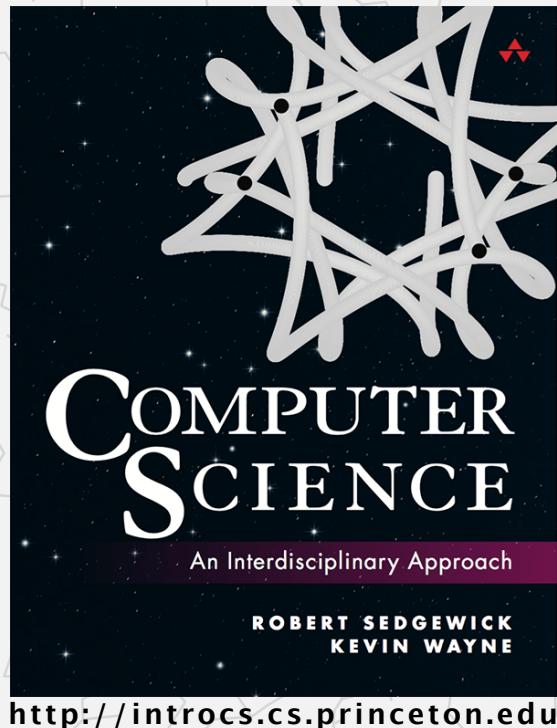
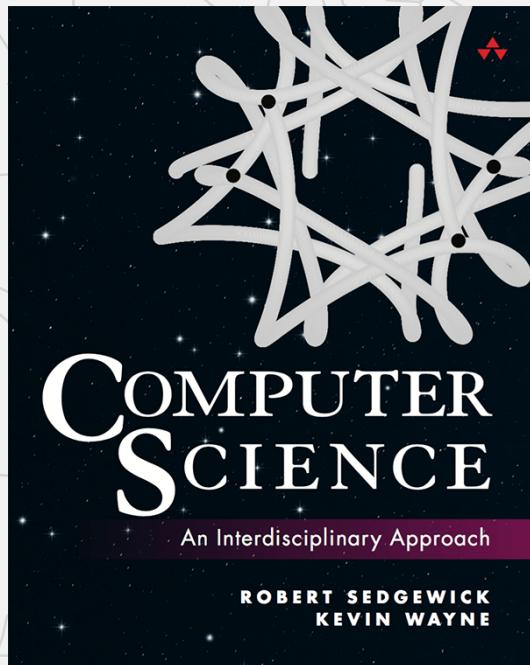


N-BODY TIPS AND TRICKS

- *n-body simulation*
- *problem decomposition*
- *the physics*
- *bugs*
- *universes*





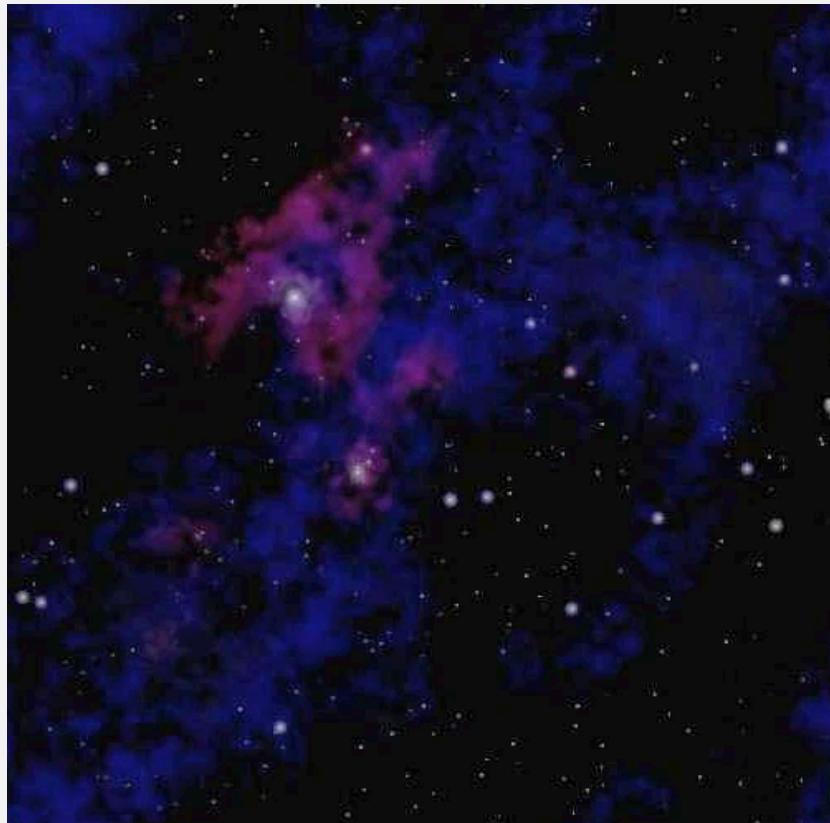
<http://introcs.cs.princeton.edu>

N-BODY TIPS AND TRICKS

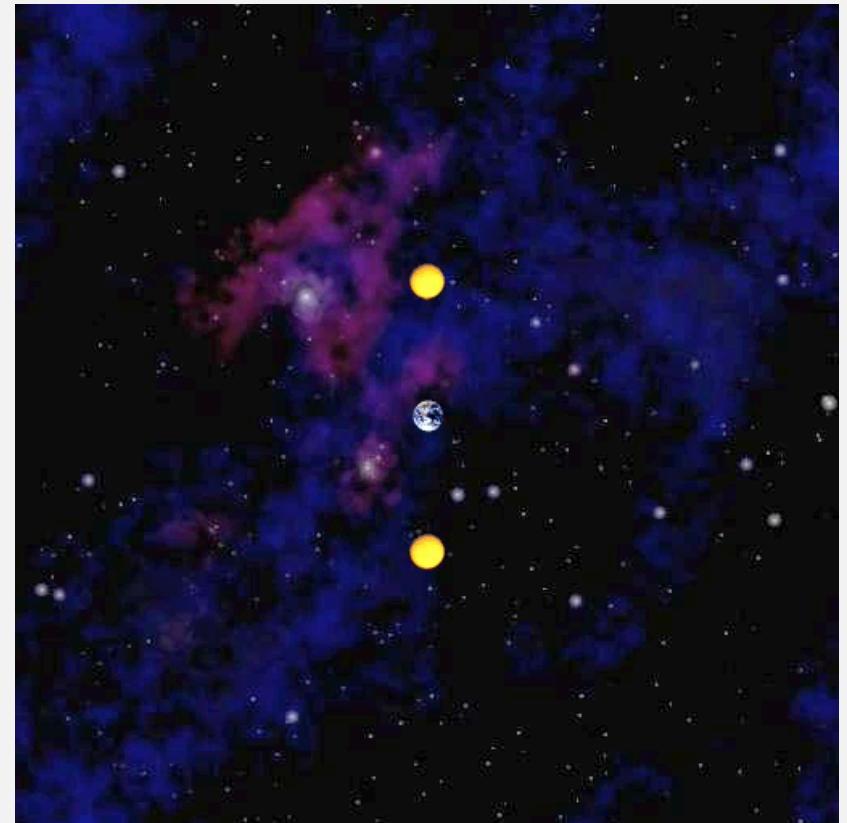
- ▶ *overview*
- ▶ *problem decomposition*
- ▶ *the physics*
- ▶ *bugs*
- ▶ *universes*

N-body simulation

Simulate the motion of n bodies, subject to Newton's laws.



