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5.4 REGULAR EXPRESSIONS

- *regular expressions*
- *REs and NFAs*
- *NFA simulation*
- *NFA construction*
- *applications*

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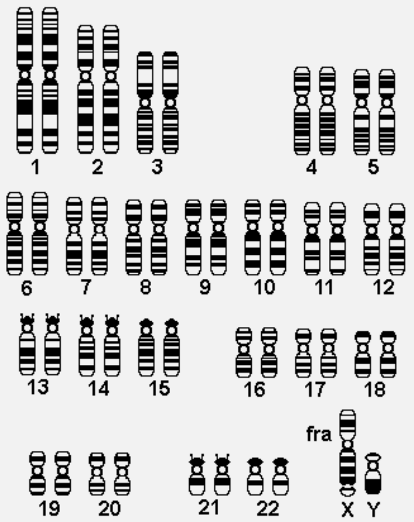
Pattern matching

Substring search. Find a single string in text.

Pattern matching. Find one of a specified set of strings in text.

Ex. [genomics]

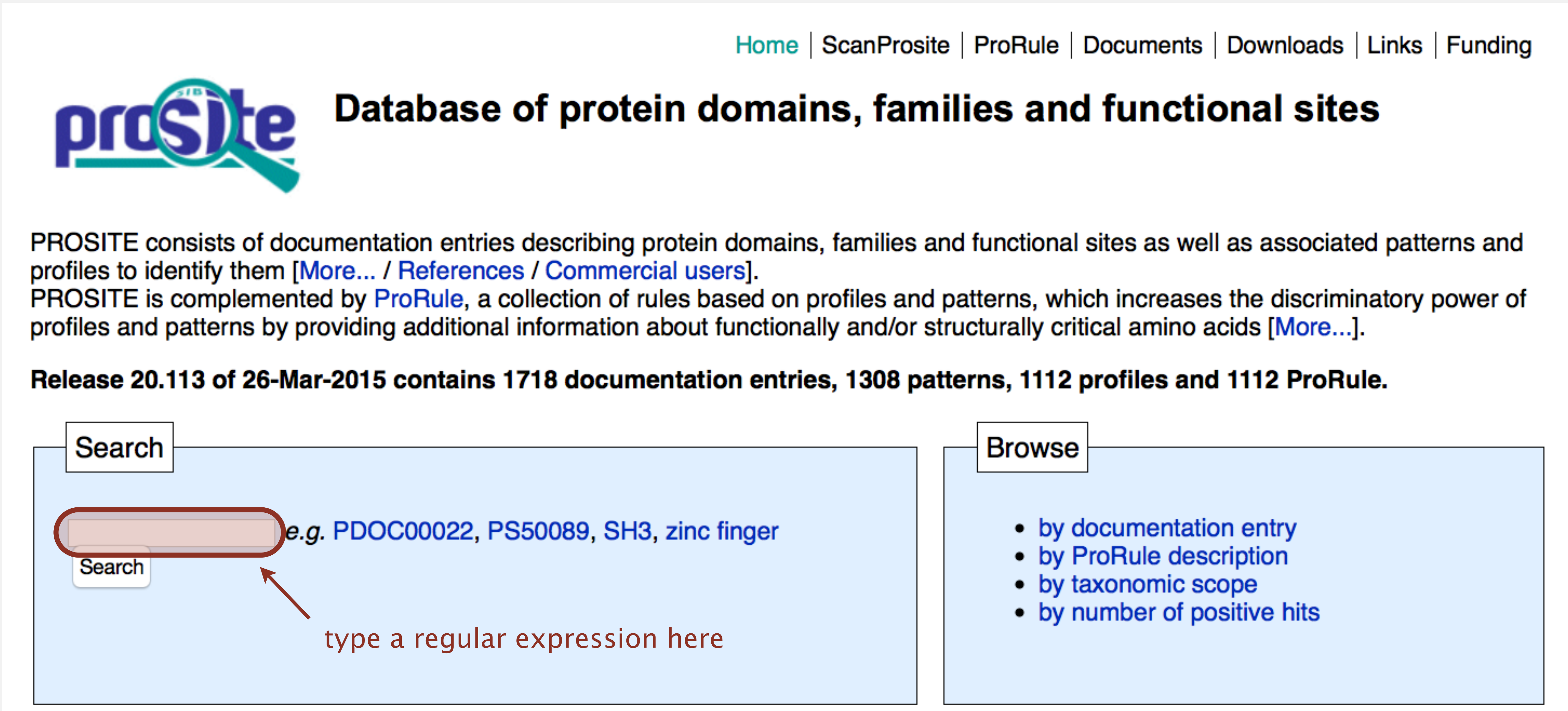
- Fragile X syndrome is a common cause of mental retardation.
- A human's genome is a string.
- It contains triplet repeats of CGG or AGG, bracketed by GCG at the beginning and CTG at the end.
- Number of repeats is variable and is correlated to syndrome.



pattern GCG(CGG|AGG)*CTG

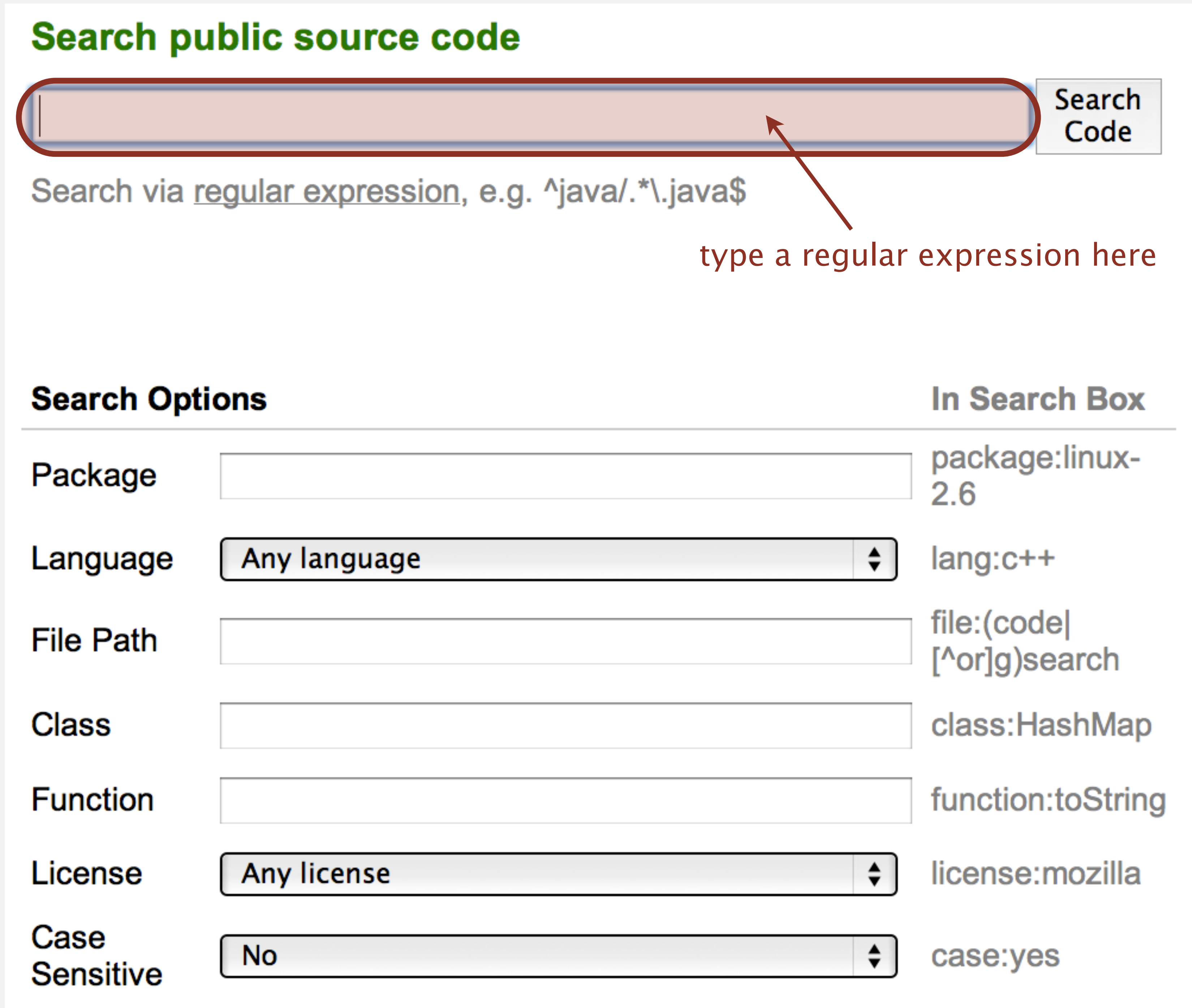
text GCGGCGTGTGTGCGAGAGAGTGGGTTTAAAGCTGGCGCGGAGGCGGCTGGCGCGGAGGCTG

