $t \text{ list}$
- (2) if $e1 : t$ and $e2 : t \text{ list}$
then $(e1 :: e2) : t \text{ list}$

More examples:

$(1 + 2) :: (3 + 4) :: []$: int list

$(2 :: []) :: (5 :: 6 :: []) :: []$: int list list

$[[2]; [5; 6]]$: int list list

(Remember that the 3rd example is an abbreviation for the 2nd)

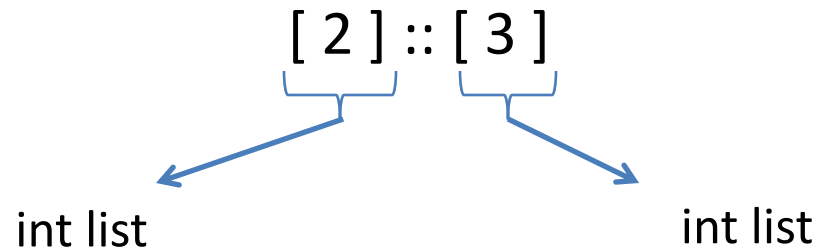
Another Example

What type does this have?

[2] :: [3]

Another Example

What type does this have?



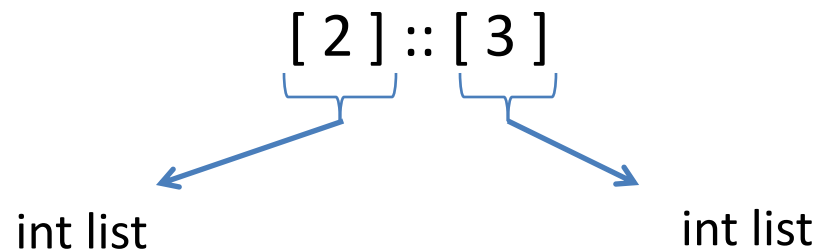
```
# [2] :: [3];;
```

```
Error: This expression has type int but an  
       expression was expected of type  
       int list
```

```
#
```

Another Example

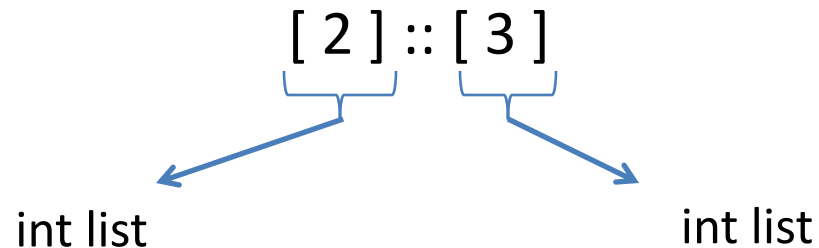
What type does this have?



Give me a simple fix that makes the expression type check?

Another Example

What type does this have?



Give me a simple fix that makes the expression type check?

Either: $2 :: [3]$: int list

Or: $[2] :: [[3]]$: int list list

Analyzing Lists

Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

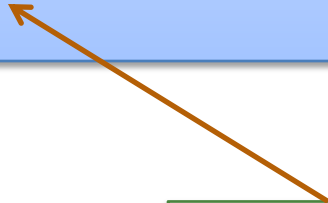
```
(* return Some v, if v is the first list element;  
   return None, if the list is empty *)  
  
let head (xs : int list) : int option =
```

Analyzing Lists

Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

```
(* return Some v, if v is the first list element;  
   return None, if the list is empty *)
```

```
let head (xs : int list) : int option =  
  match xs with  
  | [] ->  
  | hd :: _ ->
```



we don't care about the contents of the tail of the list so we use the underscore

Analyzing Lists

Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

```
(* return Some v, if v is the first list element;  
   return None, if the list is empty *)  
  
let head (xs : int list) : int option =  
  match xs with  
  | [] -> None  
  | hd :: _ -> Some hd
```

This function isn't recursive -- we only extracted a small, fixed amount of information from the list -- the first element