`planets.txt`



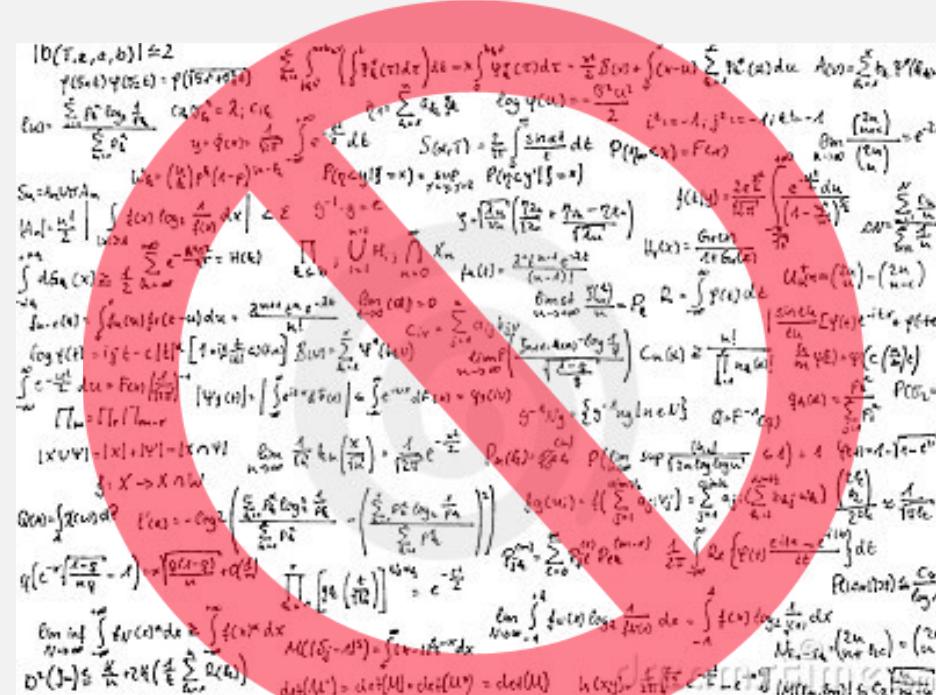
`3body.txt`

Physics and math

Newton's law of gravity. $F = \frac{G m_1 m_2}{r^2}$

Newton's second law of motion. $F = m a$

“Leapfrog” method. For numerical integration of differential equations.



$$|D(t, x, a, b)| \approx 2$$

$$\gamma(s, u) \gamma(s, v) = \gamma(s + s, u + v)$$

$$\sum_{k=0}^{n-1} \int_{t_k}^{t_{k+1}} \left(\int_s^{s+k} \gamma(\tau) d\tau \right) ds = s \int_s^{s+n} \gamma(\tau) d\tau - \frac{s}{2} \int_s^{s+n} \gamma'(u) du \quad A_{n,0} = \sum_{k=0}^{n-1} h_k \gamma'(h_k)$$

$$L_{n,0} = \frac{\sum_{k=0}^{n-1} h_k \log \frac{h_k}{2}}{\sum_{k=0}^{n-1} h_k^2} \quad c_0 = 2; c_1 = \int_0^{\infty} \sum_{k=0}^{n-1} a_k \frac{u^k}{k!} \log u du \quad \log \gamma(u) = -\frac{c_1 u^2}{2}$$

$$S(x, t) = \frac{t}{2} \int_{x-t}^{x+t} dt \quad P(\{y < x\}) = F(x) \quad \frac{\partial}{\partial x} S(x, t) = \frac{t}{2} e^{-2x}$$

$$W_k = \left(\frac{h}{k} \right)^k (e - e^{-h})^{n-k} \quad P(y < y' | y = x) = \sup_{y' > y} P(y < y' | y = x)$$

$$S_n = \lambda_n V \lambda_n \quad \int_{x-h}^x \frac{f(x)}{f(u)} \log \frac{f(x)}{f(u)} du \leq E \quad g^{-1}(g - c) =$$

$$M_n = \frac{h_n}{2} \int_{x-h_n}^x \prod_{k=0}^{n-1} \sum_{j=0}^{2^k} H_{j, k} \prod_{m=0}^{2^k-1} X_m \quad f(x) = \frac{2 \pi E}{(4 - 2x)^{1/2}} \quad U(x) = \frac{G(x)}{2 \pi \sin(2x)}$$

$$\int_x^y d\ln_k(x) \geq \frac{k}{2} h_n \quad f(x) = \frac{2^{n-1} \pi^{1/2} e^{-2x}}{(4 - 2x)^{1/2}}$$

$$I_n = \int_{x-h_n}^x \int_u^y f(u) f(x-u) du + \frac{2^{n-1} \pi^{1/2} e^{-2x}}{h_n!} \quad D_{n,0} = 0 \quad \int_{x-h_n}^x \int_u^y \sigma_{n,0} f(u) du + P_n = 2 \int_x^y p(x) dx \quad U_n = \left(\frac{h_n}{2} \right)^2$$

$$\log f(x) = \log x - c_1 x^2 \left[1 + \frac{c_1}{2} \log x + \dots \right] \quad C_{n,0} = \sum_{j=0}^{n-1} \sum_{i=0}^{2^j-1} \frac{H_{i,j} \log^j x}{i!}$$

$$\int_x^y e^{-\frac{u^2}{2}} du = F(x) \frac{e^{-x^2/2}}{\sqrt{\pi}} \quad \left[q_1(u) + \int_u^y e^{-v^2/2} dv \right] = q_1(y) \quad C_n(x) = \frac{h_n!}{\prod_{m=0}^{n-1} (m+1)!} \frac{h_n^{2n}}{n!} \gamma(c) \frac{c}{c-1}$$

$$\prod_{m=0}^{n-1} (m+1) = \prod_{m=0}^{n-1} \Gamma(m+1) \quad \left[q_1(u) + \int_u^y e^{-v^2/2} dv \right] = q_1(y) \quad q_{n,0} = \frac{P_n}{2^{n-1} \pi^{1/2}} \quad P_{n,0} =$$

$$|XUV| = |X| + |Y| - |X \cap Y| \quad \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^{n-1} \frac{1}{h_k} \log \left(\frac{x}{h_k} \right) = \frac{1}{2} \int_0^\infty e^{-t^2/2} dt \quad P_n(x) = \frac{c_0}{2^{n-1} \pi^{1/2}} \quad P_{n,0} = \frac{c_0}{2^{n-1} \pi^{1/2}} \quad P_{n,0} = \frac{c_0}{2^{n-1} \pi^{1/2}} \quad P_{n,0} = \frac{c_0}{2^{n-1} \pi^{1/2}}$$