Prosite (computational biochemistry)



http://prosite.expasy.org

Google code search



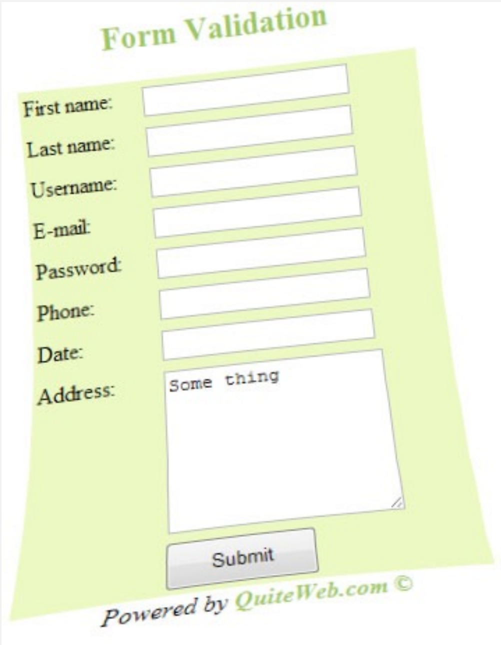
http://code.google.com/p/chromium/source/search

Pattern matching: applications

Test if a string matches some pattern.

- Scan for virus signatures.
- Process natural language.
- Specify a programming language.
- Access information in digital libraries.
- Search genome using Prosite patterns.
- Validate forms (dates, email, URL, credit card).
- Filter text (spam, NetNanny, Carnivore, malware).

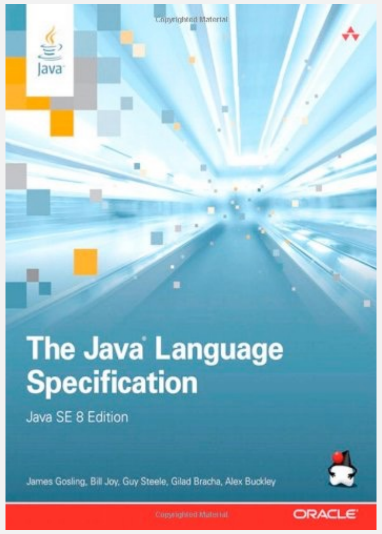
...



Parse text files.

- Compile a Java program.
- Crawl and index the Web.
- Read data stored in ad hoc input file format.
- Create Java documentation from Javadoc comments.

...



Regular expressions

A regular expression is a notation to specify a set of strings.

typically infinite

operation	order	example RE	matches	does not match
concatenation	3	A A B A A B	A A B A A B	*every other string*
or	4	A A | B A A B	A A B A A B	*every other string*
closure	2	A B * A	A A A B B B B B B B B A	A B A B A B A
parentheses	1	A (A | B) A A B	A A A A B A B A A B	*every other string*
		(A B) * A	A A B A B A B A B A B A	A A A B B A

Regular expressions: quiz 1

Which one of the following strings is not matched by the regular expression (A B | C * D) * ?

A. A B A B A B

B. C D C C D D D D

C. A B C C D A B

D. A B D A B C C A B D

Regular expression shortcuts

Additional operations further extend the utility of REs.

operation	example RE	matches	does not match
wildcard	`.U.U.U.`	CUMULUS JUGULUM	SUCCUBUS TUMULTUOUS
character class	`[A-Za-z][a-z]*`	word Capitalized	camelCase 4illegal
one or more	`A(BC)+DE`	ABCDE ABCBCDE	ADE BCDE
exactly k	`[0-9]{5}-[0-9]{4}`	08540-1321 19072-5541	111111111 166-54-111

Note. These operations are useful but not essential.

Ex. `[A-E]+` is shorthand for `(A|B|C|D|E)(A|B|C|D|E)*`

Regular expression examples

RE notation is surprisingly expressive.

regular expression	matches	does not match
`.*SPB.*` *(substring search)*	RASPBERRY CRISPBREAD	SUBSPACE SUBSPECIES
`[0-9]{3}-[0-9]{2}-[0-9]{4}` *(U.S. Social Security numbers)*	166-11-4433 166-45-1111	11-55555555 8675309
`[a-z]+@([a-z]+\.)+(edu|com)` *(simplified email addresses)*	wayne@princeton.edu rs@princeton.edu	spam@nowhere
`[$_A-Za-z][$_A-Za-z0-9]*` *(Java identifiers)*	ident3 PatternMatcher	3a ident#3

REs play a well-understood role in the theory of computation.

Regular expressions: quiz 2

Which of the following REs match genes:

(1) alphabet is { A, C, G, T }
(2) length is a multiple of 3
(3) starts with ATG (a start codon)
(4) ends with TAG or TAA or TTG (a stop codon)

A. ATG((A|C|G|T)(A|C|G|T)(A|C|G|T))*(TAG|TAA|TTG)

B. ATG([ACGT]{3})*(TAG|TAA|TTG)

C. Both A and B.

D. Neither A nor B.

Regular expressions to the rescue

http://xkcd.com/208

Regular expression caveat

Writing a RE is like writing a program.

- Need to understand programming model.
- Can be easier to write than read.
- Can be difficult to debug.

> *“ Some people, when confronted with a problem, think ‘I know I'll use regular expressions.’ Now they have two problems. ”*
>
> *— Jamie Zawinski (flame war on alt.religion.emacs)*

Bottom line. REs are amazingly powerful and expressive; using them can be amazingly complex and error-prone.

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5.4 Regular Expressions

- regular expressions
- **REs and NFAs**
- NFA simulation
- NFA construction
- applications

Duality between REs and DFAs

RE. Concise way to describe a set of strings.

DFA. Machine to recognize whether a given string is in a given set.

Kleene's theorem.

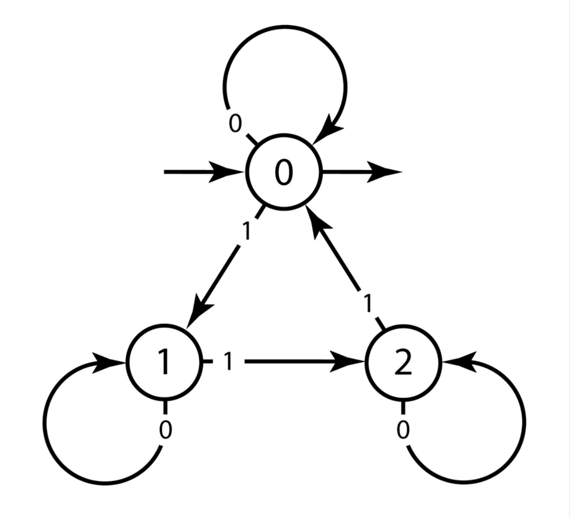
- For any DFA, there exists a RE that describes the same set of strings.
- For any RE, there exists a DFA that recognizes the same set of strings.

RE

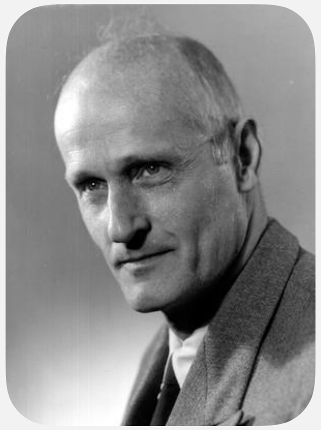
0* | (0*10*10*10*)*

number of 1s is a multiple of 3

DFA



number of 1s is a multiple of 3



Stephen Kleene
Princeton Ph.D. 1934

Pattern matching implementation: basic plan (first attempt)

Ken Thompson
Turing Award '83

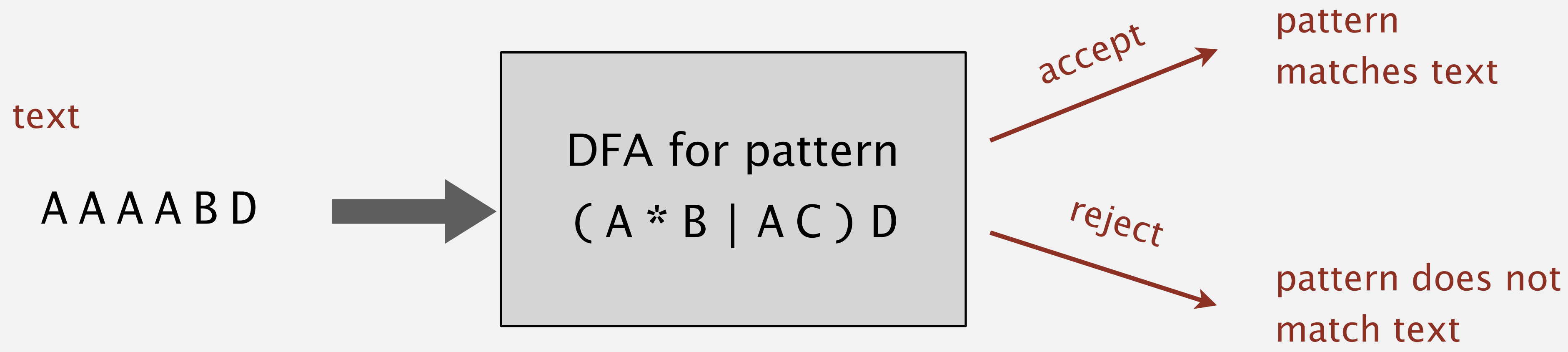
Overview is the same as for KMP.

