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5.5 DATA COMPRESSION

- *introduction*
- *run-length coding*
- *Huffman compression*
- *LZW compression*

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- *introduction*
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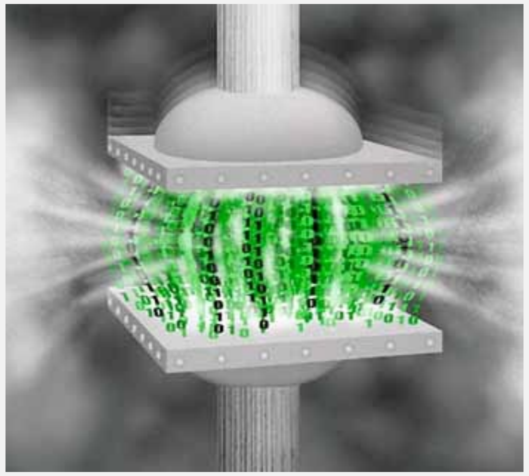
Data compression

Compression reduces the size of a file:

- To save space when storing it.
- To save time when transmitting it.
- Most files have lots of redundancy.



Who needs compression?

- Moore's law: # transistors on a chip doubles every 18–24 months.
- Parkinson's law: data expands to fill space available.
- Text, images, sound, video, …

> " *Everyday, we create 2.5 quintillion bytes of data—so much that 90% of the data in the world today has been created in the last two years alone.* " — *IBM report on big data (2011)*

Basic concepts ancient (1950s), best technology recently developed.

Applications

Generic file compression.

- Files: Gzip, bzip2, 7z.
- Archivers: PKZIP.
- File systems: NTFS, ZFS, HFS+, ReFS, GFS.

Multimedia.

- Images: GIF, JPEG.
- Sound: MP3.
- Video: MPEG, DivX™, HDTV.

Communication.

- ITU-T T4 Group 3 Fax.
- V.42bis modem.
- Skype, Google hangout.

Databases. Google, Facebook, NSA, ...

Lossless compression and expansion

Message. Bitstream B we want to compress.

Compress. Generates a "compressed" representation $C(B)$.

Expand. Reconstructs original bitstream B.

uses fewer bits
(you hope)

Compression ratio. Bits in $C(B)$ / bits in B.

Ex. 50–75% or better compression ratio for natural language.

Food for thought

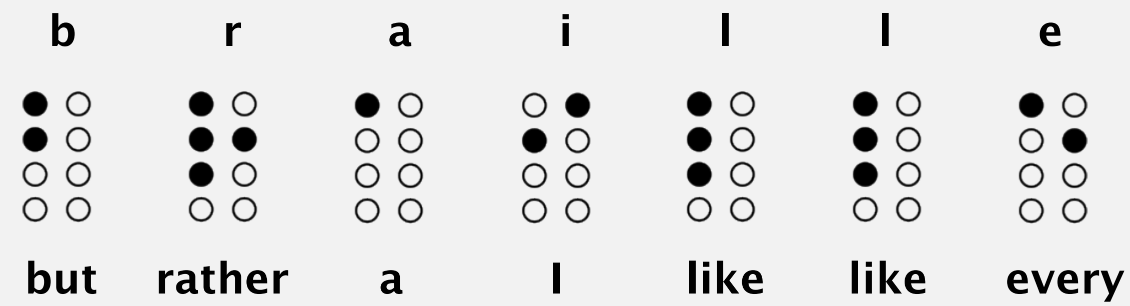
Data compression has been omnipresent since antiquity:

- Number systems.
- Natural languages.
- Mathematical notation.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

has played a central role in communications technology,

- Grade 2 Braille.
- Morse code.
- Telephone system.



and is part of modern life.

- JPEG.
- MP3.
- MPEG.

Q. What role will it play in the future?

Data representation: genomic code

Genome. String over the alphabet `{ A, T, C, G }`.

Goal. Encode an n-character genome: `A T A G A T G C A T A G ...`

Standard ASCII encoding.

- 8 bits per char.
- $8\,n$ bits.

char	hex	binary
'A'	41	01000001
'T'	54	01010100
'C'	43	01000011
'G'	47	01000111

Two-bit encoding.

- 2 bits per char.
- $2\,n$ bits (25% compression ratio).

char	binary
'A'	00
'T'	01
'C'	10
'G'	11

Fixed-length code. k-bit code supports alphabet of size 2^k.

Amazing but true. Some genomic databases in 1990s used ASCII.

Reading and writing binary data

Binary standard input. Read bits from standard input.

`public class BinaryStdIn`

`boolean`	`readBoolean()`	*read 1 bit of data and return as a* `boolean` *value*
`char`	`readChar()`	*read 8 bits of data and return as a* `char` *value*
`char`	`readChar(int r)`	*read* `r` *bits of data and return as a* `char` *value*
[similar methods for `byte` *(8 bits);* `short` *(16 bits);* `int` *(32 bits);* `long` *and* `double` *(64 bits)]*		
`boolean`	`isEmpty()`	*is the bitstream empty?*
`void`	`close()`	*close the bitstream*

Binary standard output. Write bits to standard output

`public class BinaryStdOut`

`void`	`write(boolean b)`	*write the specified bit*
`void`	`write(char c)`	*write the specified 8-bit* `char`
`void`	`write(char c, int r)`	*write the* `r` *least significant bits of the specified* `char`
[similar methods for `byte` *(8 bits);* `short` *(16 bits);* `int` *(32 bits);* `long` *and* `double` *(64 bits)]*		
`void`	`close()`	*close the bitstream*

Writing binary data

Date representation. Three different ways to represent 12/31/1999.

A character stream (StdOut)

```
StdOut.print(month + "/" + day + "/" + year);
```

00110001	00110010	00101111	00110111	00110001	00101111	00110001	00111001	00111001	00111001
1	2	/	3	1	/	1	9	9	9

80 bits

Three ints (BinaryStdOut)

```
BinaryStdOut.write(month);
BinaryStdOut.write(day);
BinaryStdOut.write(year);
```

00000000 00000000 00000000 00001100	00000000 00000000 00000000 00011111	00000000 00000000 00000111 11001111
12	31	1999

96 bits

A 4-bit field, a 5-bit field, and a 12-bit field (BinaryStdOut)

```
BinaryStdOut.write(month, 4);
BinaryStdOut.write(day, 5);
BinaryStdOut.write(year, 12);
```

1100	11111	011111001111	000
12	31	1999	

21 bits (+ 3 bits for byte alignment at close)

Binary dumps

Q. How to examine the contents of a bitstream?

Standard character stream

```
% more abra.txt
ABRACADABRA!
```

Bitstream represented as 0 and 1 characters

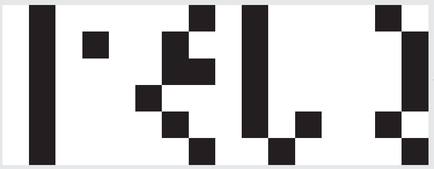
```
% java BinaryDump 16 < abra.txt
0100000101000010
0101001001000001
0100001101000001
0100010001000001
0100001001010010
0100000100100001
96 bits
```