$$Q(x) = \int_X Q(x) d\mu \quad P(x) = -\log \left(\frac{\sum_{j=0}^{n-1} a_j \log \frac{x}{h_j}}{\sum_{j=0}^{n-1} h_j} - \left(\frac{\sum_{j=0}^{n-1} a_j h_j}{\sum_{j=0}^{n-1} h_j} \right)^2 \right) \quad J_{n,0} = \left(\sum_{j=0}^{n-1} a_j v_j \right) \frac{\log h_n}{h_n} \quad \frac{d}{dt} \int_{x-t}^{x+t} dt = \frac{1}{2} \int_{x-t}^{x+t} \left(\frac{2x}{t} \right) \frac{e^{-t^2/2}}{t} dt \quad P_{n,0} = \frac{c_0}{2^{n-1} \pi^{1/2}}$$

$$q(e^{-\frac{|x-y|}{2}} - 1) = \frac{q(x-y)}{2} + O(\frac{1}{n}) \quad \prod_{k=0}^{n-1} \left[\frac{1}{h_k} \left(\frac{x}{h_k} \right)^{a_k} \right] = e^{-\frac{x^2}{2}} \quad \int_{x-h_n}^x \int_u^y \left(\frac{2x}{t} \right) \frac{e^{-t^2/2}}{t} dt = \frac{c_0}{2^{n-1} \pi^{1/2}}$$

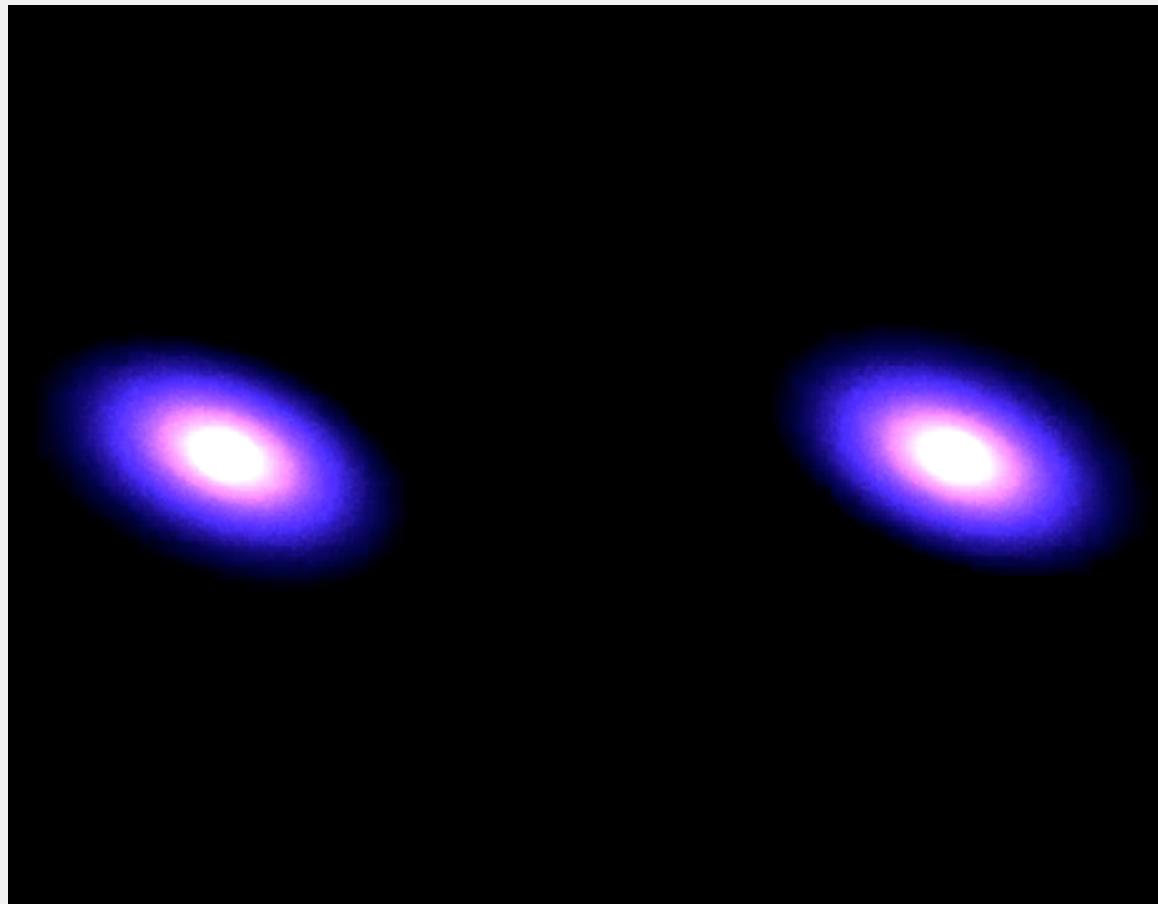
$$\liminf_{n \rightarrow \infty} \frac{1}{n} \int_X f(x) d\mu \geq \int_X f(x) dx \quad M((U_j, U)) = \int_{U_j \times U} f(x) dx \quad \lim_{n \rightarrow \infty} \int_{x-h_n}^x \int_u^y \log \frac{1}{t} dt = \int_{x-h_n}^x \int_u^y \log \frac{1}{t} dt \quad M_{n,0} = \left(\frac{2h_n}{\pi} \right)^2 \quad M_{n,0} = \left(\frac{2h_n}{\pi} \right)^2$$

$$D^*(J) \leq \frac{K}{n} \log \left(\frac{1}{n} \sum_{k=0}^{n-1} Q(h_k) \right) \quad d\mu(M) = d\mu(U) + d\mu(U') = d\mu(U) \quad h(x) = \int_{\Omega} \int_{\Omega'} \int_{\Omega''} \int_{\Omega'''} \int_{\Omega''''} \int_{\Omega'''''} \int_{\Omega''''''} \int_{\Omega'''''''}$$

don't worry (this is not a math or physics course)

Context

Applications. Cosmology, semiconductors, fluid dynamics,



http://www.youtube.com/watch?v=ua7YIN4eL_w

Programming goals

- Use standard input, standard output, and standard drawing for I/O.
- Use parallel arrays.
- Decompose a large program into small, manageable steps.



Before you begin

Carefully read assignment specification; skim checklist.

Check that standard libraries are available to Java.

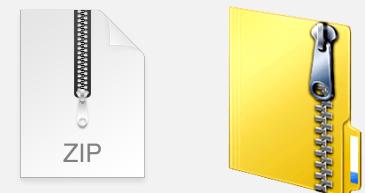
- Already configured if you used auto-installer
- Remember to use javac-introcs and java-introcs in Terminal

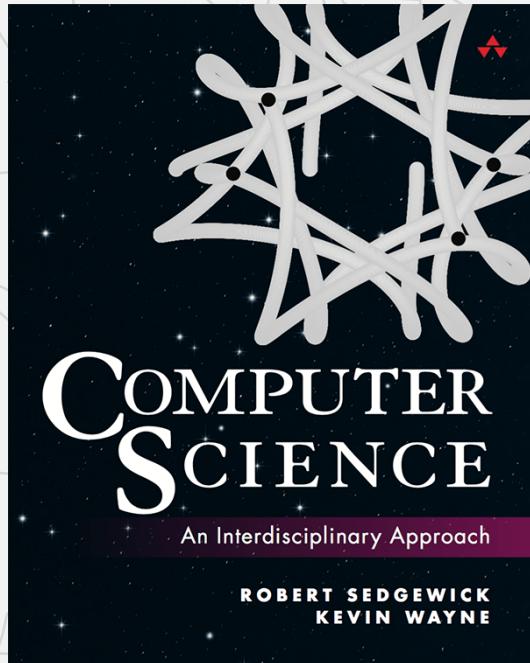
Useful programs from lecture/precept.