- No backup in text input stream.
- Linear-time guarantee.

Underlying abstraction. Deterministic finite state automata (DFA).

Basic plan. [apply Kleene's theorem]

- Build DFA from RE.
- Simulate DFA with text as input.



Bad news. Basic plan is infeasible (DFA may have exponential # of states).

Pattern matching implementation: basic plan (revised)

Ken Thompson
Turing Award '83

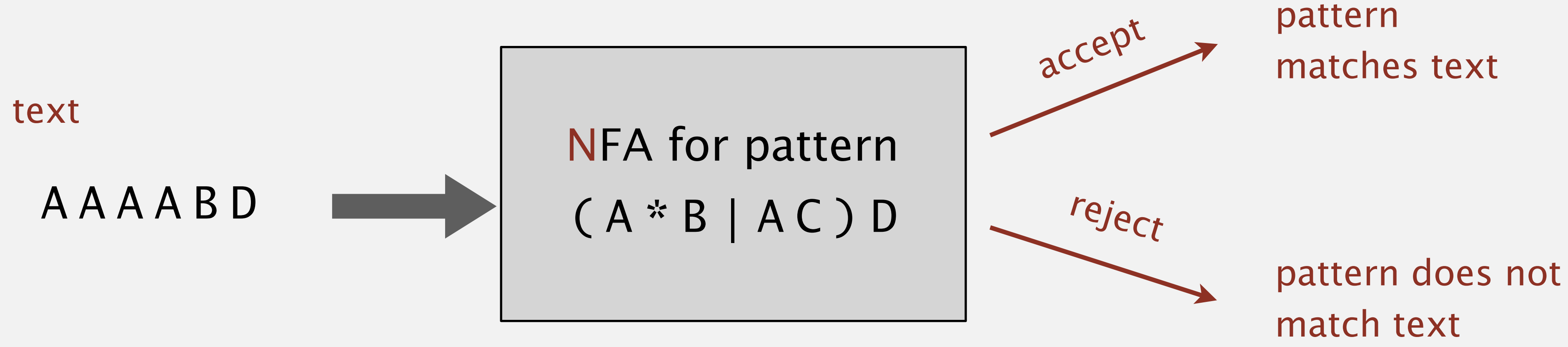
Overview is similar to KMP.

- No backup in text input stream.
- Quadratic-time guarantee (linear-time typical).

Underlying abstraction. Nondeterministic finite state automata (NFA).

Basic plan. [apply Kleene's theorem]

- Build NFA from RE.
- Simulate NFA with text as input.



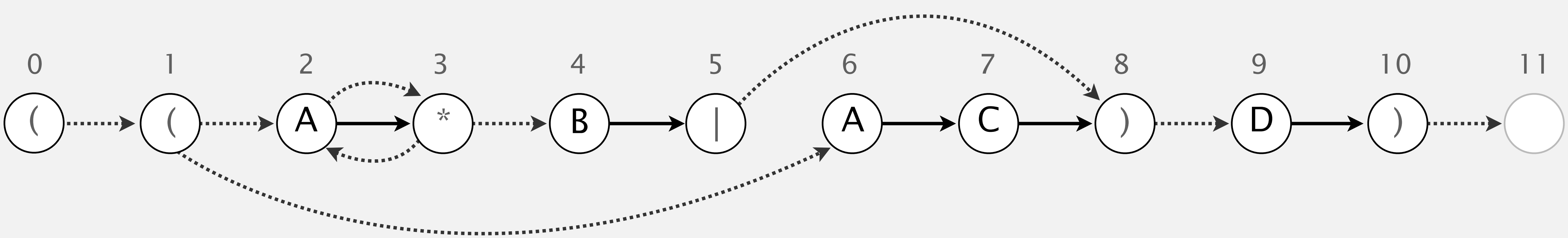
Q. What is an NFA?

Nondeterministic finite-state automata

Regular-expression-matching NFA.

- We assume RE enclosed in parentheses.
- One state per RE character (start = 0, accept = m).
- Match transition (change state and scan to next text char).
- Dashed ε-transition (change state, but don't scan text).
- Accept if any sequence of transitions ends in accept state.

after scanning all text characters



NFA corresponding to the pattern `( ( A * B | A C ) D )`

Nondeterministic finite-state automata

Q. Is A A A A B D matched by NFA?

A. Yes, because **some** sequence of legal transitions ends in state 11.

A A A A B D

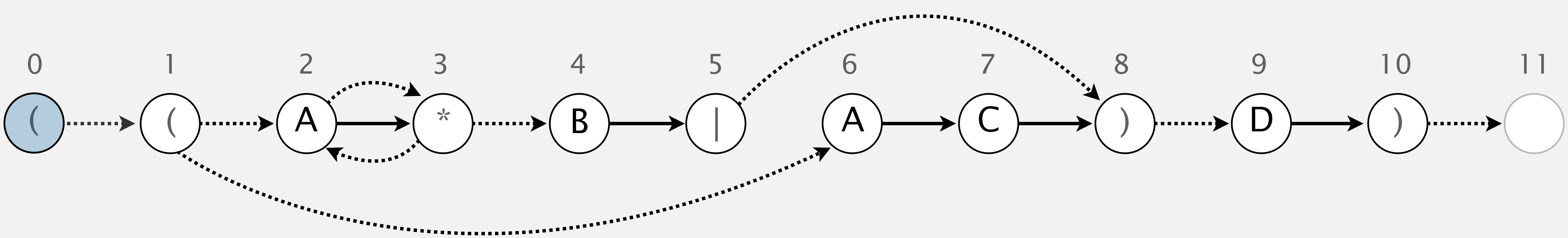
0 → 1 → 2 → 3 → 2 → 3 → 2 → 3 → 2 → 3 → 4 → 5 → 8 → 9 → 10 → 11

match transition: scan to next input character and change state

ϵ-transition: change state with no match

accept state reached and all text characters scanned: pattern found

A A A A B D



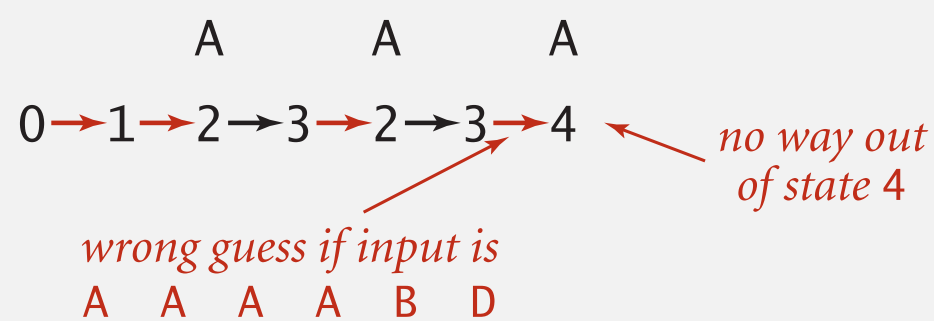
NFA corresponding to the pattern ((A * B | A C) D)

Nondeterministic finite-state automata

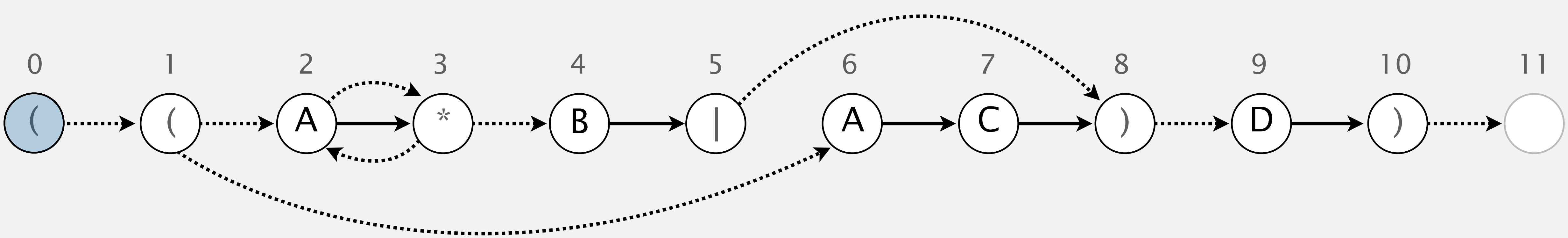
Q. Is A A A A B D matched by NFA?