Bitstream represented with hex digits

```
%  java HexDump 4 < abra.txt
41 42 52 41
43 41 44 41
42 52 41 21
12 bytes
```

Bitstream represented as pixels in a `Picture`

```
% java PictureDump 16 6 < abra.txt
```



16-by-6 pixel window, magnified

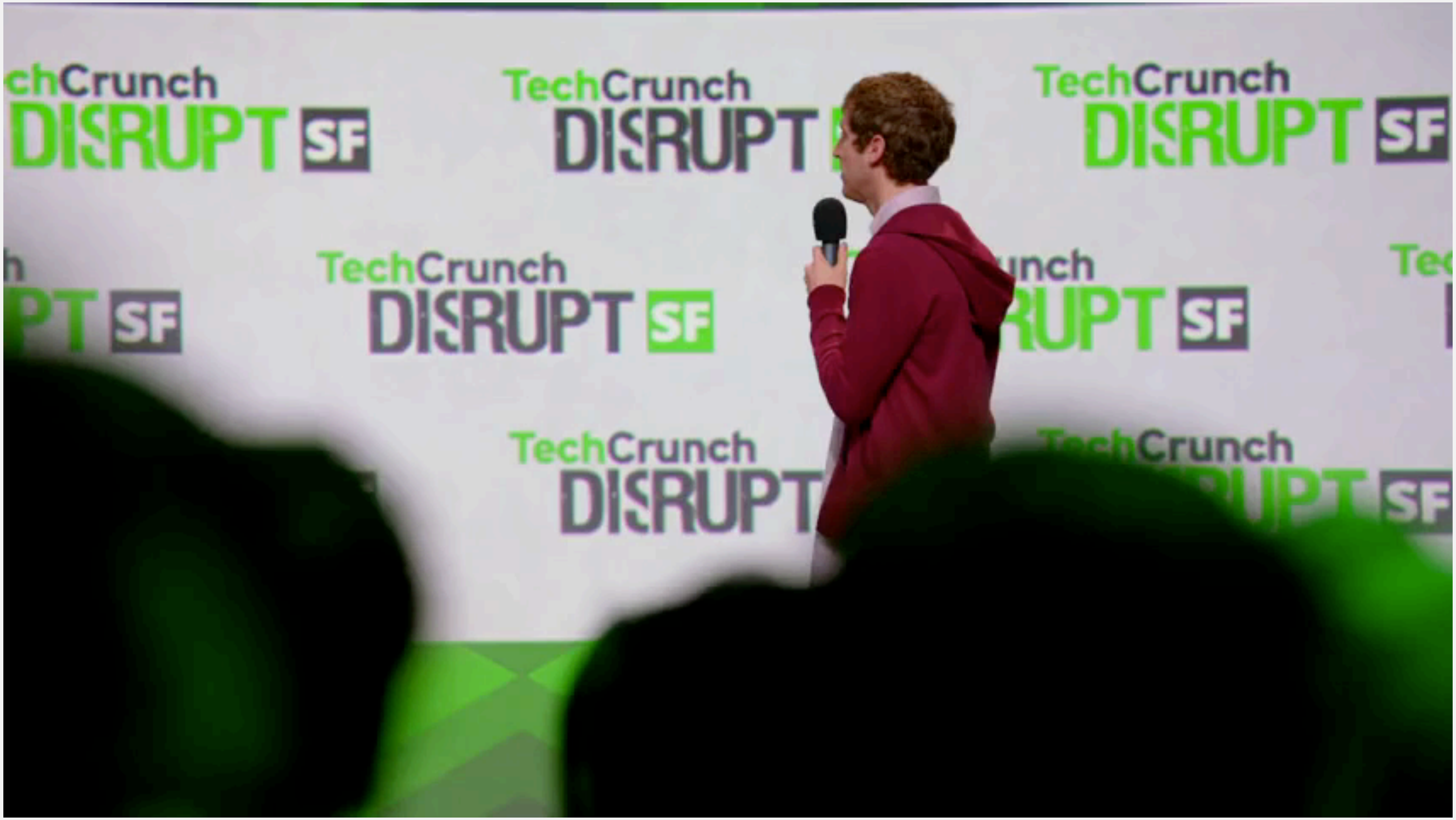
```
96 bits
```

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SP	!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

Hexadecimal-to-ASCII conversion table

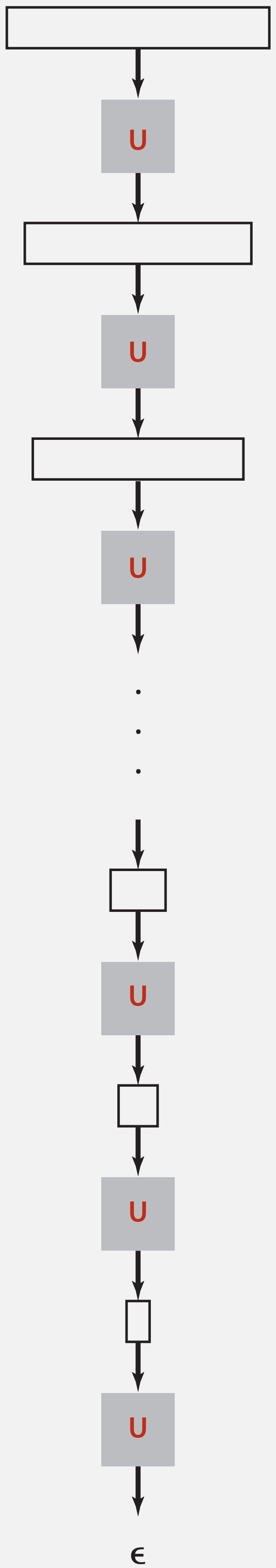
Universal data compression

Pied Piper. Claims 3.8:1 lossless compression of arbitrary data.



Universal data compression

Proposition. No algorithm can compress every bitstring.



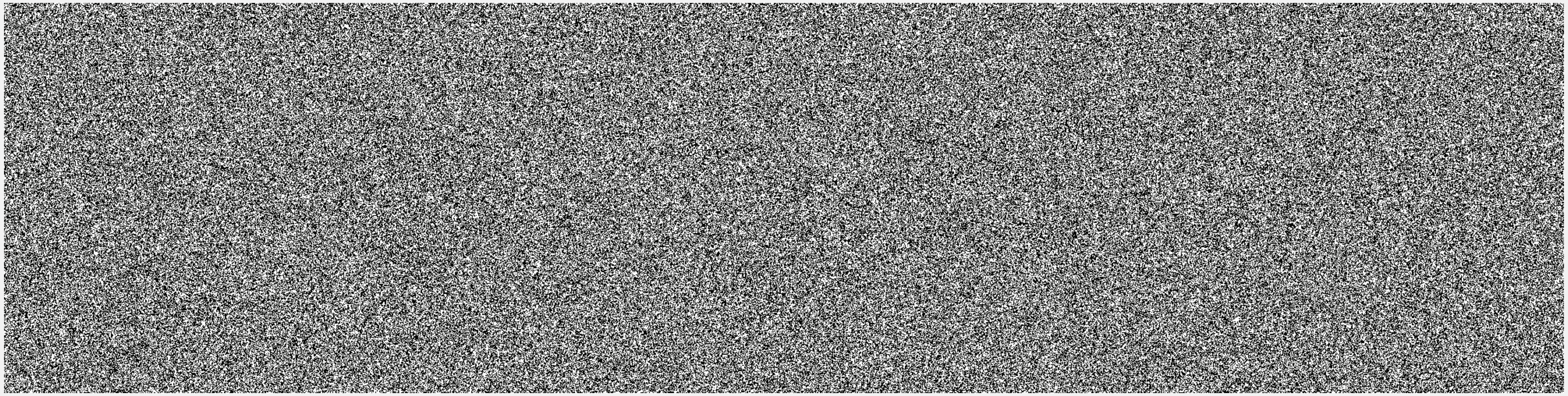
Pf 1. [by contradiction]

