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# 5.3 SUBSTRING SEARCH

- *introduction*
- *brute force*
- *Knuth–Morris–Pratt*
- *Boyer–Moore*


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- *introduction*
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# Substring search

Goal. Find pattern of length $m$ in a text of length $n$.

typically $n \gg m$

*pattern* → `N E E D L E`

*text* → `I N A H A Y S T A C K N E E D L E I N A`

*match*

# Substring search applications

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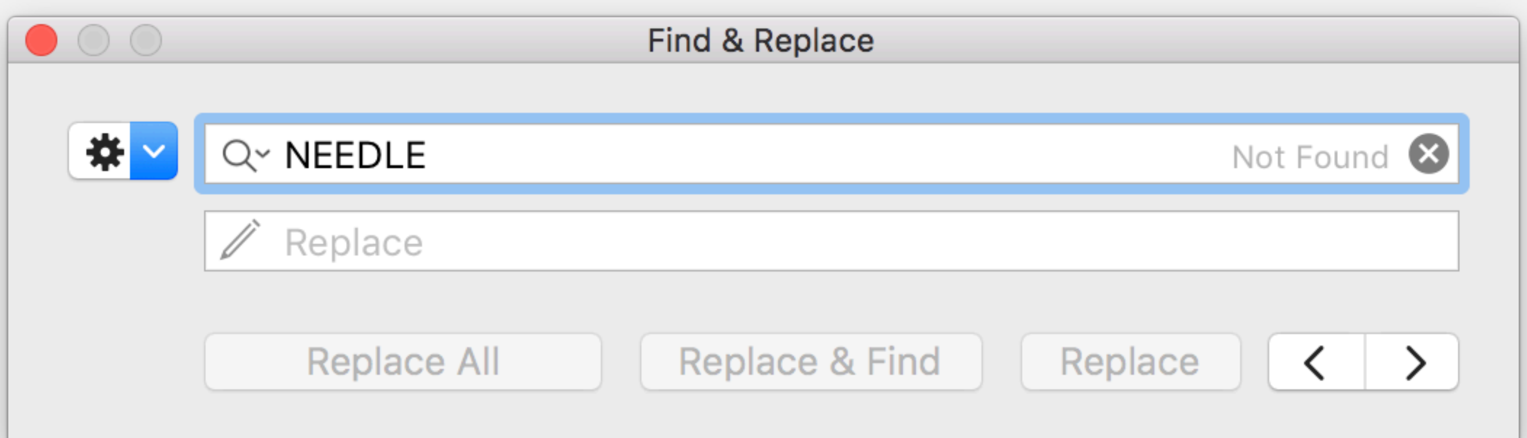
*match*



# Substring search applications

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*text* → I N A H A Y S T A C K N E E D L E I N A

*match*

Computer forensics. Search memory or disk for signatures, e.g., all URLs or RSA keys that the user has entered.




http://citp.princeton.edu/memory

# Substring search applications

Goal. Find pattern of length $m$ in a text of length $n$.

typically $n \gg m$

*pattern* → N E E D L E

*text* → I N A H A Y S T A C K N E E D L E I N A

*match*

Identify patterns indicative of spam.

- PROFITS
- LOSE WE1GHT
- herbal Viagra
- There is no catch.
- This is a one-time mailing.
- This message is sent in compliance with spam regulations.

# Substring search applications

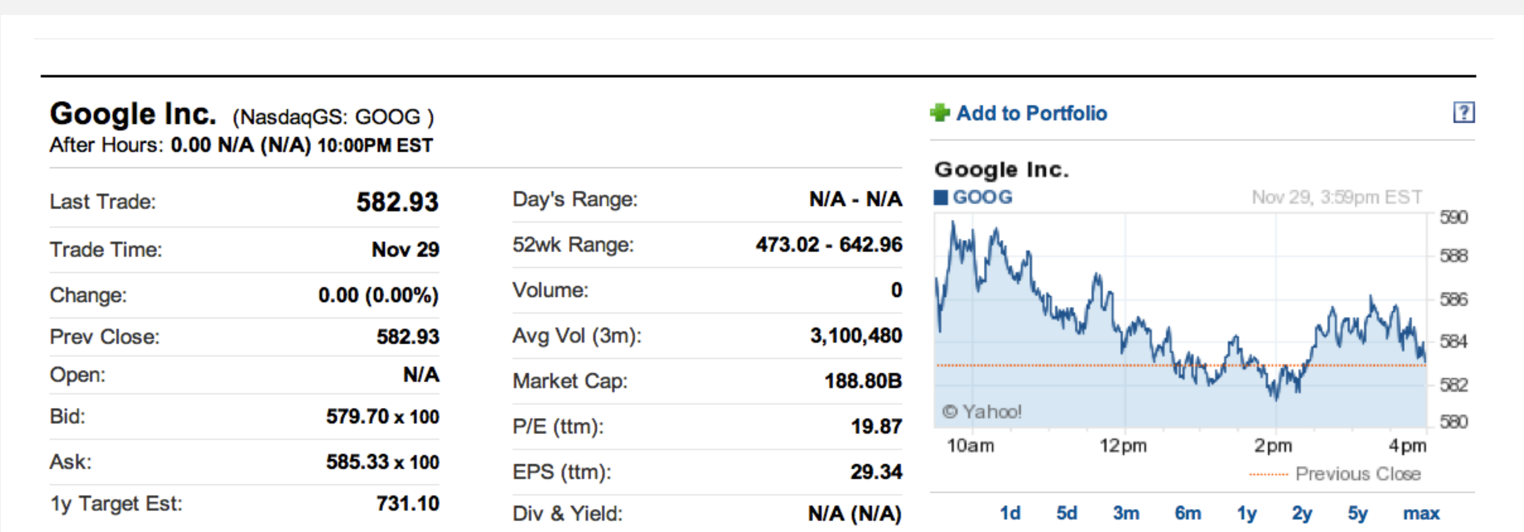
Web scraping. Extract relevant data from web page.