A. Yes, because some sequence of legal transitions ends in state 11.

[even though some sequences end in wrong state or get stuck]



A A A A B D



NFA corresponding to the pattern ((A * B | A C) D)

Nondeterministic finite-state automata

Q. Is A A A C matched by NFA?

A. No, because no sequence of legal transitions ends in state 11.

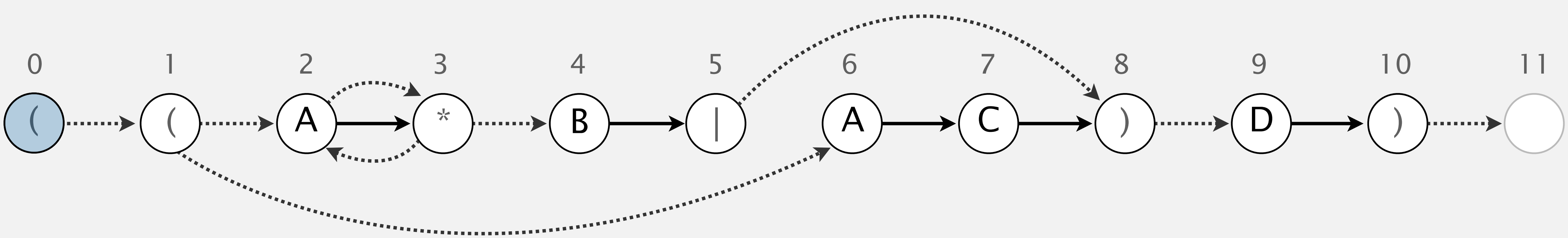
[must argue about all possible sequences (not just the one below)]

A A A A C

0 → 1 → 2 → 3 → 2 → 3 → 2 → 3 → 2 → 3 → 4

no way out of state 4

A A A A C

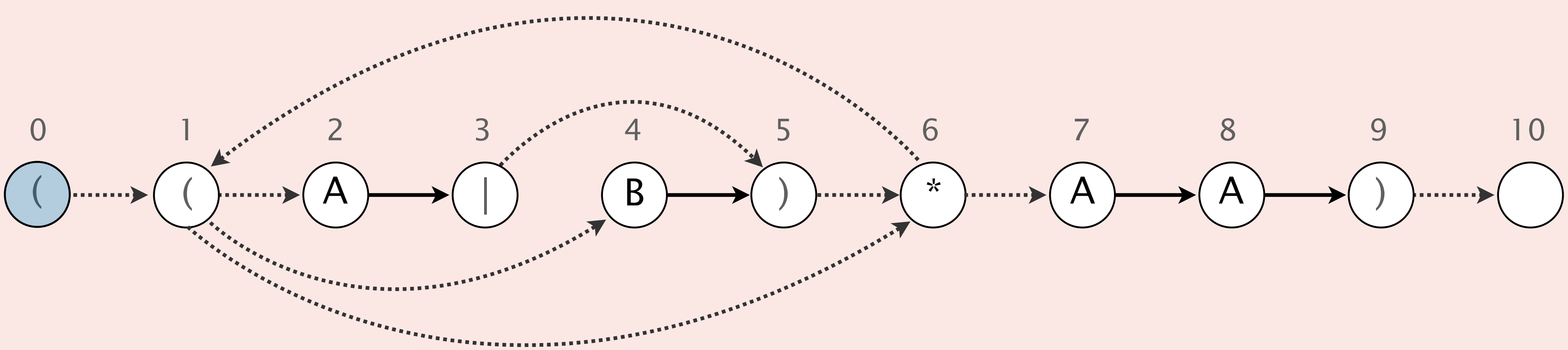


NFA corresponding to the pattern ((A * B | A C) D)

Regular expressions: quiz 3

Which of the following strings are matched by the NFA?

- **A.** B A A A A
- **B.** A A B A A B A A
- **C.** Both A and B.
- **D.** Neither A nor B.



Nondeterminism

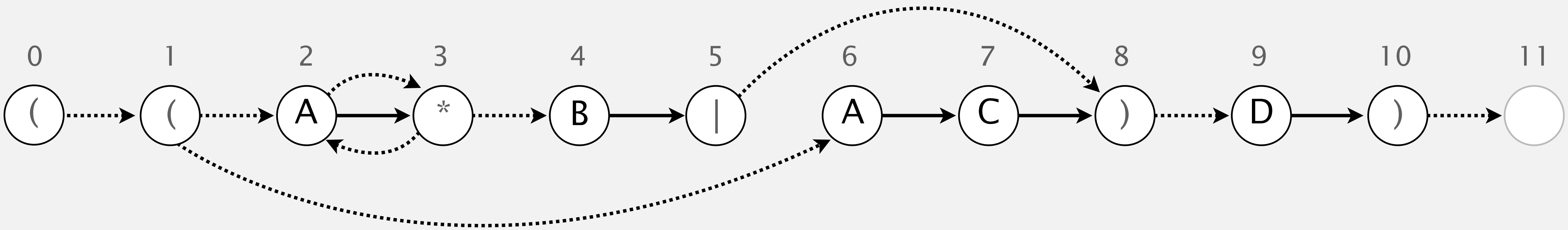
Q. How to determine whether a string is matched by an automaton?

DFA. Deterministic ⇒ easy (at each step, only one applicable transition).

NFA. Nondeterministic ⇒ hard (at each step, can be several applicable transitions; machine "guesses" the correct one!)

Q. How to simulate NFA?

A. Systematically consider all possible transition sequences. [stay tuned]



NFA corresponding to the pattern ((A * B | A C) D)

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NFA representation

State names. Integers from 0 to m.

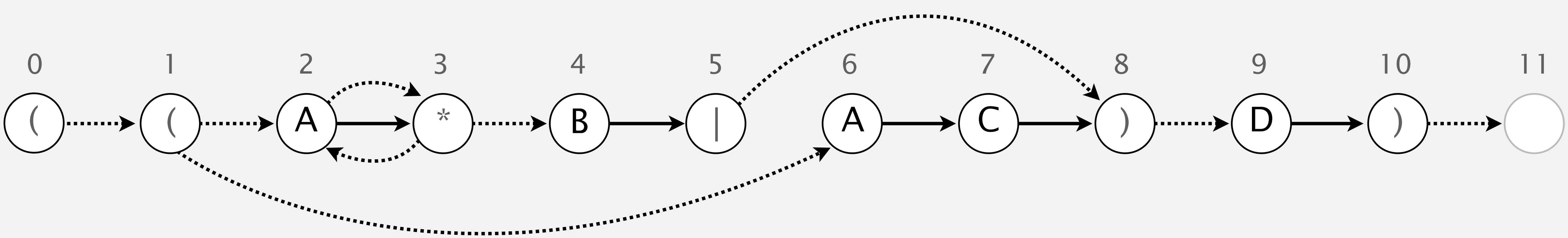
number of symbols in RE

Match-transitions. Keep regular expression in array re[].

	0	1	2	3	4	5	6	7	8	9	10
re[]	(	(	A	*	B	|	A	C	)	D	)

ε-transitions. Store in a digraph G.