Pf 2. [by counting]

- Suppose your algorithm that can compress all $1{,}000$-bit strings.
- 2^{1000} possible bitstrings with $1{,}000$ bits.
- Only $1 + 2 + 4 + \ldots + 2^{998} + 2^{999}$ can be encoded with ≤ 999 bits.
- Similarly, only 1 in 2^{499} bitstrings can be encoded with ≤ 500 bits!

Optimal data compression is undecidable

```
% java RandomBits | java PictureDump 2000 500
```



```
1000000 bits
```

A difficult file to compress: one million (pseudo-) random bits

```
public class RandomBits
{
   public static void main(String[] args)
   {
      int x = 11111;
      for (int i = 0; i < 1000000; i++)
      {
         x = x * 314159 + 218281;
         BinaryStdOut.write(x > 0);
      }
      BinaryStdOut.close();
   }
}
```

Rdenudcany in Enlgsih lnagugae

Q. How much redundancy in the English language?

A. Quite a bit.

> " *... randomising letters in the middle of words [has] little or no effect on the ability of skilled readers to understand the text. This is easy to denmtrasote. In a pubiltacion of New Scnieitst you could ramdinose all the letetrs, keipeng the first two and last two the same, and reibadailty would hadrly be aftcfeed. My ansaylis did not come to much beucase the thoery at the time was for shape and senqeuce retigcionon. Saberi's work sugsegts we may have some pofrweul palrlael prsooscers at work. The resaon for this is suerly that idnetiyfing coentnt by paarllel prseocsing speeds up regnicoiton. We only need the first and last two letetrs to spot chganes in meniang.* " — Graham Rawlinson

The gaol of data cmperisoson is to inetdify rdenudcany and epxloit it.

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Run-length encoding (RLE)

Simple type of redundancy in a bitstream. Long runs of repeated bits.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 ← 40 bits

$\underbrace{1\,1\,1\,1\,1\,1\,1}_{\text{run of length 7}}$

Representation. 4-bit counts to represent alternating runs of 0s and 1s: 15 0s, then 7 1s, then 7 0s, then 11 1s.

$\underbrace{1\,1\,1\,1}_{15}\;\underbrace{0\,1\,1\,1}_{7}\;\underbrace{0\,1\,1\,1}_{7}\;\underbrace{1\,0\,1\,1}_{11}$ ← 16 bits (instead of 40)

Q. How many bits to store the counts?

A. Typically 8 bits (but 4 on this slide for brevity).

Q. What to do when run length exceeds max count?

A. Intersperse runs of length 0.

Applications. JPEG, ITU-T T4 Group 3 Fax, ...

Run-length encoding: Java implementation

```
public class RunLength
{
   private static final int R   = 256;
   private static final int lgR = 8;

   public static void compress()
   {  /* see textbook */  }

   public static void expand()
   {
      boolean bit = false;
      while (!BinaryStdIn.isEmpty())
      {
         int run = BinaryStdIn.readInt(lgR);
         for (int i = 0; i < run; i++)
            BinaryStdOut.write(bit);
         bit = !bit;
      }
      BinaryStdOut.close();
   }
}
```

maximum run-length count

number of bits per count

initial runs are 0

read 8-bit count from standard input

write run of 0s or 1s to standard output

flip bit (for next run)

pad 0s for byte alignment

Data compression: quiz 1

What is the best compression ratio achievable from run-length encoding when using 8-bit counts?

- **A.** 1 / 256
- **B.** 1 / 16
- **C.** 8 / 255
- **D.** 1 / 8
- **E.** 16 / 255

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- Huffman compression
- LZW compression

David Huffman

Variable-length codes

Key idea. Use different number of bits to encode different characters.

Ex. Morse code: • • • — — — • • •

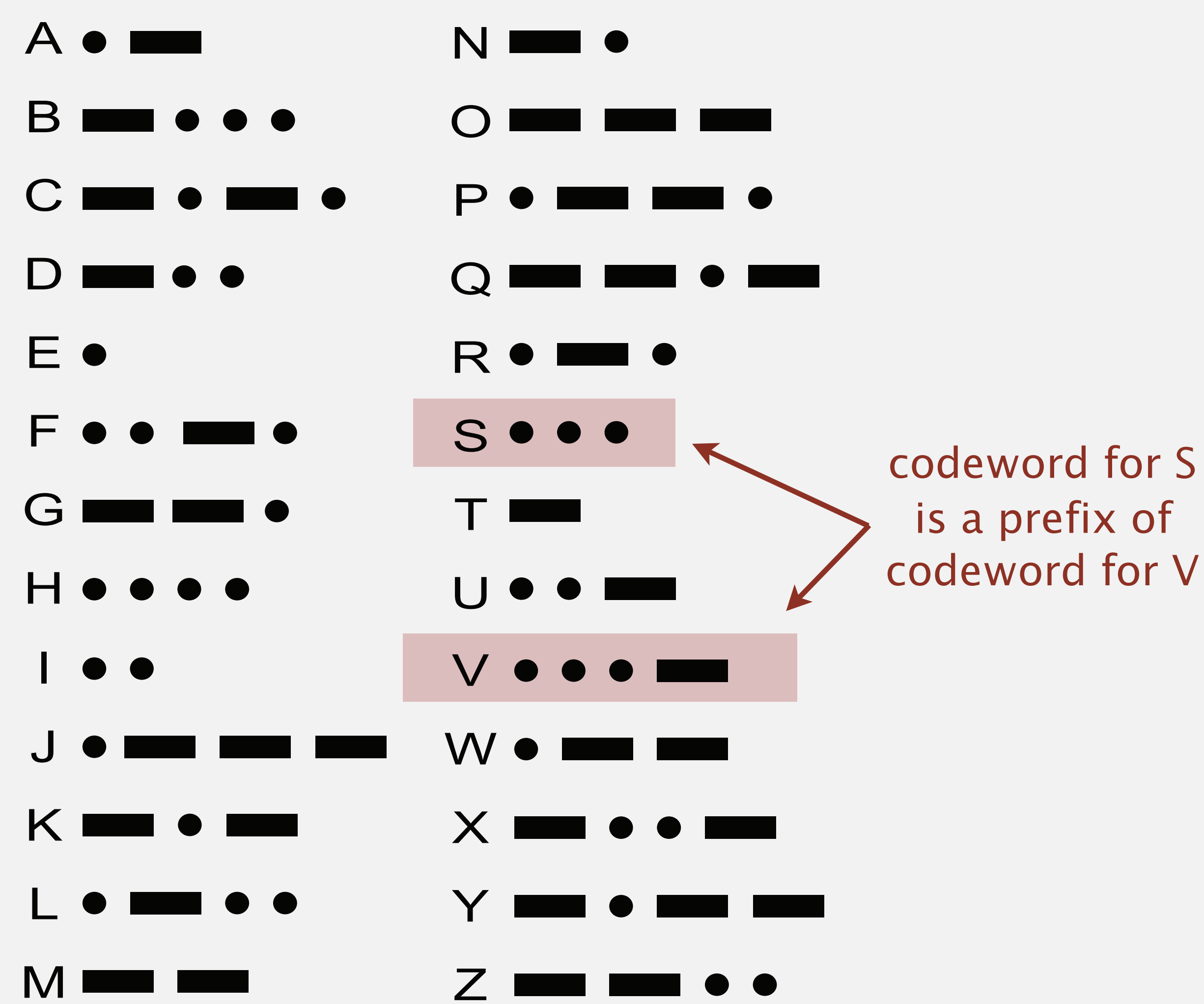
Issue. Ambiguity.