A more interesting example

```
(* Given a list of pairs of integers,  
   produce the list of products of the pairs
```

```
prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
```

```
*)
```

A more interesting example

```
(* Given a list of pairs of integers,  
   produce the list of products of the pairs
```

```
   prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
```

```
*)
```

```
let rec prods (xs : (int * int) list) : int list =
```

A more interesting example

```
(* Given a list of pairs of integers,  
    produce the list of products of the pairs
```

```
    prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
```

```
*)
```

```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
  | [] ->  
  | (x,y) :: tl ->
```

A more interesting example

```
(* Given a list of pairs of integers,  
   produce the list of products of the pairs
```

```
   prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
```

```
*)
```

```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
  | [] -> []  
  | (x,y) :: tl ->
```



A more interesting example

```
(* Given a list of pairs of integers,  
    produce the list of products of the pairs
```

```
    prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
```

```
*)
```

```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
  | [] -> []  
  | (x,y) :: tl -> ?? :: ??
```



the result type is int list, so we can speculate
that we should create a list

A more interesting example

```
(* Given a list of pairs of integers,  
   produce the list of products of the pairs
```

```
   prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
```

```
*)
```

```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
  | [] -> []  
  | (x,y) :: tl -> (x * y) :: ??
```



the first element is the product


A more interesting example

```
(* Given a list of pairs of integers,  
   produce the list of products of the pairs
```

```
   prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
```

```
*)
```

```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
  | [] -> []  
  | (x,y) :: tl -> (x * y) :: ??
```



to complete the job, we must compute
the products for the rest of the list

A more interesting example

```
(* Given a list of pairs of integers,  
    produce the list of products of the pairs
```

```
    prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
```

```
*)
```

```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
  | [] -> []  
  | (x,y) :: tl -> (x * y) :: prods tl
```

Three Parts to Constructing a Function

(1) Think about how to *break down* the input into cases:

```
let rec prods (xs :
  match xs with
  | [] -> ...
  | (x, y) :: tl ...
```

This assumption is called the *Induction Hypothesis*. You'll use it to prove your program correct.

(2) *Assume* the recursive call on smaller data is correct.

(3) Use the result of the recursive call to *build* correct answer.

```
let rec prods (xs : (int*int) list) : int list =
  ...
  | (x, y) :: tl -> ... prods tl ...
```

Another example: zip

(* Given two lists of integers,
return None if the lists are different lengths
otherwise stitch the lists together to create
Some of a list of pairs

```
zip [2; 3] [4; 5] == Some [(2,4); (3,5)]
```

```
zip [5; 3] [4] == None
```

```
zip [4; 5; 6] [8; 9; 10; 11; 12] == None
```

*)

(Give it a try.)

Another example: zip

```
let rec zip (xs : int list) (ys : int list)  
  : (int * int) list option =
```

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

  match (xs, ys) with
```


Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

  match (xs, ys) with
  | ([], []) ->
  | ([], y::ys') ->
  | (x::xs', []) ->
  | (x::xs', y::ys') ->
```

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y::ys') ->
  | (x::xs', []) ->
  | (x::xs', y::ys') ->
```

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y::ys') -> None
  | (x::xs', []) -> None
  | (x::xs', y::ys') ->
```

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

match (xs, ys) with
| ([], []) -> Some []
| ([], y::ys') -> None
| (x::xs', []) -> None
| (x::xs', y::ys') -> (x, y) :: zip xs' ys'
```




is this ok?

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

match (xs, ys) with
| ([], []) -> Some []
| ([], y::ys') -> None
| (x::xs', []) -> None
| (x::xs', y::ys') -> (x, y) :: zip xs' ys'
```

No! zip returns a list option, not a list!
We need to match it and decide if it is Some or None.



Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

match (xs, ys) with
| ([], []) -> Some []
| ([], y::ys') -> None
| (x::xs', []) -> None
| (x::xs', y::ys') ->
  (match zip xs' ys' with
   None -> None
  | Some zs -> (x, y) :: zs)
```



Is this ok?

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

match (xs, ys) with
| ([], []) -> Some []
| ([], y::ys') -> None
| (x::xs', []) -> None
| (x::xs', y::ys') ->
  (match zip xs' ys' with
   None -> None
  | Some zs -> Some ((x,y) :: zs))
```

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

  match (xs, ys) with
  | ([], []) -> Some []
  | (x::xs', y::ys') ->
      (match zip xs' ys' with
       None -> None
       | Some zs -> Some ((x,y) :: zs))
  | (_, _) -> None
```



Clean up.

Reorganize the cases.

Pattern matching proceeds in order.

A bad list example

```
let rec sum (xs : int list) : int =  
  match xs with  
  | hd::tl -> hd + sum tl
```

A bad list example

```
let rec sum (xs : int list) : int =  
  match xs with  
  | hd::tl -> hd + sum tl
```

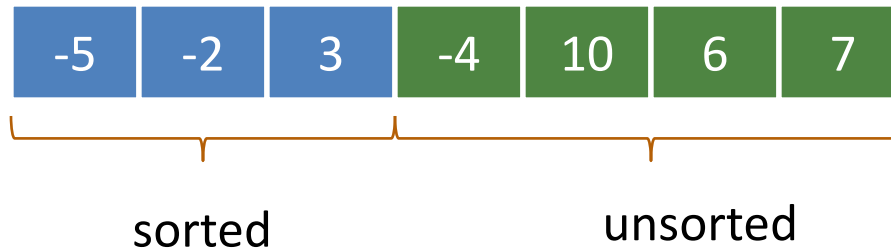
```
# Characters 39-78:  
..match xs with  
  hd :: tl -> hd + sum tl..  
Warning 8: this pattern-matching is not exhaustive.  
Here is an example of a value that is not matched: []  
val sum : int list -> int = <fun>
```

INSERTION SORT

Recall Insertion Sort

At any point during the insertion sort:

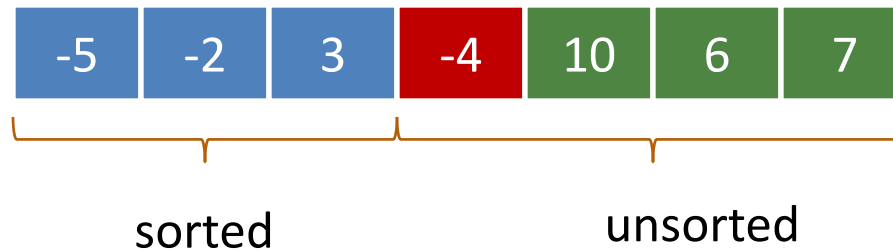
- some initial segment of the array will be sorted
- the rest of the array will be in the same (unsorted) order as it was originally



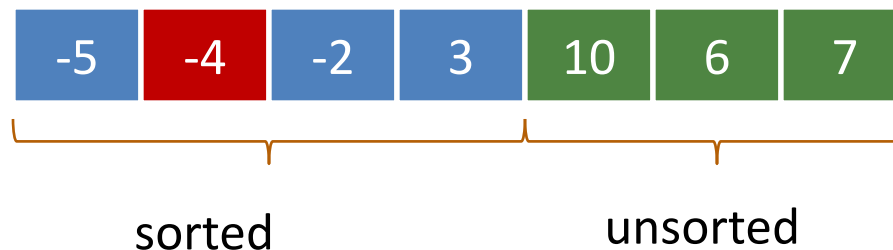
Recall Insertion Sort

At any point during the insertion sort:

- some initial segment of the array will be sorted
- the rest of the array will be in the same (unsorted) order as it was originally



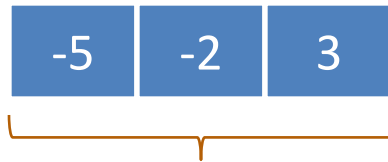
At each step, take the next item in the array and insert it in order into the sorted portion of the list



Insertion Sort With Lists

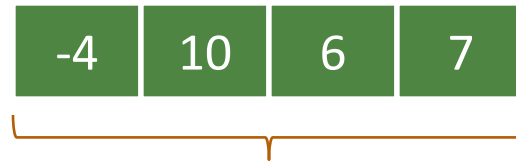
The algorithm is similar, except instead of *one array*, we will maintain *two lists*, a sorted list and an unsorted list

list 1:



sorted

list 2:



unsorted

We'll factor the algorithm:

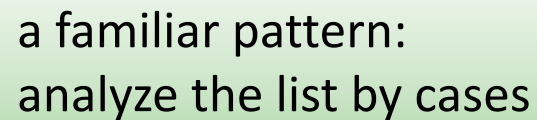
- a function to insert into a sorted list
- a sorting function that repeatedly inserts

Insert