0→1, 1→2, 1→6, 2→3, 3→2, 3→4, 5→8, 8→9, 10→11



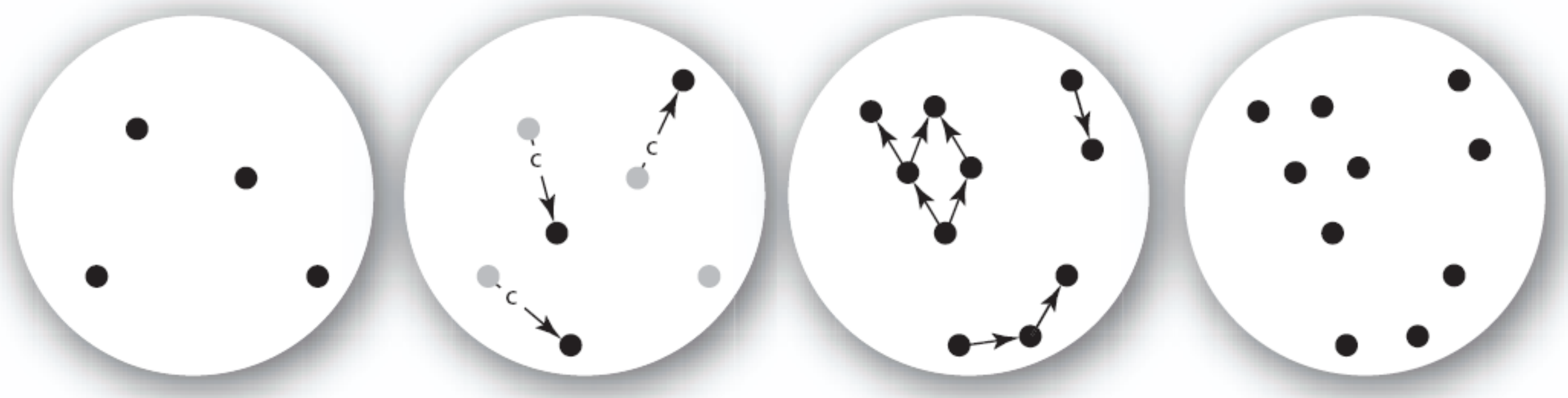
NFA corresponding to the pattern ((A * B | A C) D)

NFA simulation

Q. How to efficiently simulate an NFA?

A. Maintain set of all possible states that NFA could be in after reading in the first i text characters.

one step in simulating an NFA



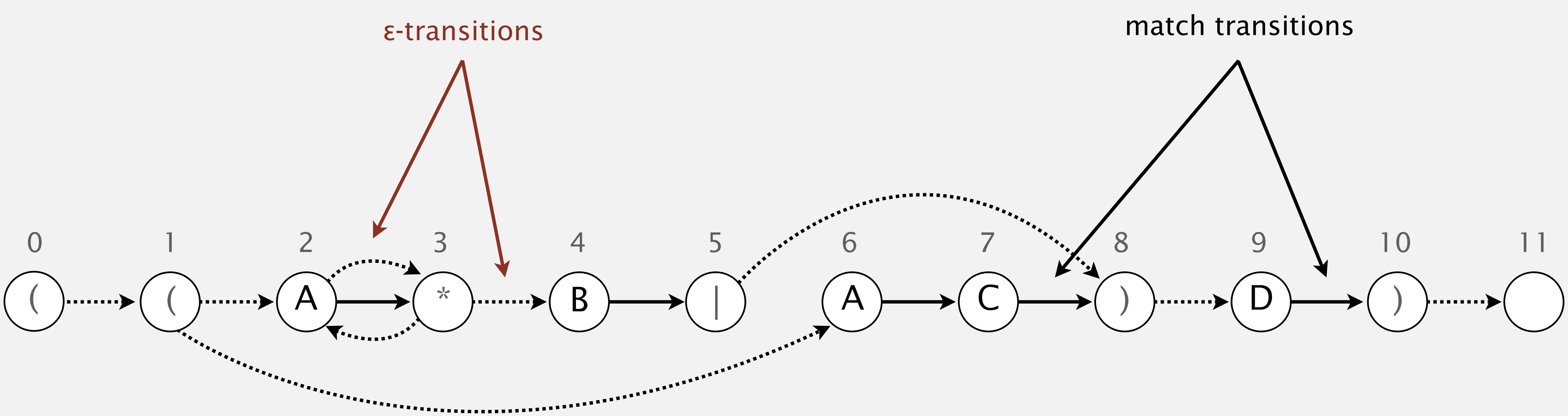
all states reachable after reading i symbols | *possible transitions on reading (i+1)st symbol c* | *possible null transitions before reading next symbol* | *all states reachable after reading i+1 symbols*

Q. How to perform reachability?

NFA simulation demo

Goal. Check whether input matches pattern.

input	A	A	B	D



NFA corresponding to the pattern ((A * B | A C) D)

Digraph reachability review

Goal. Find all vertices reachable from a given set of vertices.

recall Section 4.2

public class DirectedDFS		
	`DirectedDFS(Digraph G, int s)`	*find vertices reachable from s*
	`DirectedDFS(Digraph G, Iterable<Integer> s)`	*find vertices reachable from sources*
`boolean`	`marked(int v)`	*is v reachable from source(s)?*

Solution. Run DFS from each source, without unmarking vertices.

Performance. Runs in time proportional to $E + V$.

NFA simulation: Java implementation

```
public class NFA
{
   private char[] re;       // match transitions
   private Digraph G;       // epsilon transition digraph
   private int m;           // number of states

   public NFA(String regexp)
   {
      m  = regexp.length();
      re = regexp.toCharArray();
      G = buildEpsilonTransitionDigraph();
   }

   public boolean recognizes(String txt)
   {  /* see next slide */  }

   public Digraph buildEpsilonTransitionDigraph()
   {  /* stay tuned     */  }

}
```

stay tuned

NFA simulation: Java implementation

```
public boolean recognizes(String txt)
{
    Bag<Integer> pc = new Bag<Integer>();
    DirectedDFS dfs = new DirectedDFS(G, 0);
    for (int v = 0; v < G.V(); v++)
       if (dfs.marked(v)) pc.add(v);

   for (int i = 0; i < txt.length(); i++)
   {
    Bag<Integer> states = new Bag<Integer>();
    for (int v : pc)
    {
       if (v == m) continue;
       if ((re[v] == txt.charAt(i)) || re[v] == '.')
          states.add(v+1);
    }

    dfs = new DirectedDFS(G, states);
    pc = new Bag<Integer>();
    for (int v = 0; v < G.V(); v++)
       if (dfs.marked(v)) pc.add(v);
  }

    for (int v : pc)
       if (v == m) return true;
    return false;
}
```

states reachable from start by ε-transitions

set of states reachable after scanning past `txt.charAt(i)`

not necessarily a match (RE needs to match full text)

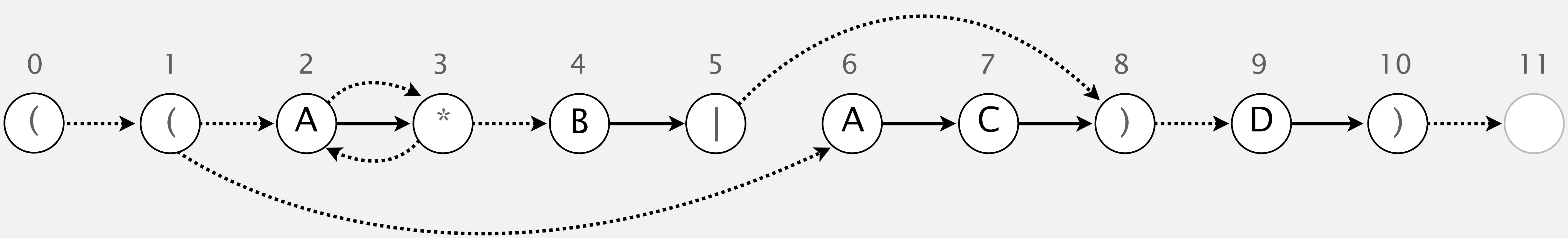
follow ε-transitions

accept if can end in state m

NFA simulation: analysis

Proposition. Determining whether an n-character text is recognized by the NFA corresponding to an m-character pattern takes time proportional to $m\,n$ in the worst case.

Pf. For each of the n text characters, we iterate through a set of states of size no more than m and run DFS on the graph of ε-transitions.
[The NFA construction we will consider ensures the number of edges ≤ $3m$.]



NFA corresponding to the pattern ((A * B | A C) D)

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5.4 Regular Expressions

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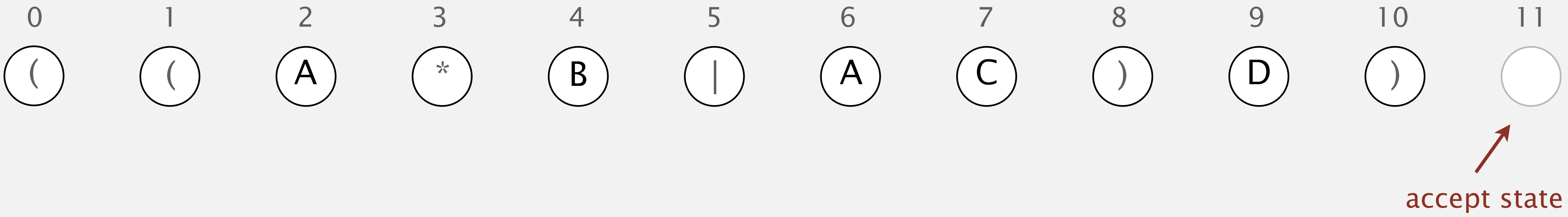
Building an NFA corresponding to an RE

States. Include a state for each symbol in the RE, plus an accept state.