S O S ?

V Z E ?

E E J I E ?

E E W N I ?



In practice. Use a short gap to separate characters.

Variable-length codes

Q. How do we avoid ambiguity?

A. Ensure that no codeword is a prefix of another.

Ex 1. Fixed-length code.

Ex 2. Append special “stop” character to each codeword.

Ex 3. General prefix-free code.

Codeword table

key	*value*
!	101
A	0
B	1111
C	110
D	100
R	1110

Compressed bitstring

011111110011001000111111100101 ← *30 bits*

A B RA CA DA B RA !

Codeword table

key	*value*
!	101
A	11
B	00
C	010
D	100
R	011

Compressed bitstring

11000111101011100110001111101 ← *29 bits*

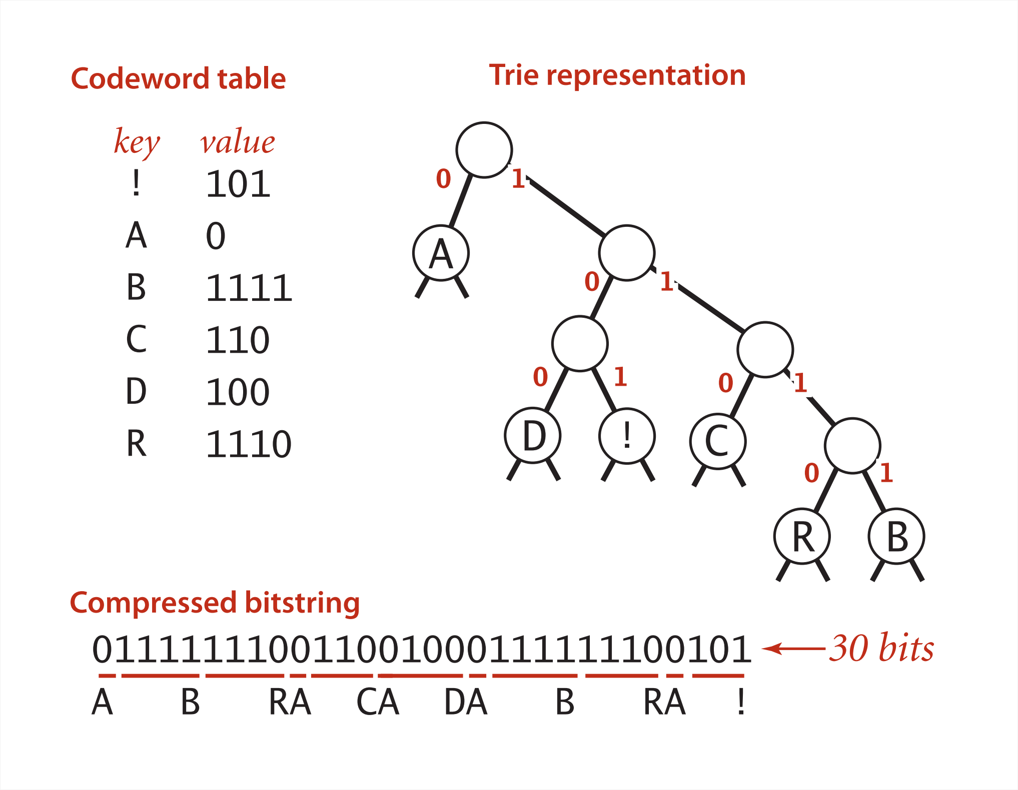
A B R A C A D A B R A !

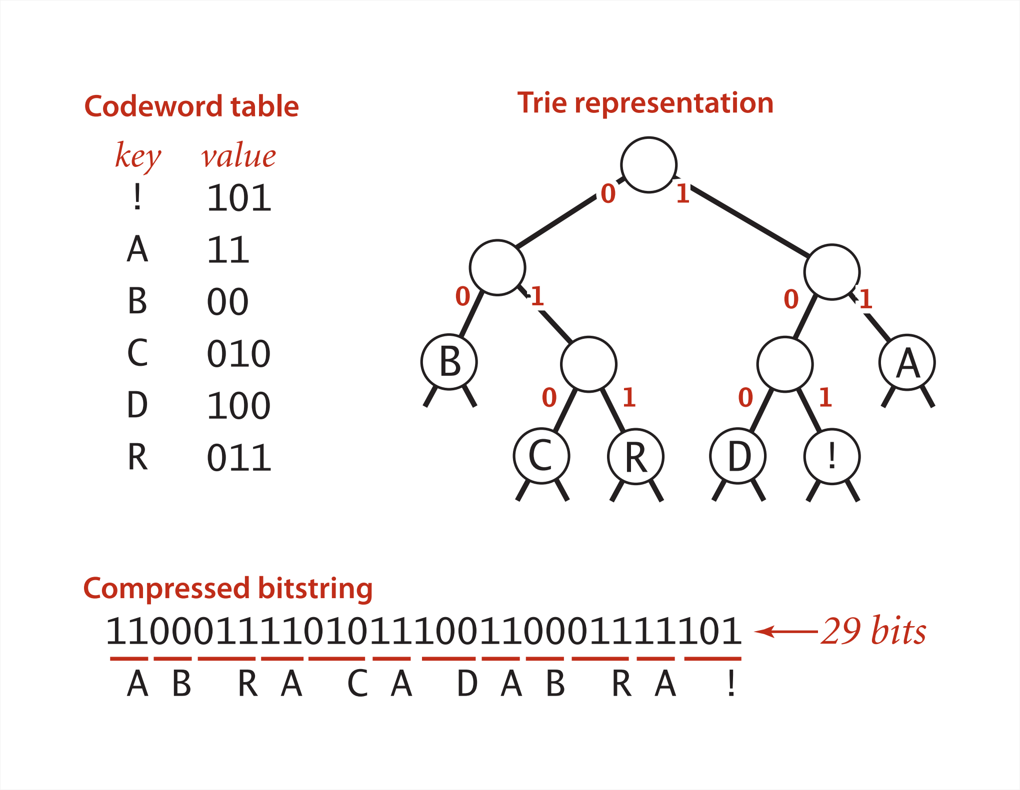
Prefix-free codes: trie representation

Q. How to represent the prefix-free code?

A. A binary trie!

- Characters in leaves.
- Codeword is path from root to leaf.