Ex. Find string delimited by `<b>` and `</b>` after first occurrence of pattern `Last Trade:`.

raw HTML

```
...
<tr>
<td class= "yfnc_tablehead1"
width= "48%">
Last Trade:
</td>
<td class= "yfnc_tabledata1">
<big><b>582.93</b></big>
</td></tr>
<td class= "yfnc_tablehead1"
width= "48%">
Trade Time:
</td>
<td class= "yfnc_tabledata1">
...
```

as rendered by browser




http://finance.yahoo.com/q?s=goog

# Web scraping: Java implementation

Java library. The `indexOf()` method in Java's `String` data type returns the index of the first occurrence of a given string, starting at a given offset.

```
public class StockQuote
{
   public static void main(String[] args)
   {
      String name = "http://finance.yahoo.com/q?s=";
      In in = new In(name + args[0]);
      String text = in.readAll();
      int start    = text.indexOf("Last Trade:", 0);
      int from     = text.indexOf("<b>",  start);
      int to       = text.indexOf("</b>", from);
      String price = text.substring(from + 3, to);
      StdOut.println(price);
   }
}
```

```
% java StockQuote goog
582.93
```

Caveat. Must update program whenever Yahoo format changes.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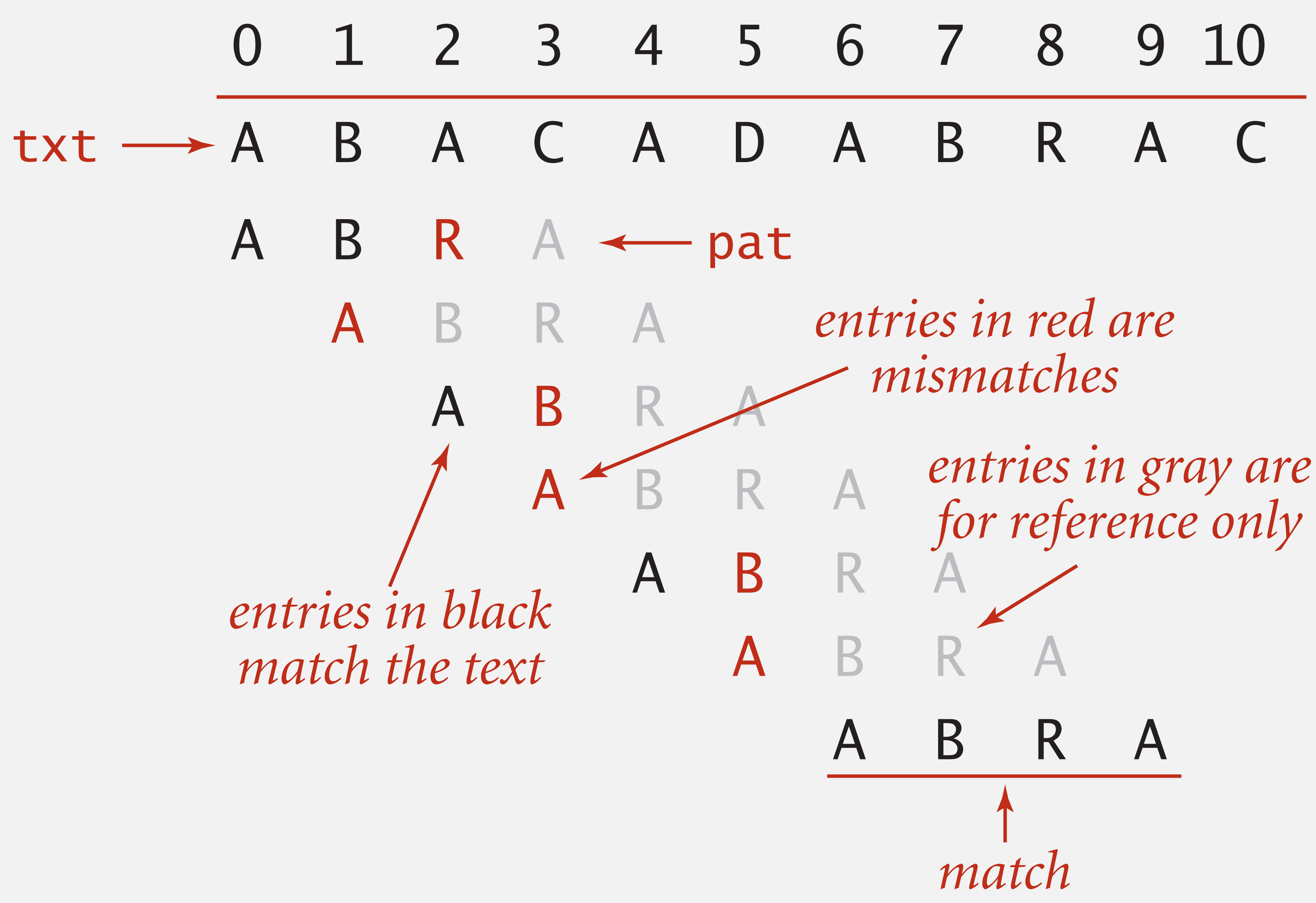
# Brute-force substring search

Check for pattern starting at each text position.



# Brute-force substring search: Java implementation

Check for pattern starting at each text position.

| i | j | i+j | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | A | B | A | C | A | D | A | B | R | A | C |
| ⋮ | | | | | | | | | | | | | |
| 4 | 3 | 7 | | | | | A | D | A | C | R | | |
| 5 | 0 | 5 | | | | | | A | D | A | C | R | |

```
public static int search(String pat, String txt)
{
   int m = pat.length();
   int n = txt.length();
   for (int i = 0; i <= n - m; i++)   ←  for each possible offset
   {
      int j;   ←  number of characters that match
      for (j = 0; j < m; j++)
         if (txt.charAt(i+j) != pat.charAt(j))
            break;
      if (j == m) return i;   ←  index in text where pattern starts
   }
   return n;   ←  not found
}
```

# Substring search: quiz 1

**What is the worst-case running time of brute-force substring search as a function of the pattern length $m$ and text length $n$ ?**

**A.** $m + n$

**B.** $m^2$

**C.** $m\,n$

**D.** $n^2$

# Algorithmic challenges in substring search

**Fundamental algorithmic challenge.** Linear-time guarantee.

```
Now is the time for all people to come to the aid of their party. Now is the time for
all good people to come to the aid of their party. Now is the time for many good people
to come to the aid of their party. Now is the time for all good people to come to the
aid of their party. Now is the time for a lot of good people to come to the aid of
their party. Now is the time for all of the good people to come to the aid of their
party. Now is the time for all good people to come to the aid of their party. Now is
the time for each good person to come to the aid of their party. Now is the time for
all good people to come to the aid of their party. Now is the time for all good
Republicans to come to the aid of their party. Now is the time for all good people to
come to the aid of their party. Now is the time for many or all good people to come to
the aid of their party. Now is the time for all good people to come to the aid of their
party. Now is the time for all good Democrats to come to the aid of their party. Now is
the time for all people to come to the aid of their party. Now is the time for all good
people to come to the aid of their party. Now is the time for many good people to come
to the aid of their party. Now is the time for all good people to come to the aid of
their party. Now is the time for a lot of good people to come to the aid of their
party. Now is the time for all of the good people to come to the aid of their party.
Now is the time for all good people to come to the aid of their attack at dawn party.
Now is the time for each person to come to the aid of their party. Now is the time for
all good people to come to the aid of their party. Now is the time for all good
Republicans to come to the aid of their party. Now is the time for all good people to
come to the aid of their party. Now is the time for many or all good people to come to
the aid of their party. Now is the time for all good people to come to the aid of their
party. Now is the time for all good Democrats to come to the aid of their party.
```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## Knuth–Morris–Pratt substring search

Intuition. Suppose we are searching in text for pattern BAAAAAAAAA.

- Suppose we match 5 chars in pattern, with mismatch on 6th char.

# Knuth–Morris–Pratt substring search

Intuition. Suppose we are searching in text for pattern B A A A A A A A A A.