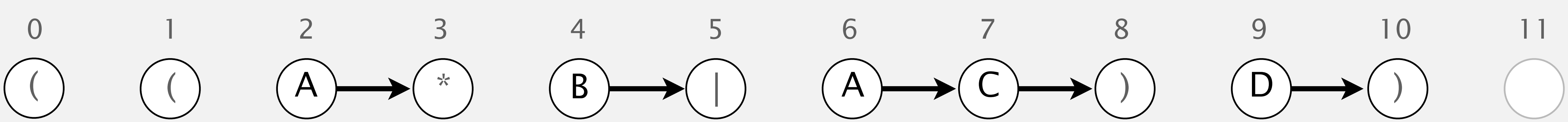
NFA corresponding to the pattern ((A * B | A C) D)

Building an NFA corresponding to an RE

Concatenation. Add match-transition edge from state corresponding to characters in the alphabet to next state.

Alphabet. `A B C D`

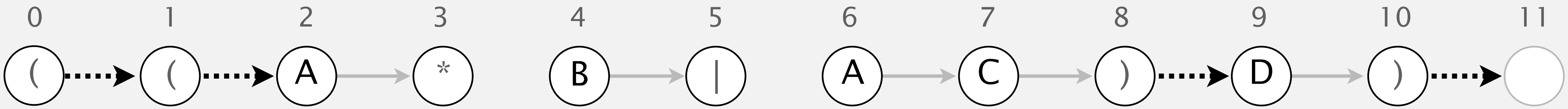
Metacharacters. `( ) . * |`



NFA corresponding to the pattern `( ( A * B | A C ) D )`

Building an NFA corresponding to an RE

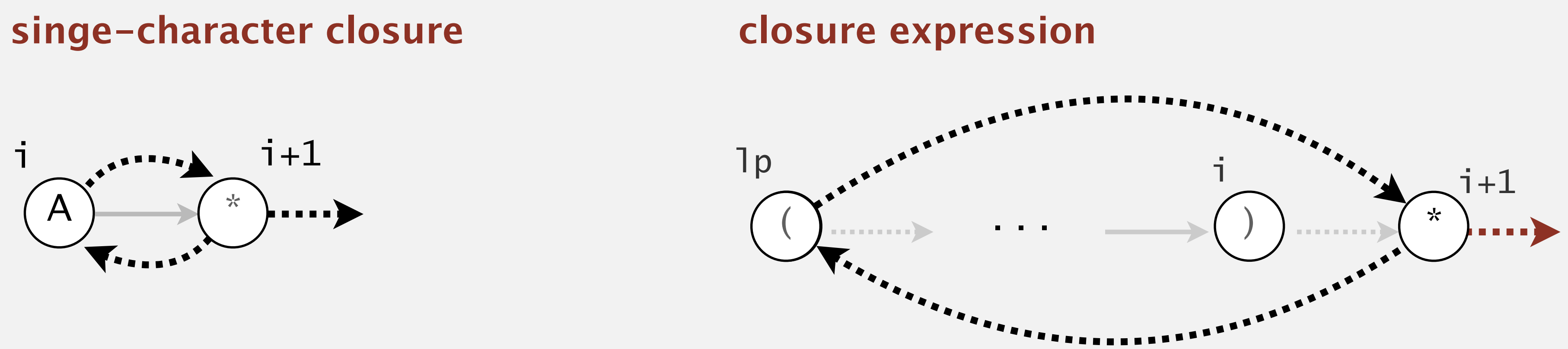
Parentheses. Add ε-transition edge from parentheses to next state.

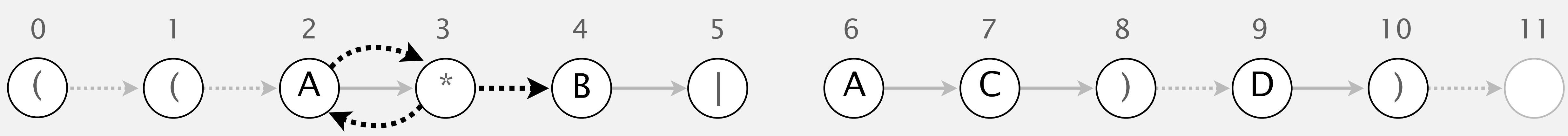


NFA corresponding to the pattern ((A * B | A C) D)

Building an NFA corresponding to an RE

Closure. Add three ε-transition edges for each * operator.



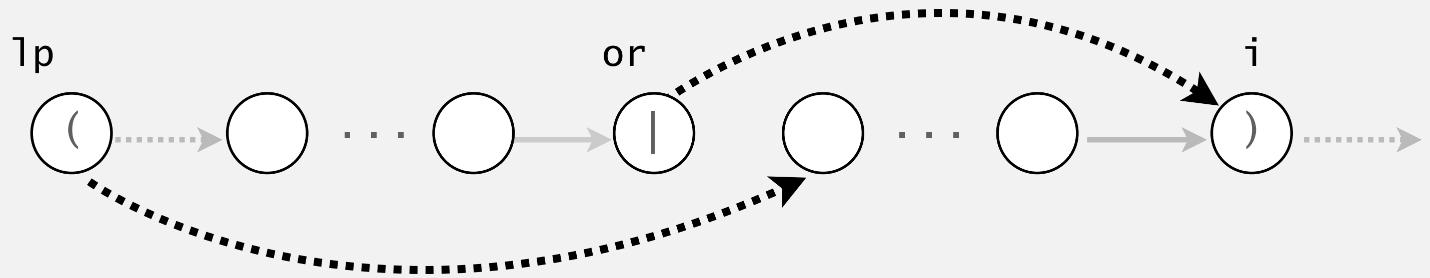


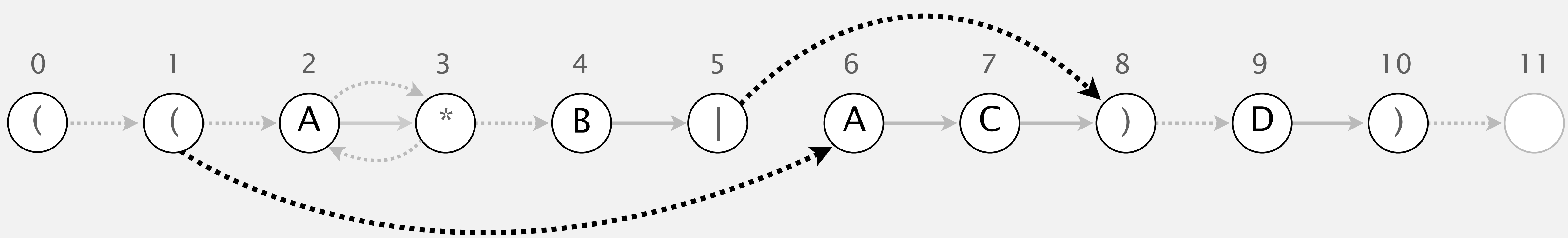
NFA corresponding to the pattern ((A * B | A C) D)

Building an NFA corresponding to an RE

2-way or. Add two ε-transition edges for each | operator.

2–way or expression





NFA corresponding to the pattern ((A * B | A C) D)

Building an NFA corresponding to an RE

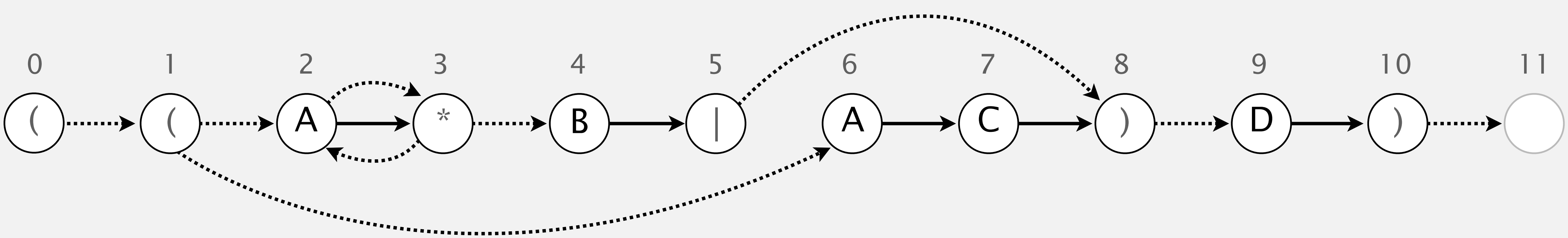
States. Include a state for each symbol in the RE, plus an accept state.

Concatenation. Add match-transition edge from state corresponding to characters in the alphabet to next state.