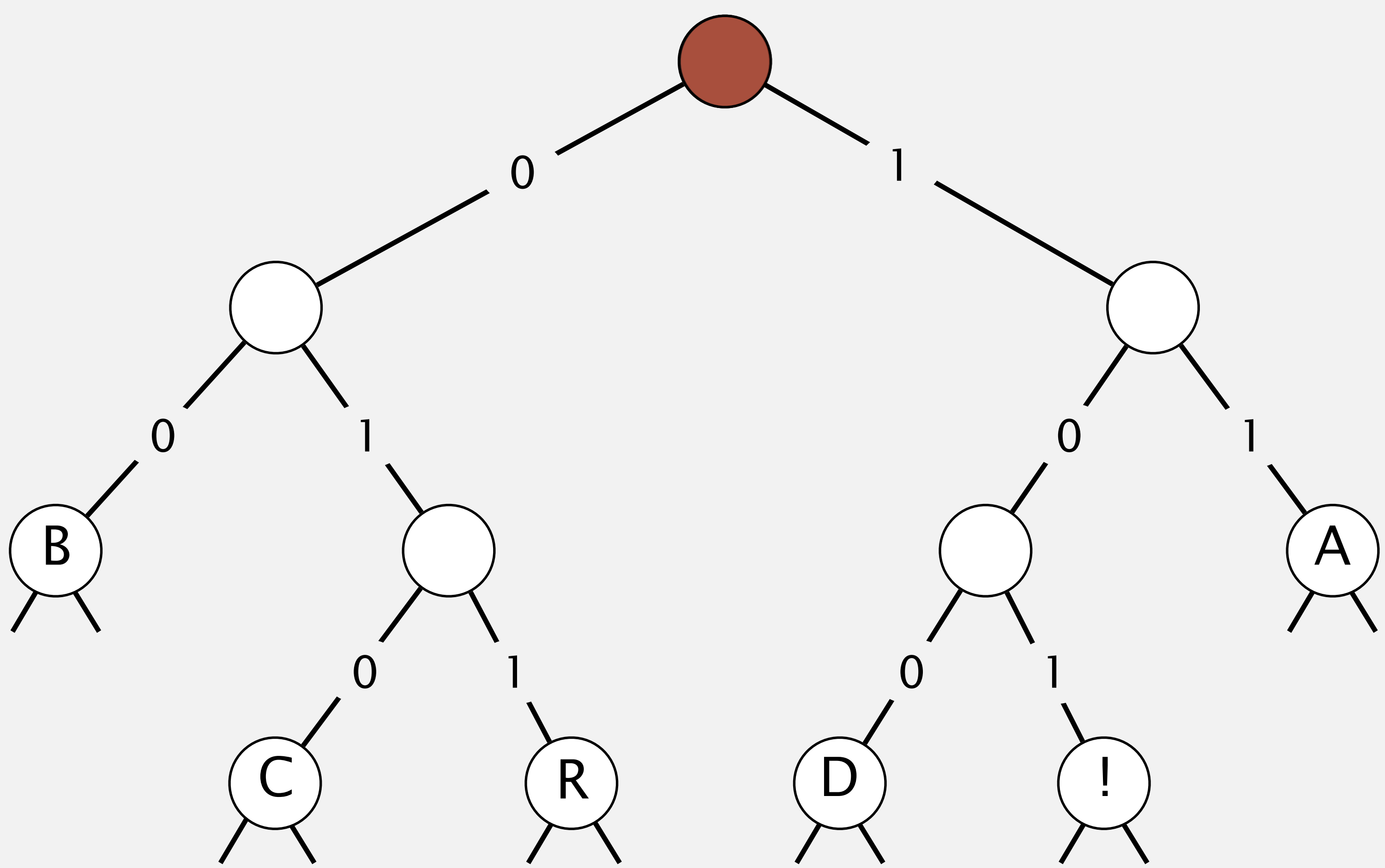


Prefix-free codes: expansion

Expansion.

- Start at root.
- Go left if bit is 0; go right if 1.
- If leaf node, write character; return to root node; repeat.

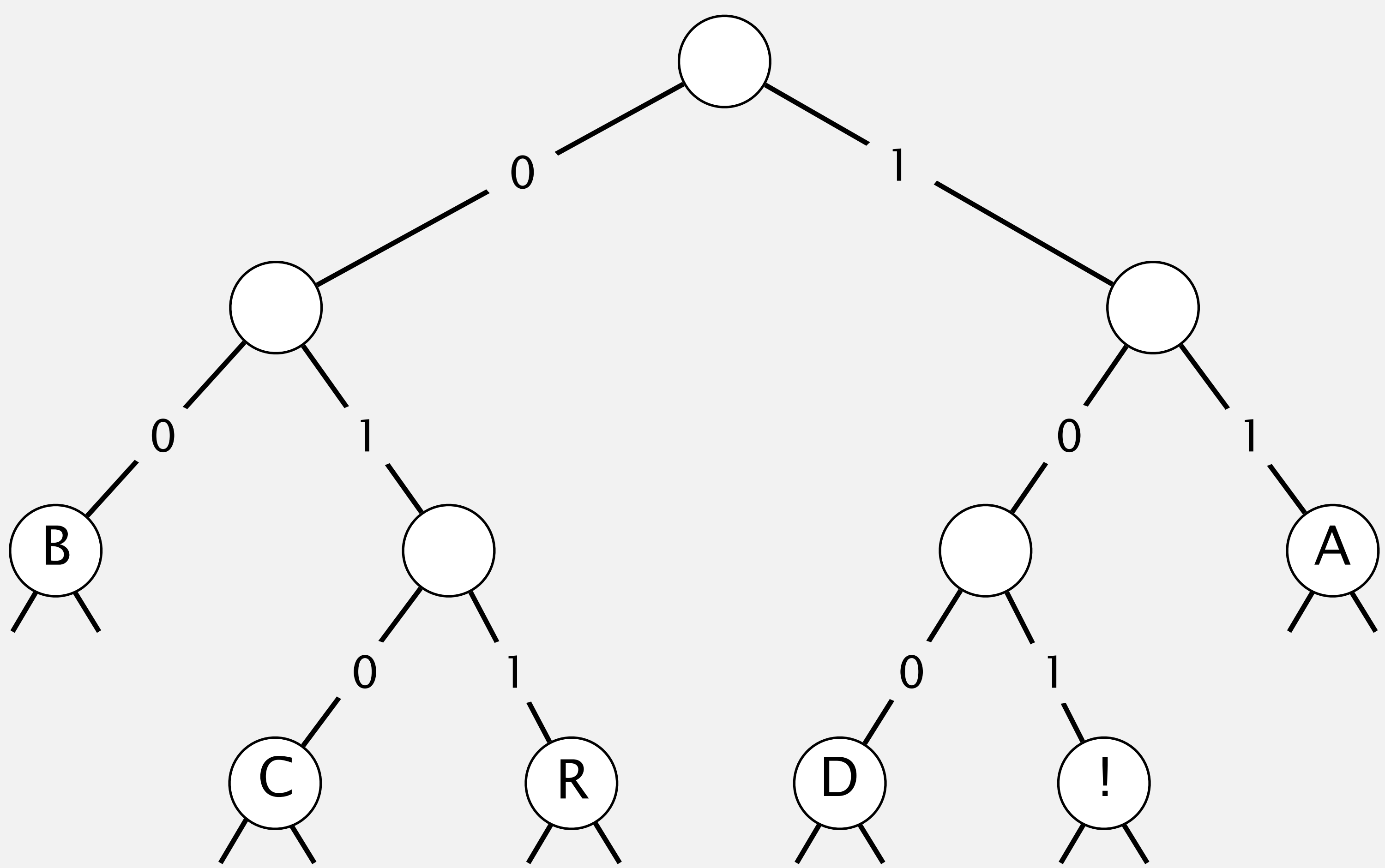
1 1	0 0	0 1 1	1 1	0 1 0	1 1	1 0 0	1 1	0 0	0 1 1	1 1	1 0 1
A	B	R	A	C	A	D	A	B	R	A	!