- Suppose we match 5 chars in pattern, with mismatch on $6^{th}$ char.
- We know previous 6 chars in text must be B A A A A B.
- Don't need to compare any text character twice.

assuming { A, B } alphabet

*text*

A B A A A A B A A A A A A A A A

*after mismatch on sixth char*

B A A A A A A A A A

*pattern*

*but no backup is needed*

B A A A A A A A A A

Knuth–Morris–Pratt algorithm. Clever method to always avoid comparing a text character more than once!

# Deterministic finite state automaton (DFA)

DFA is abstract string-searching machine.

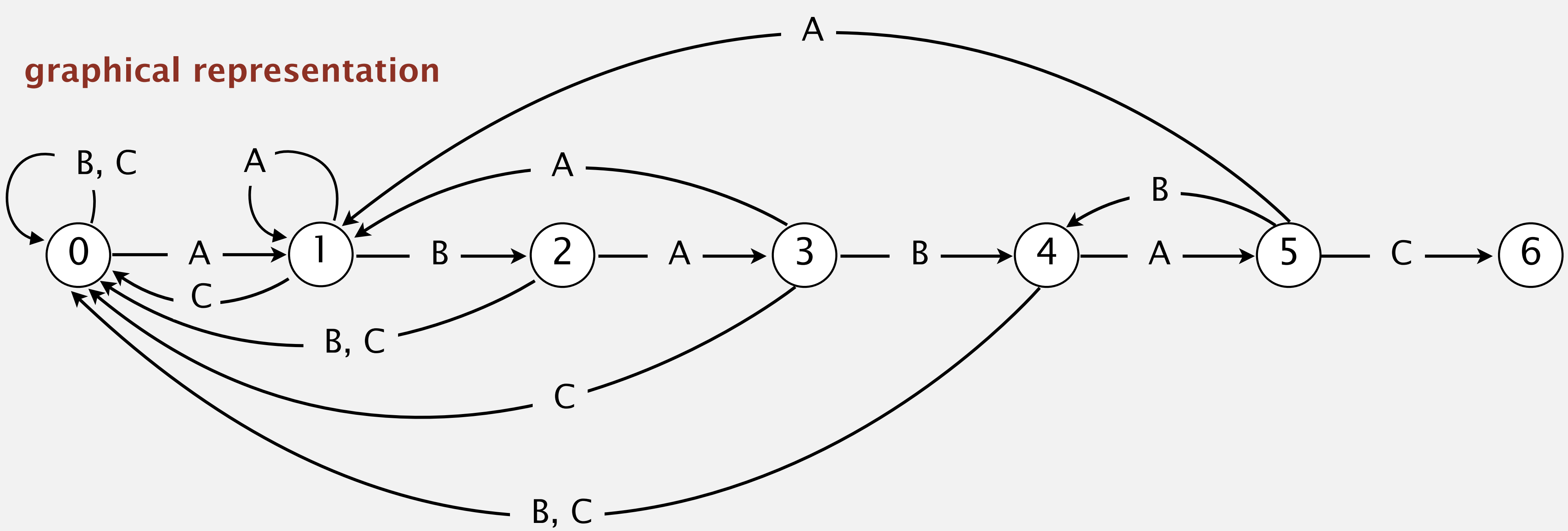
- Finite number of states (including start and halt).
- Exactly one state transition for each char in alphabet.
- Accept if sequence of state transitions leads to halt state.

**internal representation**

| | | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| `pat.charAt(j)` | | A | B | A | B | A | C |
| `dfa[][j]` | A | 1 | 1 | 3 | 1 | 5 | 1 |
| | B | 0 | 2 | 0 | 4 | 0 | 4 |
| | C | 0 | 0 | 0 | 0 | 0 | 6 |

If in state `j` reading char `c`:
    if `j` is 6 halt and accept
    else move to state `dfa[c][j]`

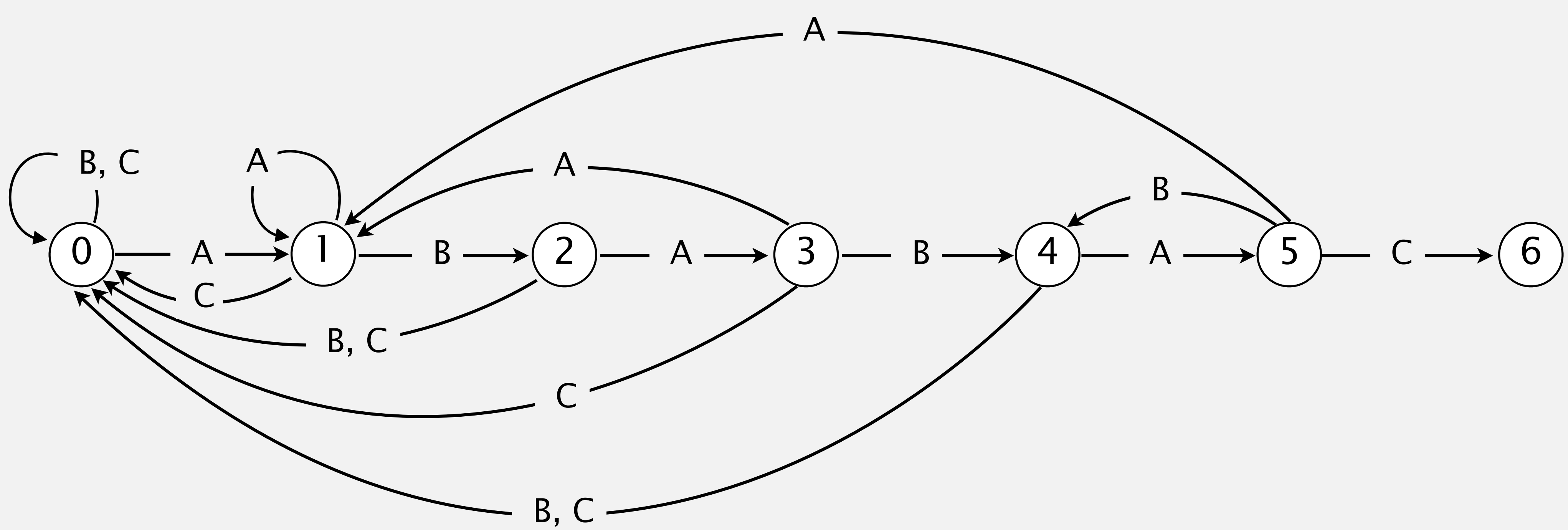
**graphical representation**



# Knuth–Morris–Pratt demo: DFA simulation

A A B A C A A B A B A C A A

| | | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| `pat.charAt(j)` | | A | B | A | B | A | C |
| | A | 1 | 1 | 3 | 1 | 5 | 1 |
| `dfa[][j]` | B | 0 | 2 | 0 | 4 | 0 | 4 |
| | C | 0 | 0 | 0 | 0 | 0 | 6 |



# Interpretation of Knuth-Morris-Pratt DFA

Q. What is interpretation of DFA state after reading in `txt[i]`?

A. State = number of characters in pattern that have been matched.

length of longest prefix of `pat[]` that is a suffix of `txt[0..i]`

Ex. DFA is in state 3 after reading in `txt[0..6]`.