```
(* insert x in to sorted list xs *)  
let rec insert (x : int) (xs : int list) : int list =
```

Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
  | [] ->  
  | hd :: tl ->
```



a familiar pattern:
analyze the list by cases

Insert

```
(* insert x in to sorted list xs *)
```

```
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
  | [] -> [x]  
  | hd :: tl ->
```



insert x into the
empty list

Insert


```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
  | [] -> [x]  
  | hd :: tl ->  
    if hd < x then  
      hd :: insert x tl
```

build a new list with:

- hd at the beginning
- the result of inserting x in to the tail of the list afterwards

Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
  | [] -> [x]  
  | hd :: tl ->  
    if hd < x then  
      hd :: insert x tl  
    else  
      x :: xs
```



put x on the front of the list,
the rest of the list follows

Insertion Sort

```
type il = int list
```

```
insert : int -> il -> il
```

```
(* insertion sort *)
```

```
let rec insert_sort(xs : il) : il =
```

Insertion Sort

```
type il = int list
```

```
insert : int -> il -> il
```

```
(* insertion sort *)
```

```
let rec insert_sort(xs : il) : il =
```

```
    let rec aux (sorted : il) (unsorted : il) : il =
```

```
        in
```

Insertion Sort

```
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =

  let rec aux (sorted : il) (unsorted : il) : il =

    in
    aux [] xs
```

Insertion Sort

```
type il = int list
```

```
insert : int -> il -> il
```

```
(* insertion sort *)
```

```
let rec insert_sort(xs : il) : il =
```

```
  let rec aux (sorted : il) (unsorted : il) : il =
```

```
    match unsorted with
```

```
    | [] ->
```

```
    | hd :: tl ->
```

```
  in
```

```
  aux [] xs
```

Insertion Sort

```
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =

  let rec aux (sorted : il) (unsorted : il) : il =
    match unsorted with
    | [] -> sorted
    | hd :: tl -> aux (insert hd sorted) tl
  in
  aux [] xs
```


A SHORT JAVA RANT

Definition and Use of Java Pairs

```
public class Pair {  
  
    public int x;  
    public int y;  
  
    public Pair (int a, int b) {  
        x = a;  
        y = b;  
    }  
}
```

```
public class User {  
  
    public Pair swap (Pair p1) {  
        Pair p2 =  
            new Pair(p1.y, p1.x);  
  
        return p2;  
    }  
}
```

What could go wrong?

A Paucity of Types

```
public class Pair {  
  
    public int x;  
    public int y;  
  
    public Pair (int a, int b) {  
        x = a;  
        y = b;  
    }  
}
```

```
public class User {  
  
    public Pair swap (Pair p1) {  
        Pair p2 =  
            new Pair(p1.y, p1.x);  
  
        return p2;  
    }  
}
```

The input **p1** to swap may be **null** and we forgot to check.
Java has no way to define a pair data structure that is *just a pair*.
How many students in the class have seen an accidental null pointer exception thrown in their Java code?

From Java Pairs to OCaml Pairs

In OCaml, if a pair may be null it is a pair option:

```
type java_pair = (int * int) option
```

From Java Pairs to OCaml Pairs

In OCaml, if a pair may be null it is a pair option:

```
type java_pair = (int * int) option
```

And if you write code like this:

```
let swap_java_pair (p:java_pair) : java_pair =  
  let (x,y) = p in  
  (y,x)
```

From Java Pairs to OCaml Pairs

In OCaml, if a pair may be null it is a pair option:

```
type java_pair = (int * int) option
```

And if you write code like this:

```
let swap_java_pair (p:java_pair) : java_pair =  
  let (x,y) = p in  
  (y,x)
```

You get a *helpful* error message like this:

```
# ... Characters 91-92:  
  let (x,y) = p in (y,x);;  
                    ^
```

```
Error: This expression has type java_pair = (int * int) option  
      but an expression was expected of type 'a * 'b
```

From Java Pairs to OCaml Pairs

```
type java_pair = (int * int) option
```

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

```
let swap_java_pair (p:java_pair) : java_pair =  
  match p with  
  | Some (x,y) -> Some (y,x)
```

From Java Pairs to OCaml Pairs

```
type java_pair = (int * int) option
```

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

```
let swap_java_pair (p:java_pair) : java_pair =  
  match p with  
  | Some (x,y) -> Some (y,x)
```

OCaml to the rescue!

```
..match p with  
  | Some (x,y) -> Some (y,x)  
Warning 8: this pattern-matching is not exhaustive.  
Here is an example of a value that is not matched:  
None
```


From Java Pairs to OCaml Pairs

```
type java_pair = (int * int) option
```

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

```
let swap_java_pair (p:java_pair) : java_pair =  
  match p with  
  | Some (x,y) -> Some (y,x)
```



An easy fix!



```
let swap_java_pair (p:java_pair) : java_pair =  
  match p with  
  | None -> None  
  | Some (x,y) -> Some (y,x)
```

From Java Pairs to OCaml Pairs

Moreover, your pairs are probably almost never null!

Defensive programming & always checking for null is
AnNOyinG

From Java Pairs to OCaml Pairs

There just isn't always some "good thing" for a function to do when it receives a bad input, like a null pointer

In OCaml, all these issues disappear when you use the proper type for a pair and that type contains no "extra junk"

```
type pair = int * int
```

Once you know OCaml, it is *hard* to write swap incorrectly

Your *bullet-proof* code is much simpler than in Java.

```
let swap (p:pair) : pair =  
  let (x,y) = p in (y,x)
```

Summary of Java Pair Rant

Java has a paucity of types

- There is no type to describe just the pairs
- There is no type to describe just the triples
- There is no type to describe the pairs of pairs
- There is no type ...

OCaml has many more types

- use option when things may be null
- do not use option when things are not null
- OCaml types describe data structures more precisely
 - programmers have fewer cases to worry about
 - entire classes of errors just go away
 - type checking and pattern analysis help prevent programmers from ever forgetting about a case

Summary of Java Pair Rant

Java has a paucity of types

- There is no type to describe just the pairs
- There is no type to describe nested types
- There is no type to describe lists
- There is no type to describe arrays

OCaml

- use `pair`
- do

SCORE: OCAML 1, JAVA 0

- `pair`
- type checking and pattern analysis help prevent programmers from ever forgetting about a case

Example problems to practice

- Write a function to sum the elements of a list
 - `sum [1; 2; 3] ==> 6`
- Write a function to append two lists
 - `append [1;2;3] [4;5;6] ==> [1;2;3;4;5;6]`
- Write a function to reverse a list
 - `rev [1;2;3] ==> [3;2;1]`
- Write a function to turn a list of pairs into a pair of lists
 - `split [(1,2); (3,4); (5,6)] ==> ([1;3;5], [2;4;6])`
- Write a function that returns all prefixes of a list
 - `prefixes [1;2;3] ==> [[]; [1]; [1;2]; [1;2;3]]`
- suffixes...