Prefix-free codes: compression

Compression.

- Method 1: start at leaf; follow path up to the root; print bits in reverse.
- Method 2: create ST of key–value pairs.



Data compression: quiz 2

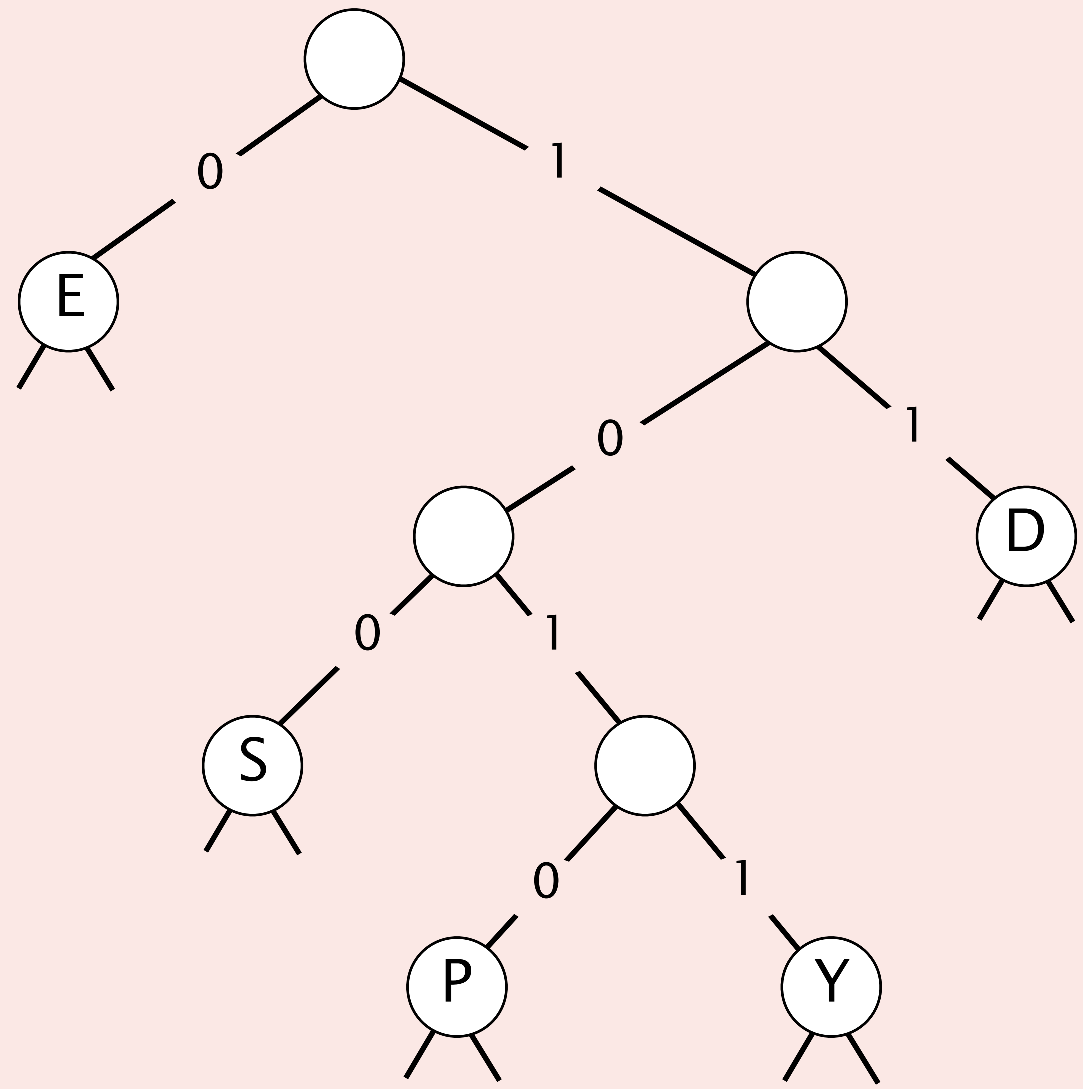
Consider the following trie representation of a prefix-free code.
Expand the compressed bitstring 1 0 0 1 0 1 0 0 0 1 1 1 0 1 1 ?

A. P E E D

B. P E S D E Y

C. S P E D

D. S P E E D Y



Huffman coding overview

Static model. Use the same prefix-free code for all messages.

Dynamic model. Use a custom prefix-free code for each message.

Compression.

- Read message.
- Build best prefix-free code for message. How? [ahead]
- Write prefix-free code.
- Compress message using prefix-free code.

Expansion.

- Read prefix-free code.
- Read compressed message and expand using prefix-free code.