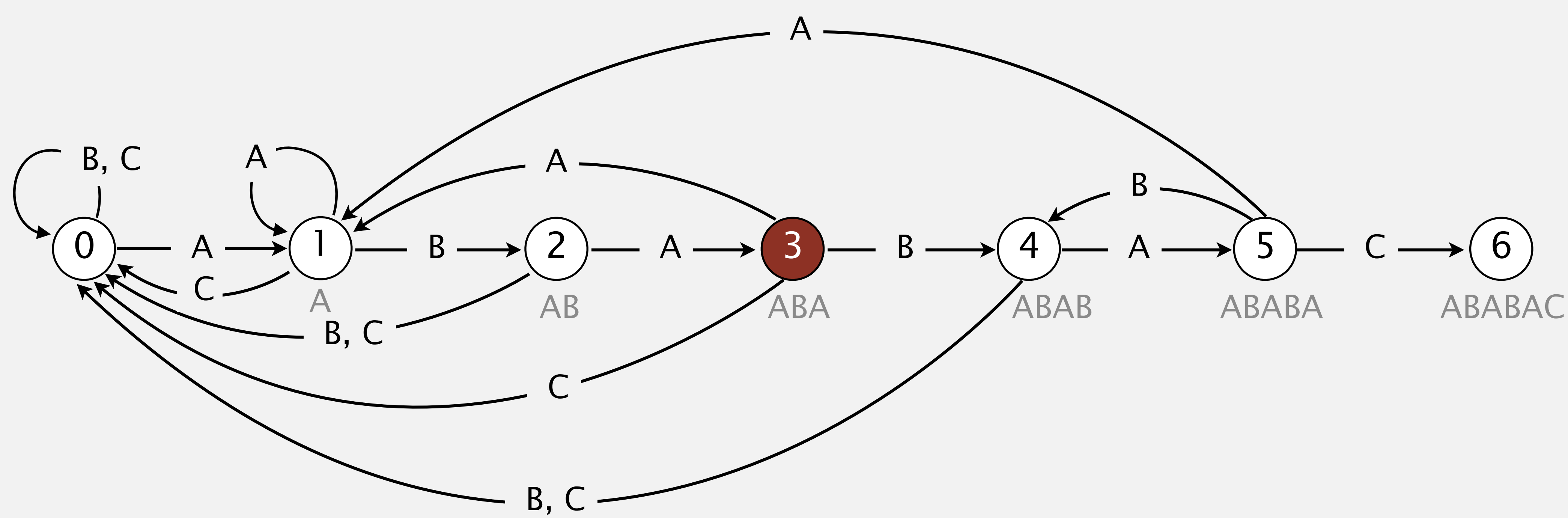
i

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| txt → | B | C | B | A | A | B | A | C | A |

suffix of `txt[0..6]`

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| pat → | A | B | A | B | A | C |