- Students.java
- BouncingBallDeluxe.java
- Distinct.java

Download the project files: sample data files and create working directory.

- Download nbody.zip from assignment specification.
(see Assignment FAQ for extracting zip file)
- Have all data files available in a folder





N-BODY TIPS AND TRICKS

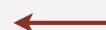
- *overview*
- *problem decomposition*
- *the physics*
- *bugs*
- *universes*

<http://introcs.cs.princeton.edu>

Decompose problem into individual steps

Develop program incrementally, decomposing into six individual steps.

1. Parse command-line arguments
2. Read universe from standard input
3. Initialize standard drawing
4. Play music on standard audio
5. Simulate the universe
 - A. Calculate net forces
 - B. Update velocities and positions
 - C. Draw universe to standard drawing
6. Print universe to standard output



physics localized to these steps
(formulas provided)

Advice. Although final code will appear in order 1–6, we recommend implementing these steps in the order 1, 2, 6, 3, 4, 5B, 5C, 5A

- Q.** Why?
A. Easier to test and debug

Start with comments

```
public class NBody {  
    public static void main(String[] args) {  
  
        // Step 1. Parse command-line arguments.  
  
        // Step 2. Read universe from standard input.  
  
        // Step 3. Initialize standard drawing.  
  
        // Step 4. Play music on standard audio.  
  
        // Step 5. Simulate the universe.  
  
        // Step 5A. Calculate net forces.  
        // Step 5B. Update velocities and positions.  
        // Step 5C. Draw universe to standard drawing.  
  
        // Step 6. Print universe to standard output.  
    }  
}
```

Command-line arguments

Step 1. Parse command-line arguments

- Read stopping time T and increment Δt from command line
- Print values of each variable (as debugging aid)

Note. Easy, but you should still test it!

```
> java-introcs NBody 10 1
```

```
tau = 10.0
```

```
dt = 1.0
```

```
> java-introcs NBody 157788000.0 25000.0
```

```
tau = 1.57788E8
```

```
dt = 25000.0
```

Standard input

Step 2. Read universe from standard input

> less planets.txt

5 ← number of bodies n

2.50e+11 ← radius of universe

1.4960e+11 0.0000e+00 0.0000e+00 2.9800e+04 5.9740e+24 earth.gif

2.2790e+11 0.0000e+00 0.0000e+00 2.4100e+04 6.4190e+23 mars.gif

data for 5.7900e+10 0.0000e+00 0.0000e+00 4.7900e+04 3.3020e+23 mercury.gif
one body

0.0000e+00 0.0000e+00 0.0000e+00 0.0000e+00 1.9890e+30 sun.gif

1.0820e+11 0.0000e+00 0.0000e+00 3.5000e+04 4.8690e+24 venus.gif

initial x- and y-position

initial x- and y-velocity

mass

image filename

This file contains the sun and the inner 4 planets of our Solar System.

optional description

Standard input

Step 2. Read universe from standard input

- Read number of bodies n from standard input
- Read $radius$ of universe standard input
- Create 6 parallel arrays, each of length n , to store the 6 pieces of information characterizing a body
- Read data associated with each body and store in parallel arrays

Hint. Recall Students.java

```
> java-introcs NBody 157788000.0 25000.0 < planets.txt
```

[no output]

Q. How to test?

A. Do Step 6 (print universe)

Standard output

Step 6. Print universe to standard output

- Write a loop to iterate over the 6 parallel arrays
- Use StdOut.printf() for formatted output (see checklist for hint)

```
> java-introcs NBody 157788000.0 25000.0 < planets.txt
```

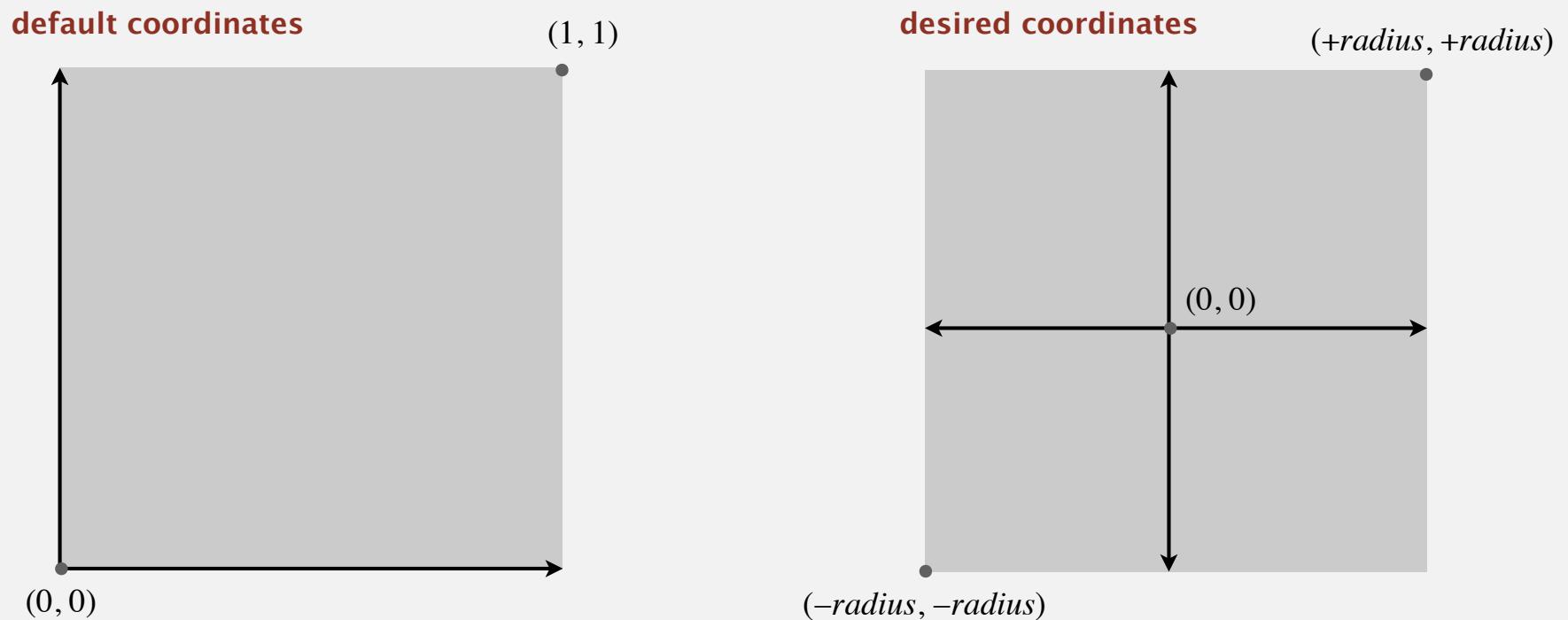
```
5
2.50e+11
1.4960e+11 0.0000e+00 0.0000e+00 2.9800e+04 5.9740e+24 earth.gif
2.2790e+11 0.0000e+00 0.0000e+00 2.4100e+04 6.4190e+23 mars.gif
5.7900e+10 0.0000e+00 0.0000e+00 4.7900e+04 3.3020e+23 mercury.gif
0.0000e+00 0.0000e+00 0.0000e+00 0.0000e+00 1.9890e+30 sun.gif
1.0820e+11 0.0000e+00 0.0000e+00 3.5000e+04 4.8690e+24 venus.gif
```

Standard drawing

Step 3. Initialize standard drawing.