Huffman trie node data type

```
private static class Node implements Comparable<Node>
{
   private final char ch;    // used only for leaf nodes
   private final int freq;   // used only by compress()
   private final Node left, right;

   public Node(char ch, int freq, Node left, Node right)
   {
      this.ch    = ch;
      this.freq  = freq;
      this.left  = left;
      this.right = right;
   }

   public boolean isLeaf()
   {  return left == null && right == null; }

   public int compareTo(Node that)
   {  return this.freq - that.freq;  }

}
```

initializing constructor

is Node a leaf?

compare nodes by frequency
(stay tuned)

Prefix-free codes: expansion

```
public void expand()
{
   Node root = readTrie();                    // read encoding trie
   int n = BinaryStdIn.readInt();             // read number of chars

   for (int i = 0; i < n; i++)                // for each encoded character i
   {
      Node x = root;                          // follow path from root to leaf
      while (!x.isLeaf())                     // to determine character
      {
         if (!BinaryStdIn.readBoolean())
            x = x.left;
         else
            x = x.right;
      }
      BinaryStdOut.write(x.ch, 8);            // write character (8 bits)
   }
   BinaryStdOut.close();
}
```

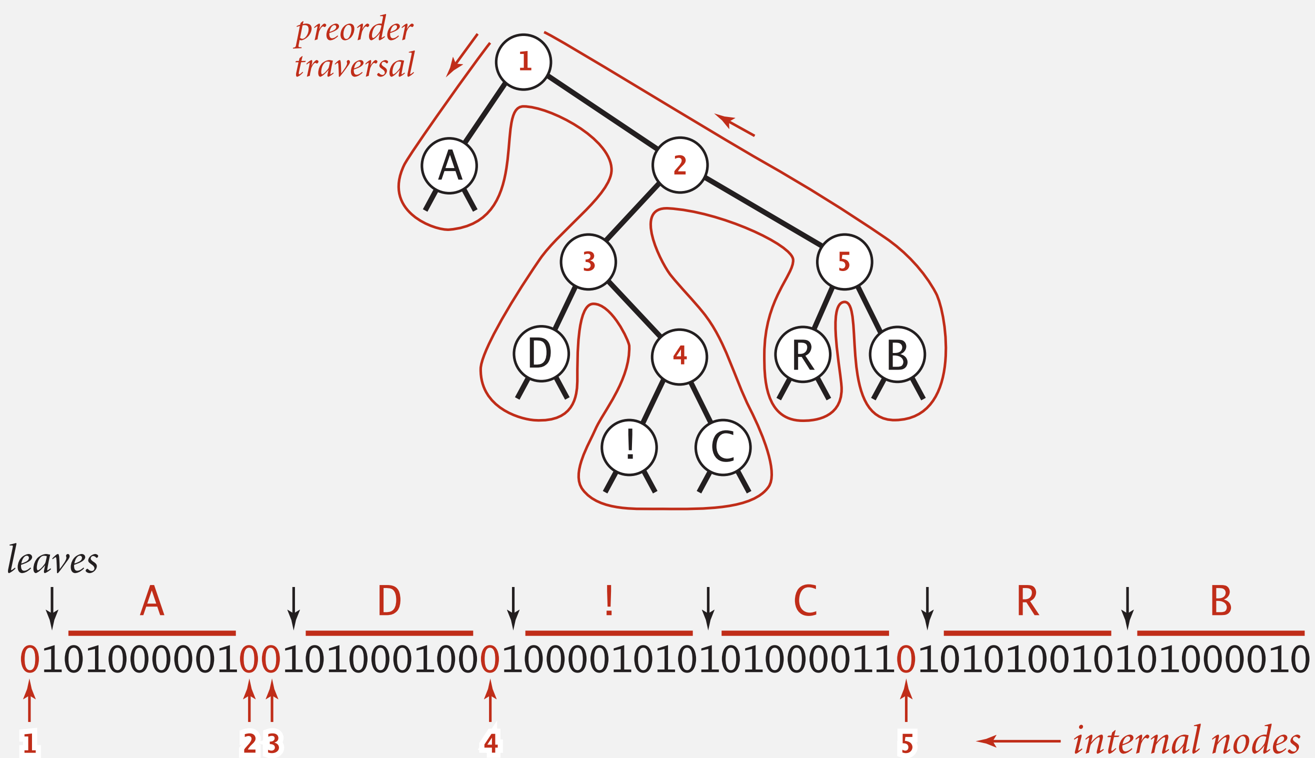
Running time. Linear in input size (number of bits).

Prefix-free codes: how to transmit

Q. How to write the trie?

A. Write preorder traversal; mark leaf nodes and internal nodes with a bit.

0 for internal nodes
1 for leaf nodes



Using preorder traversal to encode a trie as a bitstream

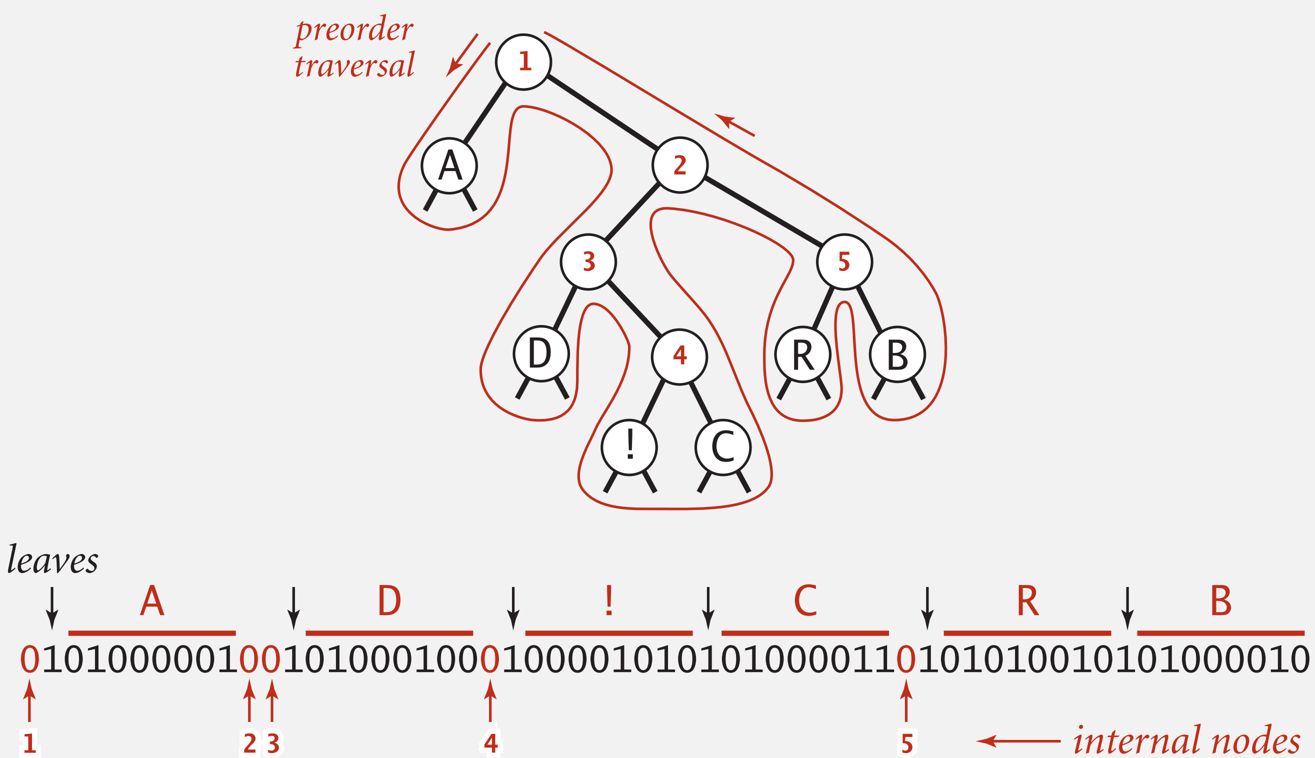
```
private static void writeTrie(Node x)
{
   if (x.isLeaf())
   {
      BinaryStdOut.write(true);
      BinaryStdOut.write(x.ch, 8);
      return;
   }
   BinaryStdOut.write(false);
   writeTrie(x.left);
   writeTrie(x.right);
}
```

Note. If message is long, overhead of transmitting trie is small.