prefix of `pat[]`



## Substring search: quiz 2

**Which state is the DFA in after processing the following input?**

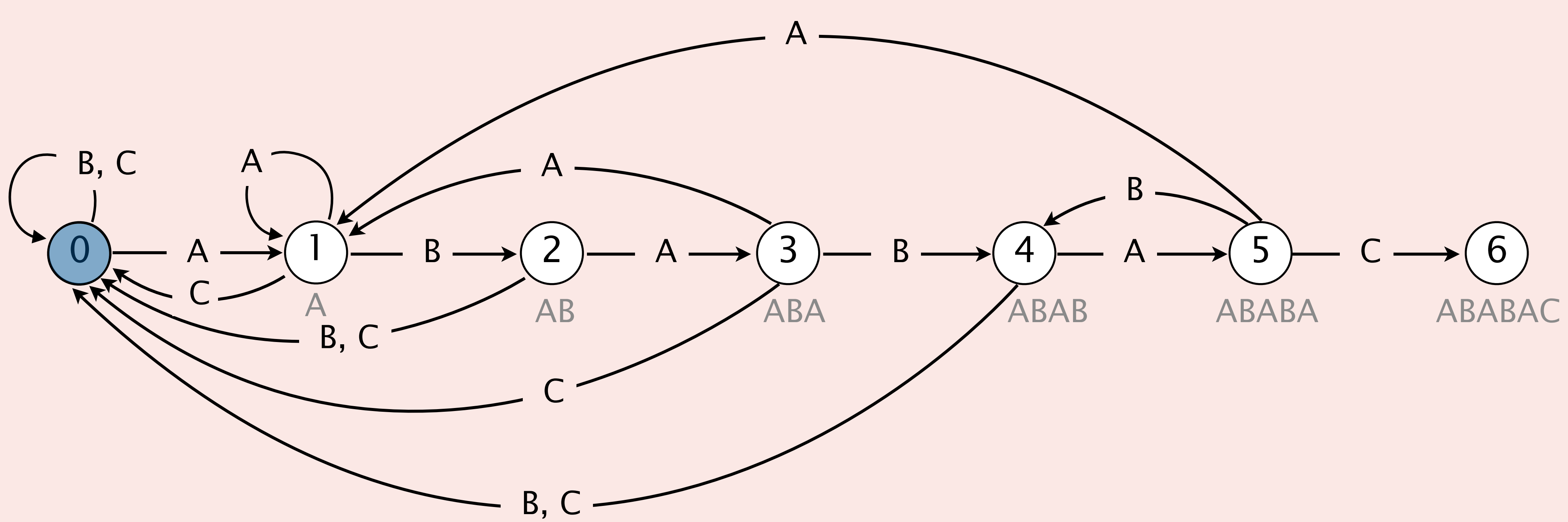
B A A B A B A B

**A.** 0

**B.** 1

**C.** 3

**D.** 4



# Substring search: quiz 3

**Which state is the DFA in after processing the following input?**

A B A A B B A B A B B A B A A B A A B A A A B A B A B A A B A A B A A B A B A B

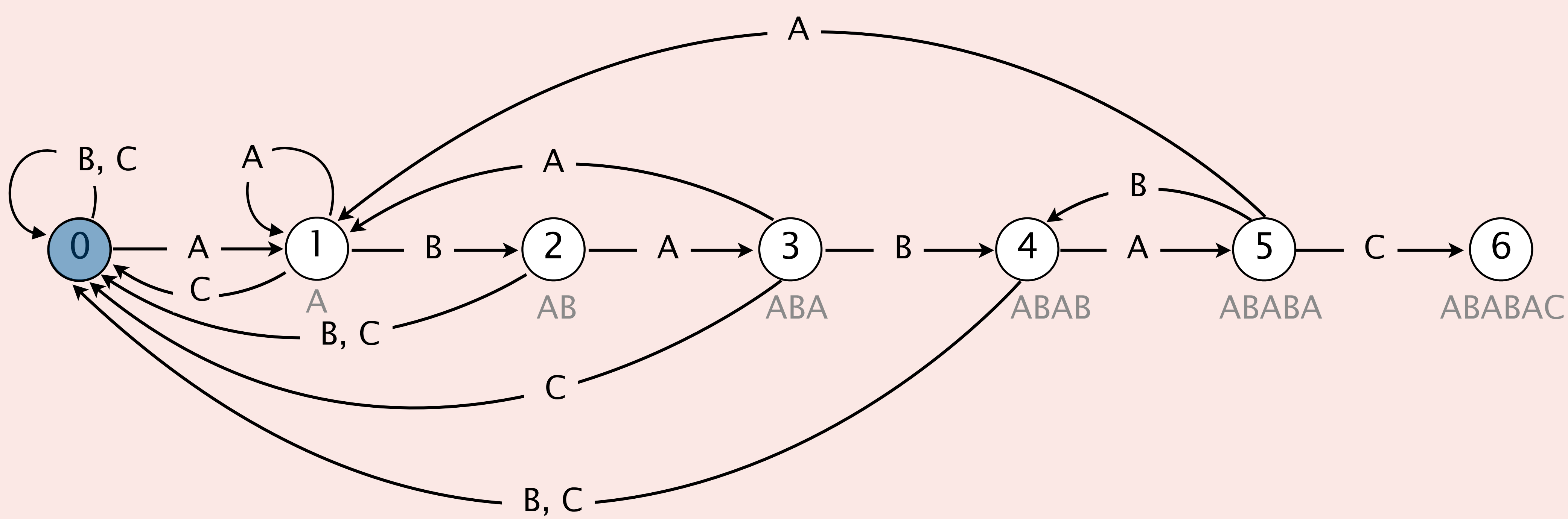
**A.** 0

**B.** 1

**C.** 3

**D.** 4

**E.** 5



# Knuth–Morris–Pratt substring search: Java implementation

## Key differences from brute-force implementation.

- Need to precompute `dfa[][]` from pattern.
- Each text character compared (at most) once.

stop on first match

```
public int search(String txt)
{
   int i, j, n = txt.length();
   for (i = 0, j = 0; i < n && j < m; i++)
      j = dfa[txt.charAt(i)][j];
   if (j == m) return i - m;
   else        return n;
}
```

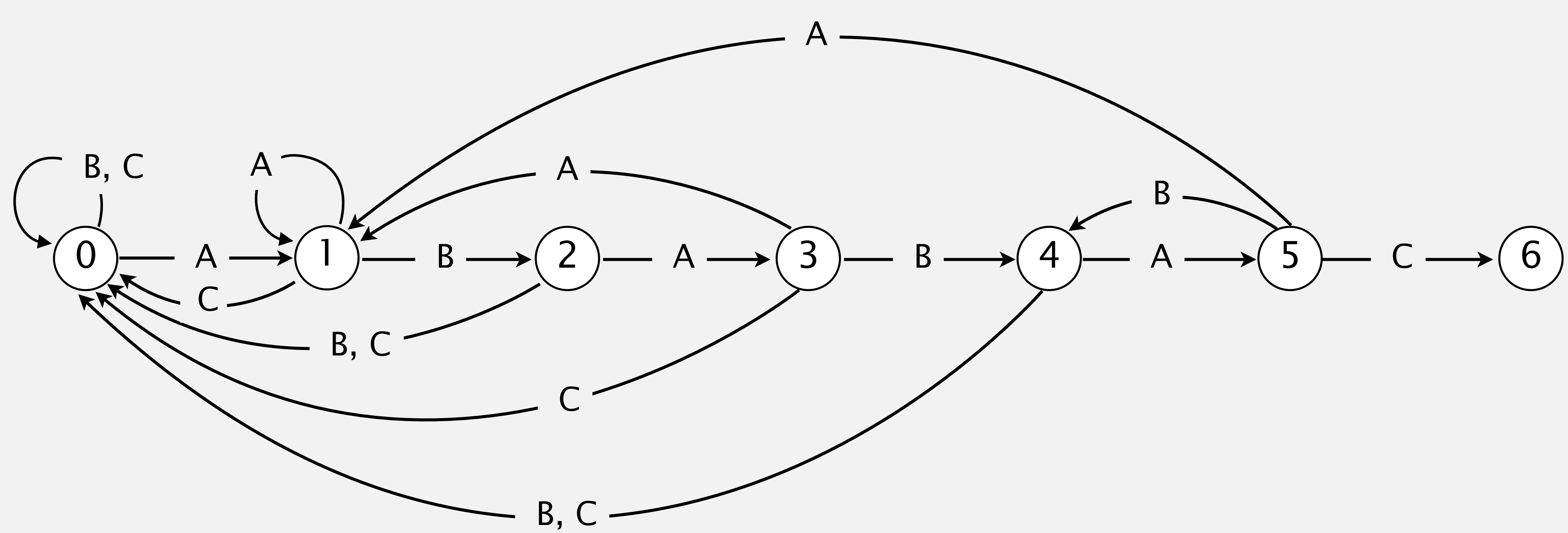
## Running time.

- Simulate DFA on text: at most $n$ character accesses.
- Build DFA: how to do efficiently? [warning: tricky algorithm ahead]

# Knuth–Morris–Pratt demo: DFA construction

| | | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| pat.charAt(j) | | A | B | A | B | A | C |
| dfa[][j] | A | 1 | 1 | 3 | 1 | 5 | 1 |
| | B | 0 | 2 | 0 | 4 | 0 | 4 |
| | C | 0 | 0 | 0 | 0 | 0 | 6 |

**Constructing the DFA for KMP substring search for A B A B A C**



# How to build DFA from pattern?

Include one state for each character in pattern (plus accept state).

| | | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| `pat.charAt(j)` | | A | B | A | B | A | C |
| `dfa[][j]` | A | | | | | | |
| | B | | | | | | |
| | C | | | | | | |

# How to build DFA from pattern?

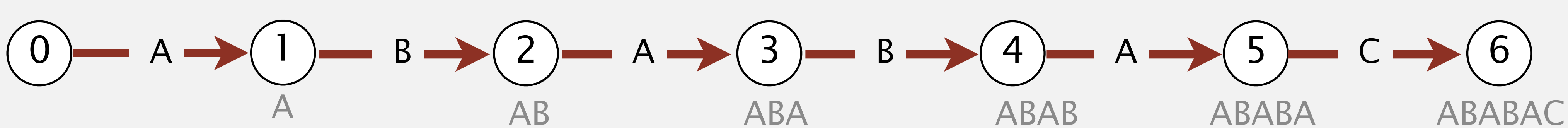
Match transition. If in state `j` and next char `c == pat.charAt(j)`, go to `j+1`.

first j characters of pattern have already been matched

next char matches

now first j +1 characters of pattern have been matched

| | | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| `pat.charAt(j)` | | A | B | A | B | A | C |
| `dfa[][j]` | A | 1 | | 3 | | 5 | |
| | B | | 2 | | 4 | | |
| | C | | | | | | 6 |



# How to build DFA from pattern?

**Mismatch transition.** If in state `j` and next char `c != pat.charAt(j)`, then the last `j-1` characters of input are `pat[1..j-1]`, followed by `c`.