- Enable double buffering by calling `StdDraw.enableDoubleBuffering()`
- Default x - and y -scale supports coordinates between 0 and 1
- Change scale to be between $-radius$ and $+radius$

Hint: `StdDraw.setXscale()` and `StdDraw.setYscale()`



Q. How to test?

Standard audio

Step 4. Play music.

- Call StdAudio.play("2001.wav")
- Easy (but optional)

Also sprach Zarathustra!

R. STRAUSS
arr. by JASON LIU
(LTS_1990)

Sehr breit. ♩ = 69

Piano

pp tremolo

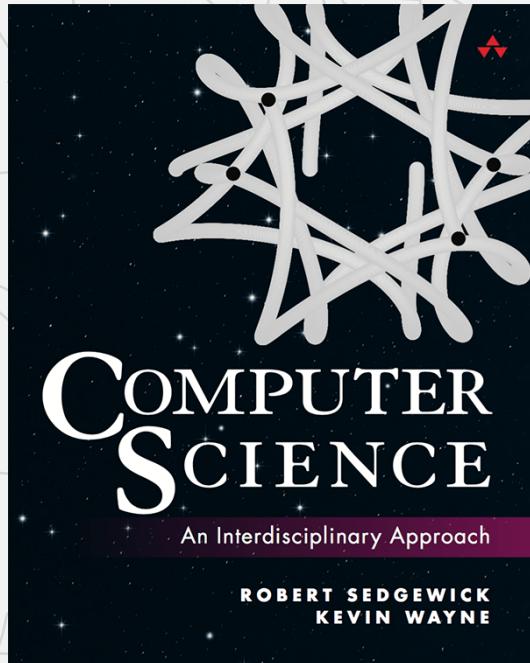
p (feierlich)

f

sempre pp

> p





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N-BODY TIPS AND TRICKS

- ▶ *overview*
- ▶ *problem decomposition*
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- ▶ *bugs*
- ▶ *universes*

The simulation loop (the "big time loop")

Step 5. Simulate the universe. At each time step t :

- A. Calculate the net force on each body
 - B. Update the velocities and positions
 - C. Draw the universe
-
- Q.** In which order should I implement these 3 sub-steps?
- A. 5B, 5C, 5A because calculating forces is hardest
-
- Q.** Can I interleave steps 5A, 5B, and 5C?
- A. No. Not only is it bad design, but it ruins the physics
(need position of all bodies at time t , not some at time $t + \Delta t$)

Hint. See `BouncingBallDeluxe.java`

Measuring time

Time loop. From $t = 0$ up to (but not including) T , incrementing by Δt

Hint. Easy, but also easy to get wrong. \Rightarrow Test!

$$T = 23.0, \Delta t = 2.5$$

$$t = 0.0$$

$$t = 2.5$$

$$t = 5.0$$

$$t = 7.5$$

$$t = 10.0$$

$$t = 12.5$$

$$t = 15.0$$

$$t = 17.5$$

$$t = 20.0$$

$$t = 22.5$$

$$T = 25.0, \Delta t = 2.5$$

$$t = 0.0$$

$$t = 2.5$$

$$t = 5.0$$

$$t = 7.5$$

$$t = 10.0$$

$$t = 12.5$$

$$t = 15.0$$

$$t = 17.5$$

$$t = 20.0$$

$$t = 22.5$$



don't include 25.0

Updating the velocities and positions

Step 5B. [for now, forces and accelerations are 0]

- Update the velocity of each body: $v_x = v_x + a_x \Delta t$, $v_y = v_y + a_y \Delta t$
- Update the position of each body: $p_x = p_x + v_x \Delta t$, $p_y = p_y + v_y \Delta t$

Warning. Cut-and-paste errors are common

Q. How to test?

A. Artificial universe that is easy to check by hand

> **java-introcs NBody 192 1 < 3body-zero-gravity.txt**

3

5.12e+02

1.9200e+02	1.9200e+02	1.0000e+00	1.0000e+00	1.0000e-30	earth.gif
5.1200e+02	1.9200e+02	2.0000e+00	1.0000e+00	1.0000e-40	venus.gif
1.9200e+02	5.1200e+02	1.0000e+00	2.0000e+00	1.0000e-50	mars.gif

Drawing the universe

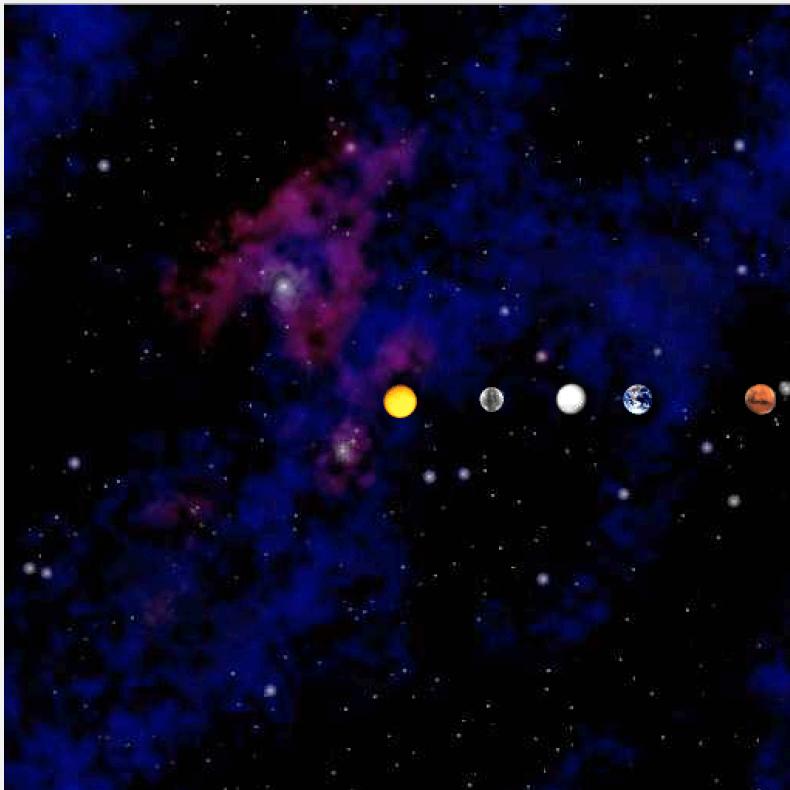
Step 5C.