Prefix-free codes: how to transmit

Q. How to read the trie?

A. Reconstruct from preorder traversal.



Using preorder traversal to encode a trie as a bitstream

```
private static Node readTrie()
{
   if (BinaryStdIn.readBoolean())
   {
      char c = BinaryStdIn.readChar(8);
      return new Node(c, 0, null, null);
   }
   Node x = readTrie();
   Node y = readTrie();
   return new Node('\0', 0, x, y);
}
```

arbitrary value
(value not used with internal nodes)

Huffman codes

Q. How to find best prefix-free code?

Huffman algorithm:

- Count frequency `freq[i]` for each char `i` in input.
- Start with one node corresponding to each char `i` (with weight `freq[i]`).
- Repeat until single trie formed:
  - select two tries with min weight `freq[i]` and `freq[j]`
  - merge into single trie with weight `freq[i] + freq[j]`

Applications:

JPEG

PDF Adobe

mp3

DivX MPEG4

gzip PKZIP

Constructing a Huffman encoding trie: Java implementation

```
private static Node buildTrie(int[] freq)
{
   MinPQ<Node> pq = new MinPQ<Node>();
       for (char i = 0; i < R; i++)
          if (freq[i] > 0)
             pq.insert(new Node(i, freq[i], null, null));

   while (pq.size() > 1)
   {
      Node x = pq.delMin();
      Node y = pq.delMin();
      Node parent = new Node('\0', x.freq + y.freq, x, y);
      pq.insert(parent);
   }

   return pq.delMin();
}
```

initialize PQ with singleton tries

merge two smallest tries

not used for internal nodes

total frequency

two subtries

Huffman compression summary

Proposition. Huffman's algorithm produces an optimal prefix-free code.

Pf. See textbook.

no prefix-free code
uses fewer bits

Two-pass implementation (for compression).

- Pass 1: tabulate character frequencies; build trie.
- Pass 2: encode file by traversing trie (or symbol table).

Running time (for compression). Using a binary heap $\Rightarrow$ $n + R \log R$.

Running time (for expansion). Using a binary trie $\Rightarrow$ n.

input size

alphabet size

Q. Can we do better (in terms of compression ratio)? [stay tuned]

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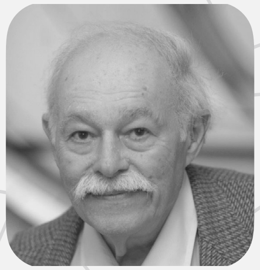
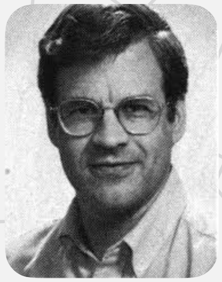

Abraham Lempel


Jacob Ziv


Terry Welch

Statistical methods

Static model. Same model for all texts.

- Fast.
- Not optimal: different texts have different statistical properties.
- Ex: ASCII, Morse code.

Dynamic model. Generate model based on text.

- Preliminary pass needed to generate model.
- Must transmit the model.
- Ex: Huffman code.

Adaptive model. Progressively learn and update model as you read text.

- More accurate modeling produces better compression.
- Decoding must start from beginning.
- Ex: LZW.

LZW compression demo

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A	
matches	A	B	R	A	C	A	D	A B		R A		B R		A B R			A	
value	41	42	52	41	43	41	44	81		83		82		88			41	80

LZW compression for A B R A C A D A B R A B R A B R A

key	value
⋮	⋮
A	41
B	42
C	43
D	44
⋮	⋮

key	value
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86

key	value
DA	87
ABR	88
RAB	89
BRA	8A
ABRA	8B

codeword table

LZW expansion demo

value	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A	D	A B	R A	B R	A B R	A	

LZW expansion for 41 42 52 41 43 41 44 81 83 82 88 41 80

key	value
⋮	⋮
41	A
42	B
43	C
44	D
⋮	⋮

key	value
81	AB
82	BR
83	RA
84	AC
85	CA
86	AD

key	value
87	DA
88	ABR
89	RAB
8A	BRA
8B	ABRA

codeword table

Which is the LZW compression for ABABABA ?

A. 41 42 41 42 41 42 80

B. 41 42 41 81 81 80

C. 41 42 81 81 41 80

D. 41 42 81 83 80

Data compression: quiz 4

Which is key data structure to implement LZW compression efficiently?

A. array

B. red–black BST

C. hash table

D. none of the above

LZW tricky case: expansion

value	41	42	81	83	80
output	A	B	A B	A B A	

need to know code for 83 before it is in codeword table!

we can deduce that the code for 83 is ABx for some character x

now, we have deduced x!

LZW expansion for 41 42 81 83 80

key	value
⋮	⋮
41	A
42	B
43	C
44	D
⋮	⋮

key	value
81	AB
82	BA
83	ABA

codeword table

LZW in the real world

Lempel–Ziv and friends.

- LZ77.
- LZ78.
- LZW.
- Deflate / zlib = LZ77 variant + Huffman.

Unix compress, GIF, TIFF, V.42bis modem: LZW. ← previously under patent

zip, 7zip, gzip, jar, png, pdf: deflate / zlib.

iPhone, Wii, Apache HTTP server: deflate / zlib.

← not patented (widely used in open source)

Apache HTTP SERVER PROJECT

Lossless data compression benchmarks

year	scheme	bits / char
1967	**ASCII**	7
1950	**Huffman**	4.7
1977	**LZ77**	3.94
1984	**LZMW**	3.32
1987	**LZH**	3.3
1987	**move-to-front**	3.24
1987	**LZB**	3.18
1987	**gzip**	2.71
1988	**PPMC**	2.48
1994	**SAKDC**	2.47
1994	**PPM**	2.34
1995	**Burrows-Wheeler**	**2.29**
1997	**BOA**	1.99
1999	**RK**	1.89

next programming assignment

data compression using Calgary corpus

Data compression summary

Lossless compression.

- Represent fixed-length symbols with variable-length codes. [Huffman]
- Represent variable-length symbols with fixed-length codes. [LZW]

Lossy compression. [not covered in this course]

- JPEG, MPEG, MP3, ...
- FFT/DCT, wavelets, fractals, ...

$$X_k = \sum_{i=0}^{n-1} x_i \cos\left[\frac{\pi}{n}\left(i + \frac{1}{2}\right) k\right]$$

Theoretical limits on compression. Shannon entropy: $H(X) = -\sum_{i}^{n} p(x_i) \lg p(x_i)$

Practical compression. Exploit extra knowledge whenever possible.