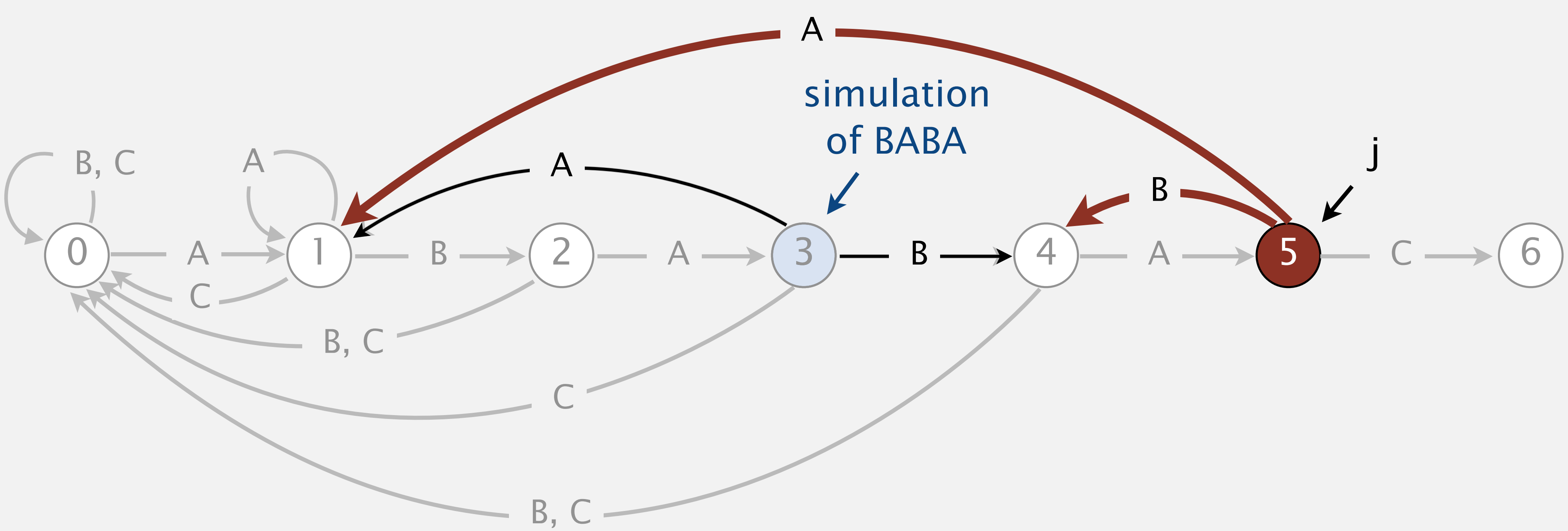
**To compute `dfa[c][j]`:** Simulate `pat[1..j-1]` on DFA and take transition `c`.
**Running time.** Seems to require `j` steps.

still under construction (!)

**Ex.** `dfa['A'][5] = 1`     `dfa['B'][5] = 4`

simulate BABAA     simulate BABAB

| j | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| pat.charAt(j) | A | B | A | B | A | C |



# How to build DFA from pattern?

**Mismatch transition.** If in state `j` and next char `c != pat.charAt(j)`, then the last `j-1` characters of input are `pat[1..j-1]`, followed by `c`.

state x

**To compute `dfa[c][j]`:** Simulate `pat[1..j-1]` on DFA and take transition `c`.
**Running time.** Takes only constant time if we maintain state x.

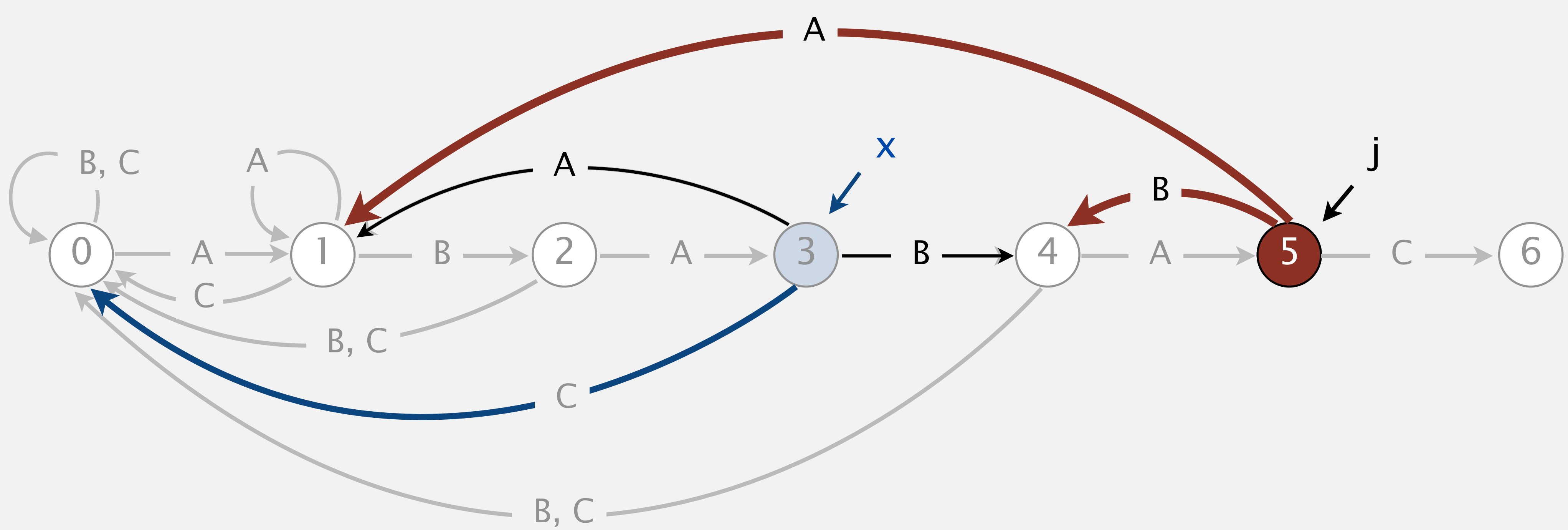
**Ex.** `dfa['A'][5] = 1`     `dfa['B'][5] = 4`     `x' = 0`

from state x,
take transition 'A'
= dfa['A'][x]

from state x,
take transition 'B'
= dfa['B'][x]

from state x,
take transition 'C'
= dfa['C'][x]