- Draw background image
- Write loop to display n bodies
- Call `StdDraw.show()` to display results on screen
- Call `StdDraw.pause(20)` to control animation speed

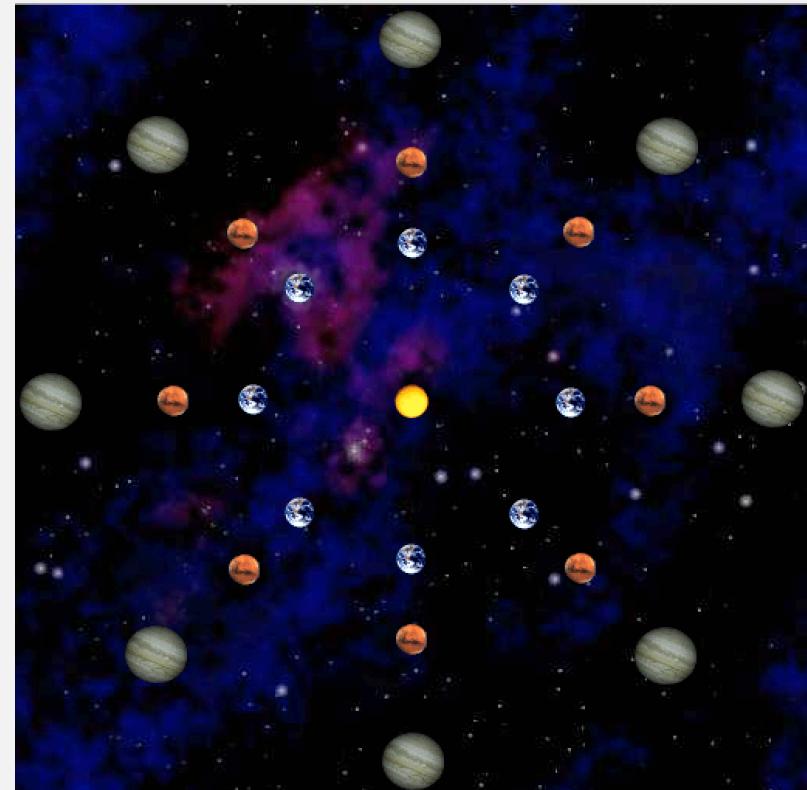
Drawing the universe

Step 5C.

- Draw background image
- Write loop to display n bodies
- Call `StdDraw.show()` to display results on screen
- Call `StdDraw.pause(20)` to control animation speed



`planets.txt`



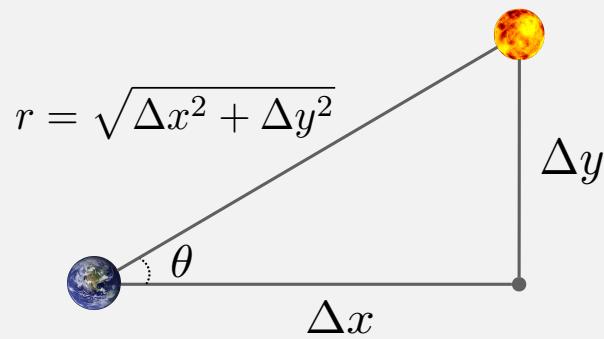
`kaleidoscope.txt`

Calculating the force (between two bodies at time t)

Step 5A.

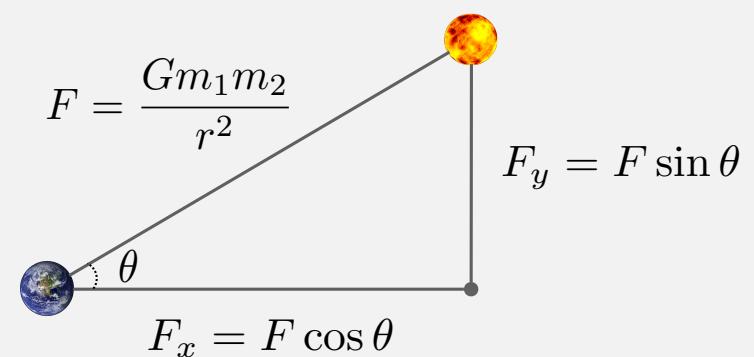
- Apply Newton's law of gravity
- A bit of high-school trig (formulas provided)

distance between two bodies



$$\cos \theta = \frac{\Delta x}{r}, \quad \sin \theta = \frac{\Delta y}{r}$$

force between two bodies



Calculating the force (between all pairs of bodies at time t)

Principle of superposition. Add all pairwise forces.

$$\vec{F}_{earth} = \vec{F}_{mars \rightarrow earth} + \vec{F}_{mercury \rightarrow earth} + \vec{F}_{sun \rightarrow earth} + \vec{F}_{venus \rightarrow earth}$$

How to implement?

- Need two extra arrays fx[] and fy[]. Why?
- Need to examine all pairs of bodies, ala Distinct.java.

Warmup. Enumerate all pairs of bodies.

n = 5

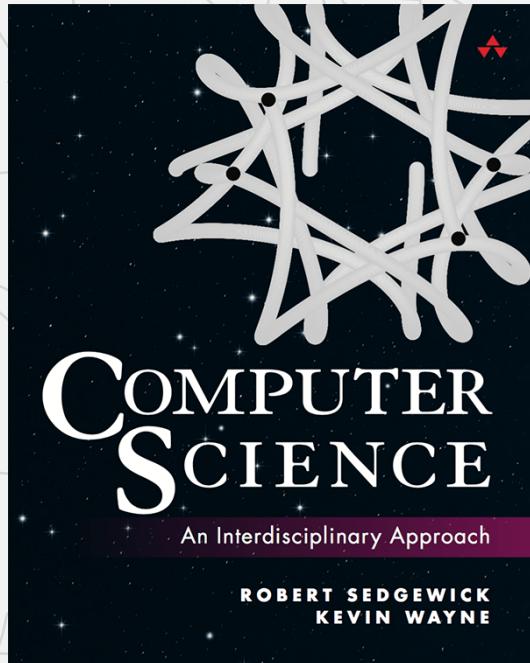
0-1 0-2 0-3 0-4
1-0 1-2 1-3 1-4
2-0 2-1 2-3 2-4
3-0 3-1 3-2 3-4
4-0 4-1 4-2 4-3

n = 4

0-1 0-2 0-3
1-0 1-2 1-3
2-0 2-1 2-3
3-0 3-1 3-2



don't include 0-0, 1-1, 2-2, or 3-3



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N-BODY TIPS AND TRICKS

- ▶ *overview*
- ▶ *problem decomposition*
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- ▶ *bugs*
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Advice - Keys to Becoming a Good Programmer

- ▶ Develop code incrementally; test after each step
- ▶ Test, test, test
- ▶ Take your time!
- ▶ Start early!
- ▶ Seek help if you get stuck
- ▶ Write outline of code (using comments) first; fill in code later

Command-line bug

```
> java-introcs NBody 157788000.0 25000.0 > planets.txt
```

```
<Ctrl-C>
```

```
> java-introcs NBody 157788000.0 25000.0 < planets.txt
```