Parentheses. Add ε-transition edge from parentheses to next state.

Closure. Add three ε-transition edges for each * operator.

2-way or. Add two ε-transition edges for each | operator.



NFA corresponding to the pattern ((A * B | A C) D)

Regular expressions: quiz 4

How would you modify the NFA below to match `((ABC*)+)` ?

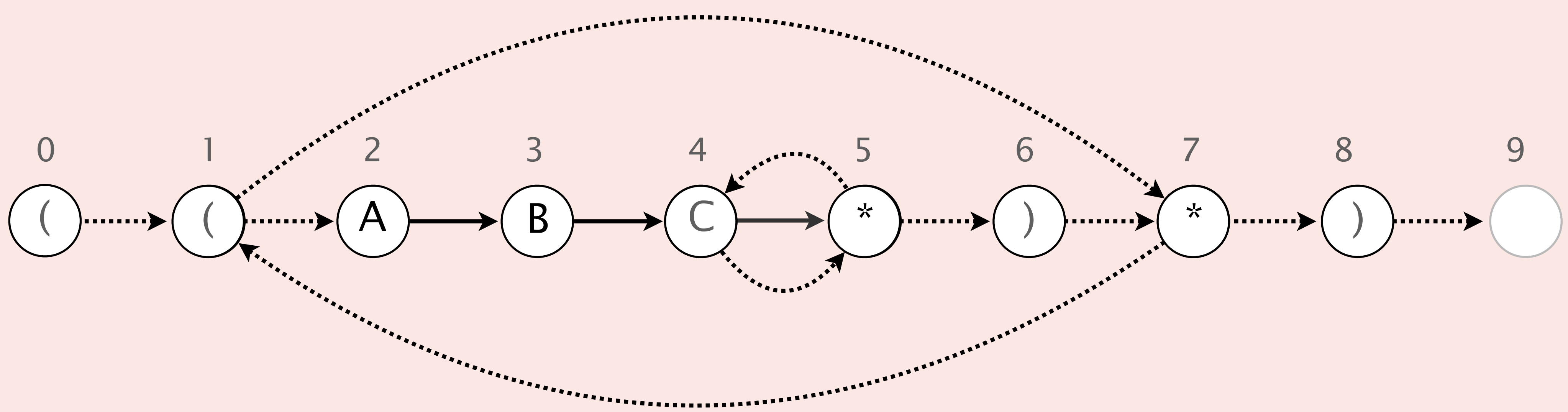
one or more occurrence

A. Remove ε-transition edge 1→7.

B. Remove ε-transition edge 7→1.

C. Remove ε-transition edges 1→7 and 7→1.

D. None of the above.



NFA corresponding to the pattern `( ( A B C * ) * )`

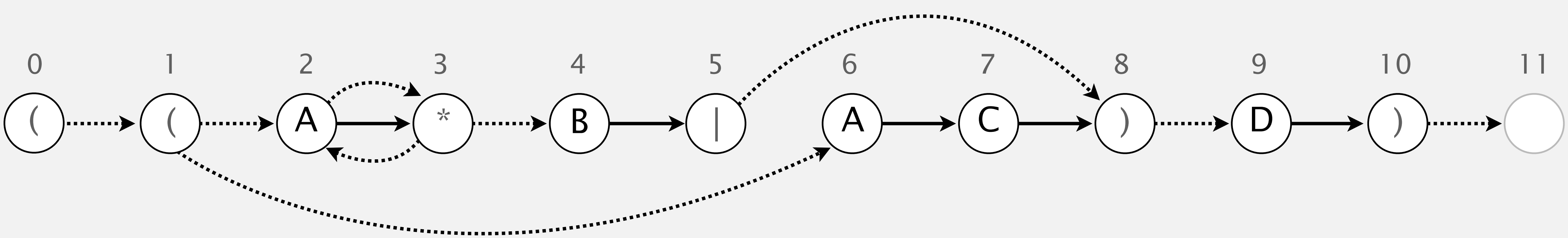
NFA construction: implementation

Goal. Write a program to build the ε-transition digraph.

Challenges. Remember left parentheses to implement closure and 2-way or; remember | symbols to implement 2-way or.

Solution. Maintain a stack.

- (symbol: push (onto stack.
- | symbol: push | onto stack.
-) symbol: pop corresponding (and any intervening |; add ε-transition edges for closure/or.



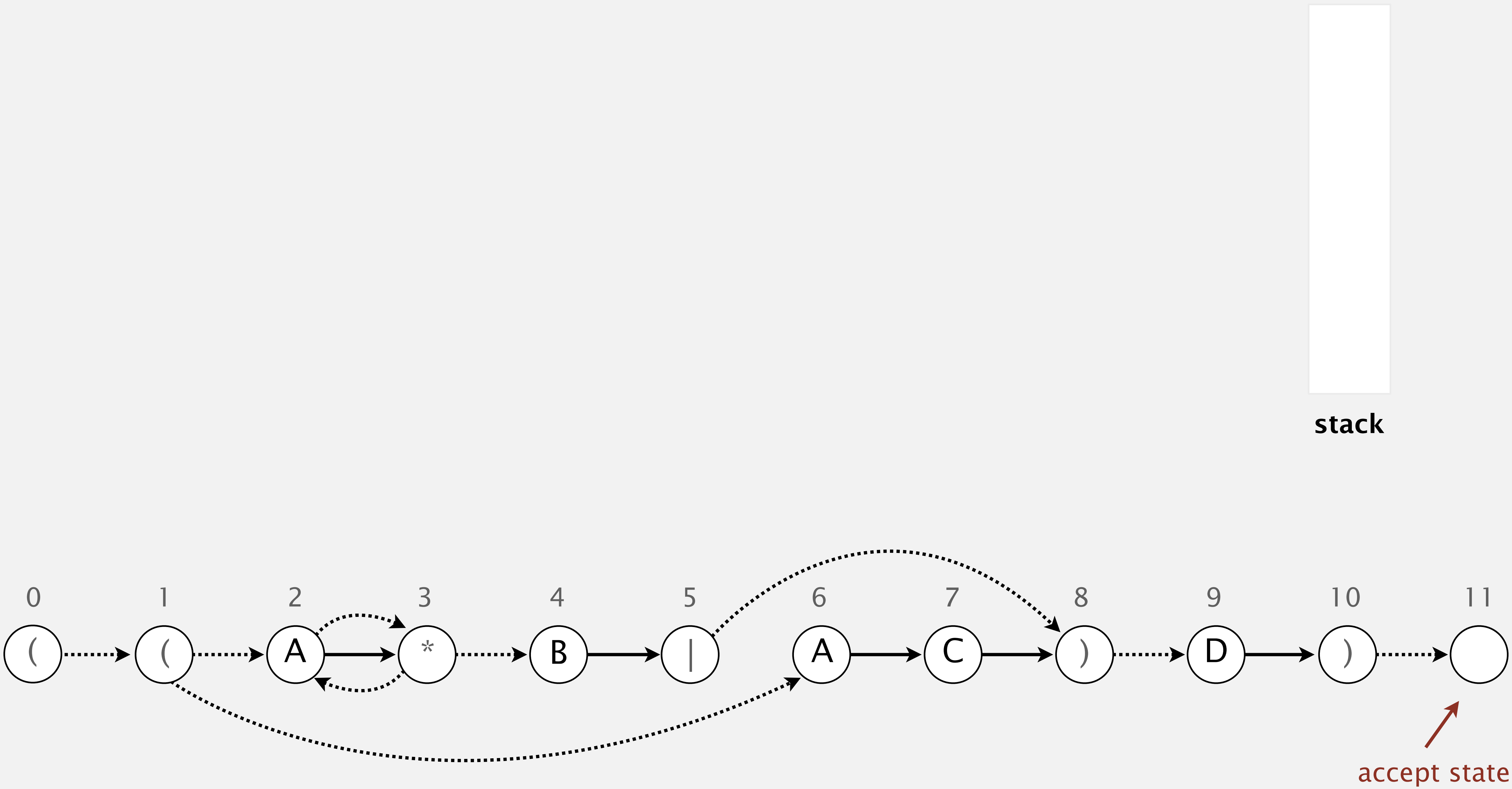
NFA corresponding to the pattern ((A * B | A C) D)

NFA construction demo

stack

((A * B | A C) D)

NFA construction demo



NFA corresponding to the pattern ((A * B | A C) D)