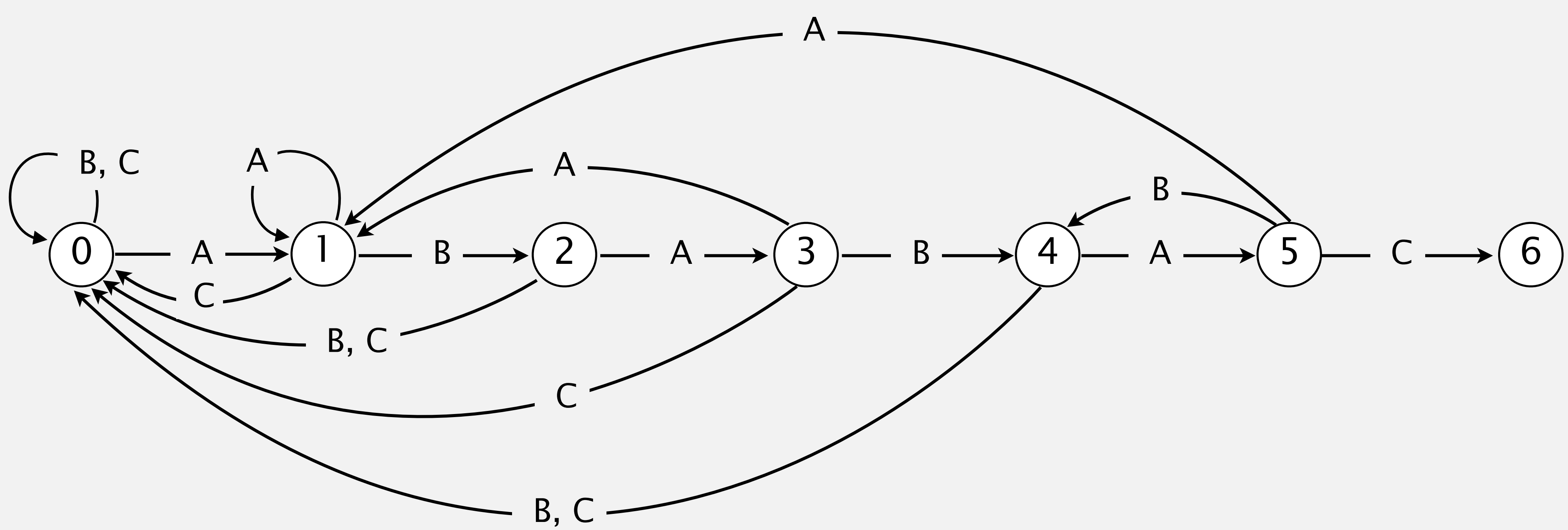
| 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| A | B | A | B | A | C |



# Knuth-Morris-Pratt demo: DFA construction in linear time

| | | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| pat.charAt(j) | | A | B | A | B | A | C |
| dfa[][j] | A | 1 | 1 | 3 | 1 | 5 | 1 |
| | B | 0 | 2 | 0 | 4 | 0 | 4 |
| | C | 0 | 0 | 0 | 0 | 0 | 6 |

**Constructing the DFA for KMP substring search for A B A B A C**



# Constructing the DFA for KMP substring search: Java implementation

For each state `j`:

- Copy `dfa[][x]` to `dfa[][j]` for mismatch case.
- Set `dfa[pat.charAt(j)][j]` to `j+1` for match case.
- Update `x`.

```
public KMP(String pat)
{
   this.pat = pat;
   m = pat.length();
   dfa = new int[R][m];
   dfa[pat.charAt(0)][0] = 1;
   for (int x = 0, j = 1; j < m; j++)
   {
      for (int c = 0; c < R; c++)
         dfa[c][j] = dfa[c][x];          // copy mismatch cases
      dfa[pat.charAt(j)][j] = j+1;       // set match case
      x = dfa[pat.charAt(j)][x];         // update restart state
   }
}
```

Running time. $m$ character accesses (but space/time proportional to $R\,m$).

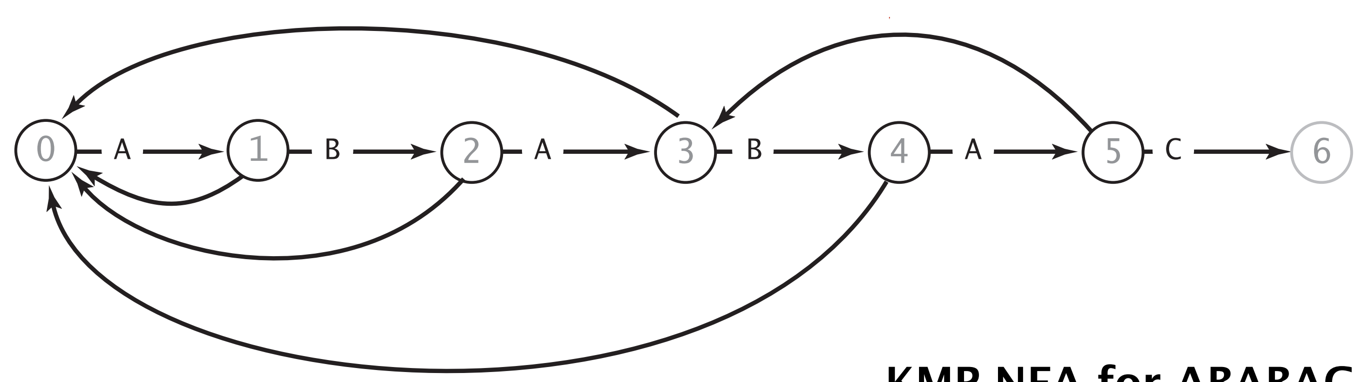
# KMP substring search analysis

Proposition. KMP substring search accesses no more than $m + n$ chars to search for a pattern of length $m$ in a text of length $n$.

Pf. Each pattern character accessed once when constructing the DFA; each text character accessed (at most) once when simulating the DFA.

Proposition. KMP constructs `dfa[][]` in time and space proportional to $R\, m$.

Larger alphabets. Improved version of KMP constructs `nfa[]` in time and space proportional to $m$.




KMP NFA for ABABAC

# Knuth–Morris–Pratt: brief history

- Independently discovered by two theoreticians and a hacker.
  - Knuth: inspired by esoteric theorem, discovered linear algorithm
  - Pratt: made running time independent of alphabet size
  - Morris: built a text editor for the CDC 6400 computer
- Theory meets practice.

SIAM J. COMPUT.
Vol. 6, No. 2, June 1977

**FAST PATTERN MATCHING IN STRINGS***

DONALD E. KNUTH†, JAMES H. MORRIS, JR.‡ AND VAUGHAN R. PRATT¶

**Abstract.** An algorithm is presented which finds all occurrences of one given string within another, in running time proportional to the sum of the lengths of the strings. The constant of proportionality is low enough to make this algorithm of practical use, and the procedure can also be extended to deal with some more general pattern-matching problems. A theoretical application of the algorithm shows that the set of concatenations of even palindromes, i.e., the language $\{\alpha\alpha^R\}^*$, can be recognized in linear time. Other algorithms which run even faster on the average are also considered.