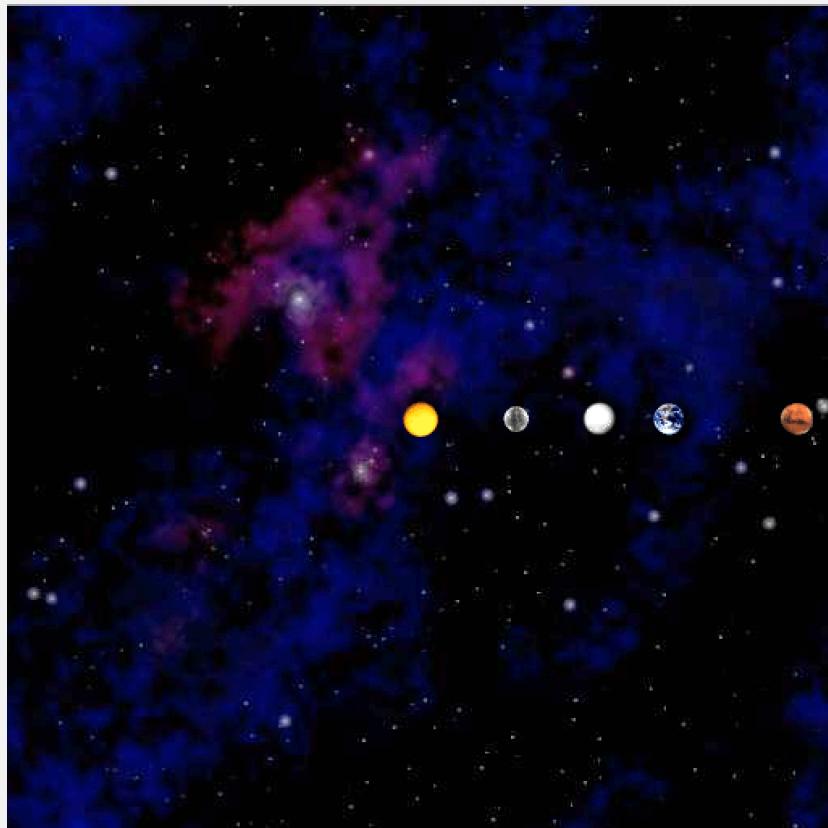
```
Exception in thread "main" java.util.NoSuchElementException
```

```
  at java.util.Scanner.throwFor(Scanner.java:907)
  at java.util.Scanner.next(Scanner.java:1530)
  at java.util.Scanner.nextInt(Scanner.java:2160)
  at java.util.Scanner.nextInt(Scanner.java:2119)
  at StdIn.readInt(StdIn.java:319)
  at NBody.main(NBody.java:54)
```

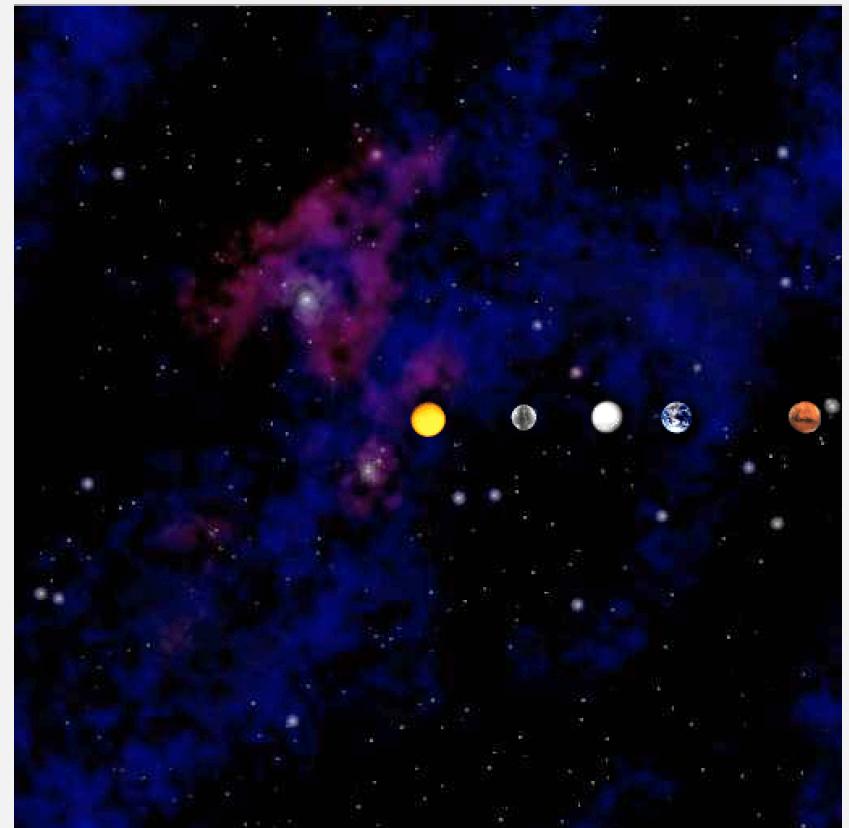
```
> less planets.txt
```

[it's empty - you erased it!]