NFA construction: Java implementation

```
private Digraph buildEpsilonTransitionDigraph() {
   Digraph G = new Digraph(m+1);
   Stack<Integer> ops = new Stack<Integer>();
   for (int i = 0; i < m; i++) {
      int lp = i;

      if (re[i] == '(' || re[i] == '|') ops.push(i);

      else if (re[i] == ')') {
         int or = ops.pop();
         if (re[or] == '|') {
            lp = ops.pop();
            G.addEdge(lp, or+1);
            G.addEdge(or, i);
         }
         else lp = or;
      }

      if (i < m-1 && re[i+1] == '*') {
         G.addEdge(lp, i+1);
         G.addEdge(i+1, lp);
      }

      if (re[i] == '(' || re[i] == '*' || re[i] == ')')
         G.addEdge(i, i+1);

   }
   return G;
}
```

left parentheses and |

2-way or

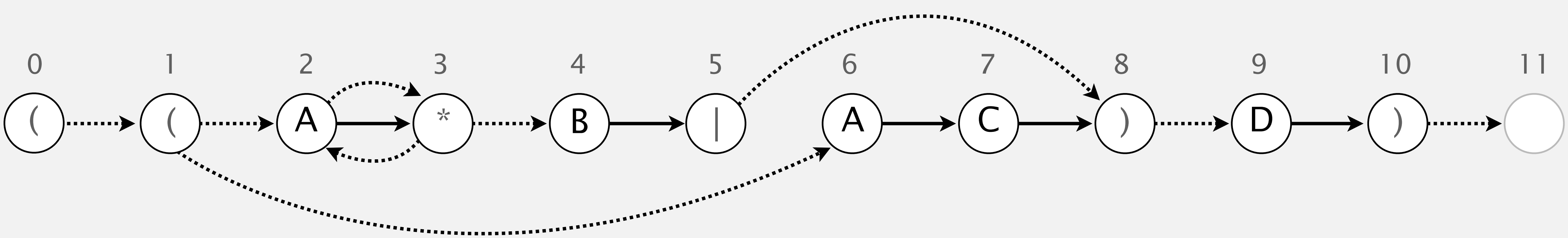
closure
(needs 1-character lookahead)

metasymbols

NFA construction: analysis

Proposition. Building the NFA corresponding to an m-character RE takes time and space proportional to m.

Pf. For each of the m characters in the RE, we add at most three ε-transitions and execute at most two stack operations.



NFA corresponding to the pattern ((A * B | A C) D)

https://algs4.cs.princeton.edu

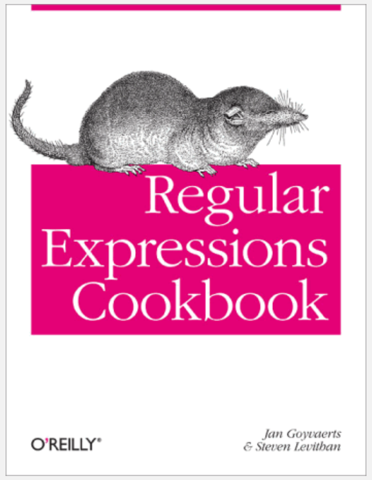
5.4 Regular Expressions

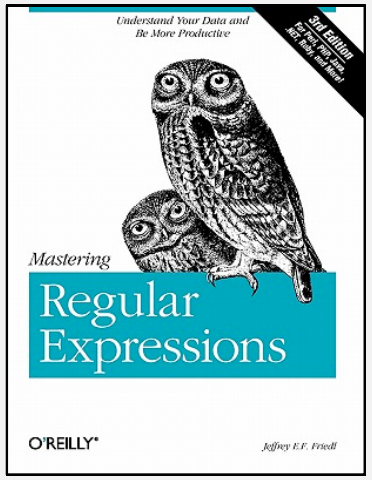
- *regular expressions*
- *REs and NFAs*
- *NFA simulation*
- *NFA construction*
- ***applications***

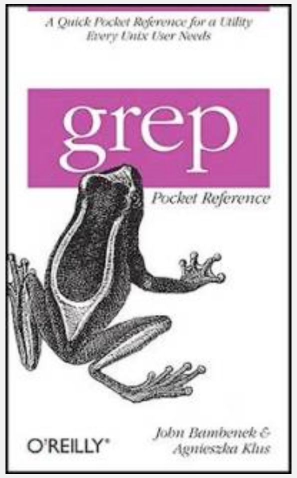
Industrial-strength grep implementation

To complete the implementation:

- Add multiway or.
- Handle metacharacters.
- Support character classes.
- Add capturing capabilities.
- Extend the closure operator.
- Error checking and recovery.
- Greedy vs. reluctant matching.







Ex. Which substring(s) should be matched by the RE `<blink>.*</blink>` ?

```
←—— reluctant ——→                ←—— reluctant ——→
<blink>text</blink>some text<blink>more text</blink>
←———————————————— greedy ————————————————→
```

Regular expressions in Java

Validity checking. Does the `input` match the `regexp` ?

Java string library. Use `input.matches(regexp)` for basic RE matching.

```
public class Validate
{
   public static void main(String[] args)
   {
      String regexp = args[0];
      String input  = args[1];
      StdOut.println(input.matches(regexp));
   }
}
```

```
% java Validate "[$_A-Za-z][$_A-Za-z0-9]*" ident123
true

% java Validate "[a-z]+@([a-z]+\.)+(edu|com)" rs@cs.princeton.edu
true

% java Validate "[0-9]{3}-[0-9]{2}-[0-9]{4}" 166-11-4433
true
```

legal Java identifier

valid email address (simplified)

Social Security number

Harvesting information

Goal. Print all substrings of input that match a RE.

```
% java Harvester "GCG(CGG|AGG)*CTG" chromosomeX.txt
GCGCGGCGGCGGCGGCGGCTG
GCGCTG
GCGCTG
GCGCGGCGGCGGAGGCGGAGGCGGCTG
```

harvest patterns from DNA

harvest links from website

```
% java Harvester "http://(\w+\.)*(\w+)" http://www.cs.princeton.edu
http://www.w3.org
http://www.cs.princeton.edu
http://drupal.org
http://csguide.cs.princeton.edu
http://www.cs.princeton.edu
http://www.princeton.edu
```

Harvesting information

RE pattern matching is implemented in Java's `java.util.regexp.Pattern` and `java.util.regexp.Matcher` classes.

```
import java.util.regex.Pattern;
import java.util.regex.Matcher;

public class Harvester
{
   public static void main(String[] args)
   {
      String regexp   = args[0];
      In in           = new In(args[1]);
      String input    = in.readAll();
      Pattern pattern = Pattern.compile(regexp);
      Matcher matcher = pattern.matcher(input);
      while (matcher.find())
      {
         StdOut.println(matcher.group());
      }
   }
}
```

`compile()` creates a `Pattern` (NFA) from RE

`matcher()` creates a `Matcher` (NFA simulator) from NFA and text

`find()` looks for the next match

`group()` returns the substring most recently found by `find()`

Regular expressions: quiz 5

What is the worst-case running time of Java's matches() method?

A. $m + n$

B. $m\, n$

C. $m\, n^2$

D. 2^n

m = pattern length

n = text length

matches

`public boolean matches(String regex)`

Tells whether or not this string matches the given regular expression.

An invocation of this method of the form *str*`.matches(`*regex*`)` yields exactly the same result as the expression

`Pattern.matches(`*regex*`, `*str*`)`

Parameters:

`regex` - the regular expression to which this string is to be matched

Returns:

`true` if, and only if, this string matches the given regular expression

Context

Abstract machines, languages, and nondeterminism.

- Basis of the theory of computation.
- Intensively studied since the 1930s.
- Basis of programming languages.

Compiler. A program that translates a program to machine code.

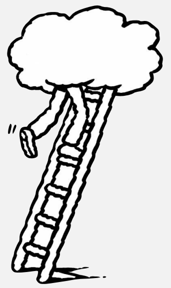
- KMP string $\Rightarrow$ DFA.
- `grep` RE $\Rightarrow$ NFA.
- `javac` Java language $\Rightarrow$ Java byte code.

	KMP	grep	Java
pattern	string	RE	program
parser	unnecessary	check if legal	check if legal
compiler output	DFA	NFA	byte code
simulator	DFA simulator	NFA simulator	JVM

Summary of pattern-matching algorithms

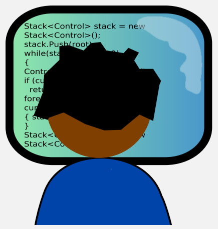
Theoretician.

- RE is a compact description of a set of strings.
- NFA is an abstract machine equivalent in power to RE.
- DFAs, NFAs, and REs have limitations.



Programmer.

- Implement substring search via DFA simulation.
- Implement RE pattern matching via NFA simulation.



You.

- Core CS principles provide useful tools that you can exploit now.
- REs and NFAs provide introduction to theory of computing.

Example of essential paradigm in computer science.

- Build the right intermediate abstractions.
- Solve important practical problems.