Don Knuth


Jim Morris


Vaughan Pratt

# Cyclic Rotation

A string $s$ is a cyclic rotation of $t$ if $s$ and $t$ have the same length and $s$ is a suffix of $t$ followed by a prefix of $t$.

| yes | yes | no |
|---|---|---|
| `ROTATEDSTRING` | `ABABABBABBABA` | `ROTATEDSTRING` |
| `STRINGROTATED` | `BABBABBABAABA` | `GNIRTSDETATOR` |

Problem. Given two binary strings $s$ and $t$, design a linear-time algorithm to determine if $s$ is a cyclic rotation of $t$.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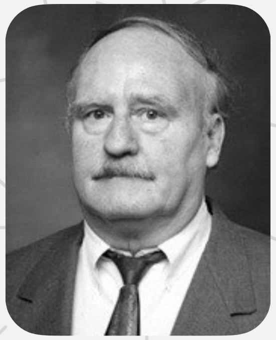

Robert Boyer

J. Strother Moore

# Boyer-Moore: mismatched character heuristic

## Intuition.

- Scan characters in pattern from right to left.
- Can skip as many as $m$ text chars when finding one not in the pattern.

| i | j | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *text* → | F | I | N | D | I | N | A | H | A | Y | S | T | A | C | K | N | E | E | D | L | E | I | N | A |
| 0 | 5 | N | E | E | D | L | E ← *pattern* | | | | | | | | | | | | | | | | | | |
| 5 | 5 | | | | | | N | E | E | D | L | E | | | | | | | | | | | | | |
| 11 | 4 | | | | | | | | | | | | N | E | E | D | L | E | | | | | | | |
| 15 | 0 | | | | | | | | | | | | | | | | N | E | E | D | L | E | | | |

*return* i = 15

**no S in pattern**

**align N in text with N in pattern**

**align N in text with N in pattern**

# Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 1. Mismatch character not in pattern.

```
                  i
before            ↓
txt     . . . . . . T L E . . . . . .
pat           N E E D L E

                      i
after                 ↓
txt     . . . . . . T L E . . . . . .
pat                   N E E D L E
```

**mismatch character T not in pattern: increment i one character beyond T**

# Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 2a. Mismatch character in pattern.

**before**

i ↓

txt . . . . . . N L E . . . . . .

pat N E E D L E

**after**

i ↓

txt . . . . . . N L E . . . . . .

pat N E E D L E

**mismatch character N in pattern: align text N with rightmost (why?) pattern N**

# Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 2b. Mismatch character in pattern (but heuristic no help).

```
                    i
before              ↓
txt     .  .  .  .  .  .  E  L  E  .  .  .  .  .  .
pat                 N  E  E  D  L  E
```

```
                 i
aligned with rightmost E?
                 ↓
txt     .  .  .  .  .  .  E  L  E  .  .  .  .  .  .
pat        N  E  E  D  L  E
```

**mismatch character E in pattern: align text E with rightmost pattern E ?**

# Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 2b. Mismatch character in pattern (but heuristic no help).

**before**

```
            i
            ↓
txt  . . . . . . E L E . . . . . .
pat        N E E D L E
```

**after**

```
              i
              ↓
txt  . . . . . . E L E . . . . . .
pat          N E E D L E
```

**mismatch character E in pattern: increment i by 1**

**Which text character is compared with the E next in Boyer–Moore?**

**A.** R (index 5)

**B.** O (index 6)

**C.** O (index 12)

**D.** O (index 13)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *text* → | B | O | O | Y | E | R | O | B | E | R | T | M | O | O | R | E | J | S |
| *pattern* → | M | O | O | R | E | | | | | | | | | | | | | |
| | | | | | M | O | O | R | E | | | | | | | | | |

# Substring search: quiz 6

**Which text character is compared with the E next in Boyer–Moore?**

**A.** O

**B.** R

**C.** E

**D.** J

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *text* → | B | O | O | Y | E | R | O | B | E | R | T | M | O | O | R | E | J | S |
| *pattern* → | M | O | O | R | E | | | | | | | | | | | | | |
| | | | | | M | O | O | R | E | | | | | | | | | |
| | | | | | | | | | M | O | O | R | E | | | | | |

# Boyer-Moore: mismatched character heuristic

Q. How much to skip?

A. Precompute index of rightmost occurrence of character c in pattern.
(–1 if character not in pattern)

```
right = new int[R];
for (int c = 0; c < R; c++)
   right[c] = -1;
for (int j = 0; j < m; j++)
   right[pat.charAt(j)] = j;
```

| | | N | E | E | D | L | E | |
|---|---|---|---|---|---|---|---|---|
| c | | 0 | 1 | 2 | 3 | 4 | 5 | right[c] |
| A | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| B | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| C | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| D | -1 | -1 | -1 | -1 | 3 | 3 | 3 | 3 |
| E | -1 | -1 | 1 | 2 | 2 | 2 | 5 | 5 |
| ... | | | | | | | | -1 |
| L | -1 | -1 | -1 | -1 | -1 | 4 | 4 | 4 |
| M | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| N | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ... | | | | | | | | -1 |

**Boyer-Moore skip table computation**

# Boyer-Moore: Java implementation

```
public int search(String txt)
{
   int n = txt.length();
   int m = pat.length();
   int skip;
   for (int i = 0; i <= n-m; i += skip)
   {
      skip = 0;
      for (int j = m-1; j >= 0; j--)
      {
         if (pat.charAt(j) != txt.charAt(i+j))
         {
            skip = Math.max(1, j - right[txt.charAt(i+j)]);
            break;
         }
      }
      if (skip == 0) return i;
   }
   return n;
}
```

compute skip value

in case other term is zero or negative

match

# Boyer-Moore: analysis

Property. Substring search with the Boyer–Moore mismatched character heuristic takes about $\sim n / m$ character compares to search for a pattern of length $m$ in a text of length $n$.

sublinear!

Worst-case. Can be as bad as $\sim m\, n$.

| i | skip | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | txt → | B | B | B | B | B | B | B | B | B | B | |
| 0 | 0 | | A | B | B | B | B | | | | | | ← pat |
| 1 | 1 | | | A | B | B | B | B | | | | | |
| 2 | 1 | | | | A | B | B | B | B | | | | |
| 3 | 1 | | | | | A | B | B | B | B | | | |
| 4 | 1 | | | | | | A | B | B | B | B | | |
| 5 | 1 | | | | | | | A | B | B | B | B | |

Boyer–Moore variant. Can improve worst case to $\sim 3\, n$ character compares by adding a KMP-like rule to guard against repetitive patterns.

# Substring search: quiz 7

**Which substring search algorithm does Java's indexOf() method use?**

**A.** Brute-force search

**B.** Knuth–Morris–Pratt

**C.** Boyer–Moore

**D.** Rabin–Karp

**indexOf**

`public int indexOf(String str)`

Returns the index within this string of the first occurrence of the specified substring.

The returned index is the smallest value k for which:

```
 this.startsWith(str, k)
```

If no such value of k exists, then -1 is returned.

**Parameters:**

`str` - the substring to search for.

**Returns:**

the index of the first occurrence of the specified substring, or -1 if there is no such occurrence.