Visual bugs

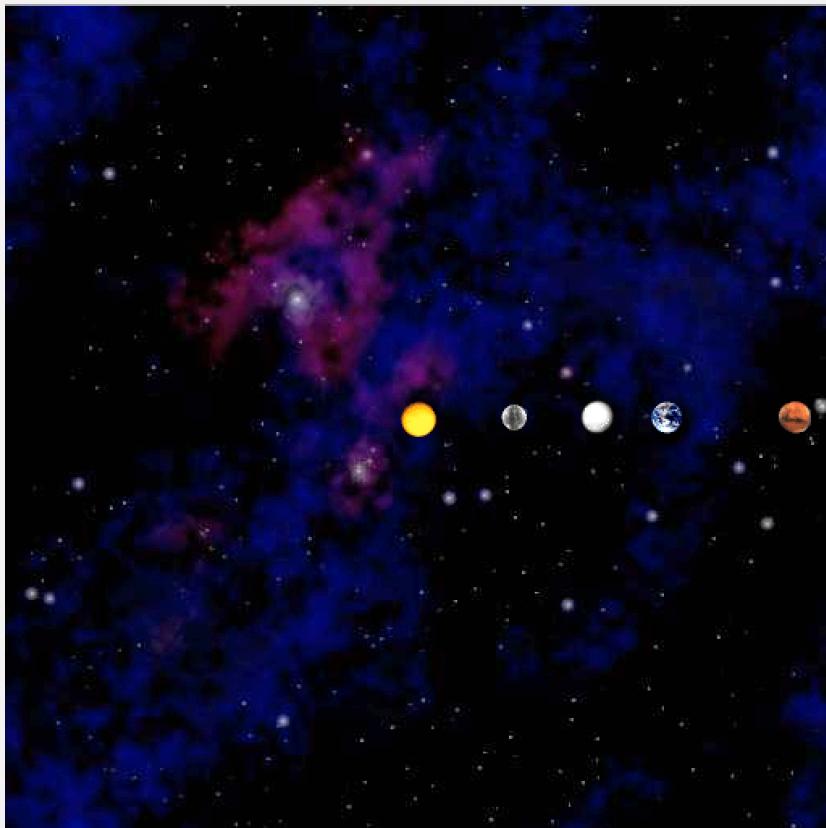


no motion

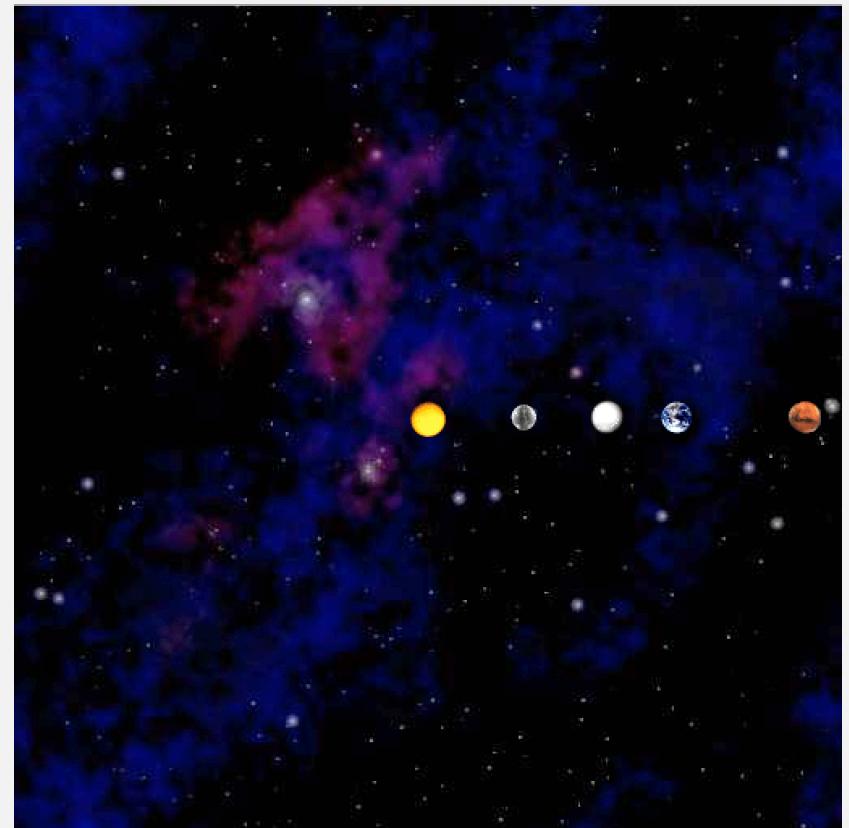


no gravity

Visual bugs

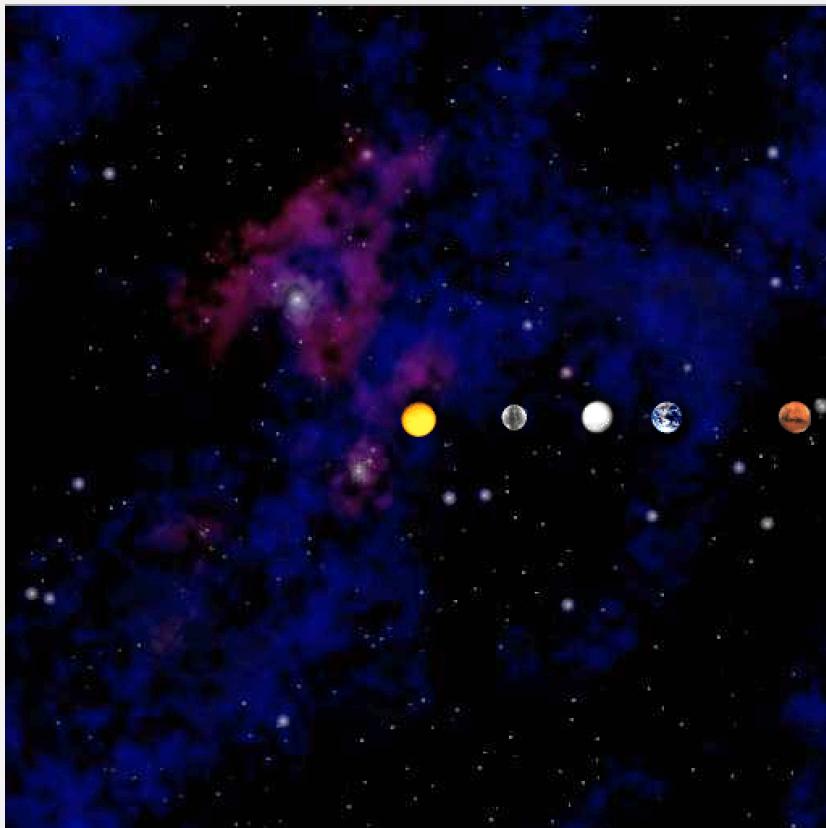


no double buffering

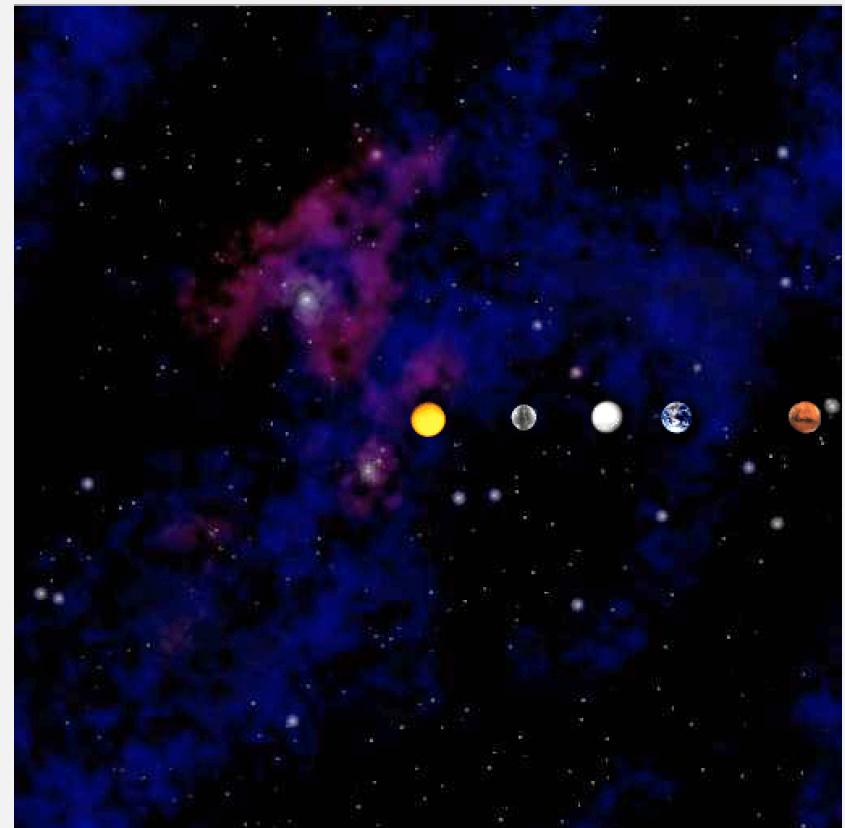


planets repel one another

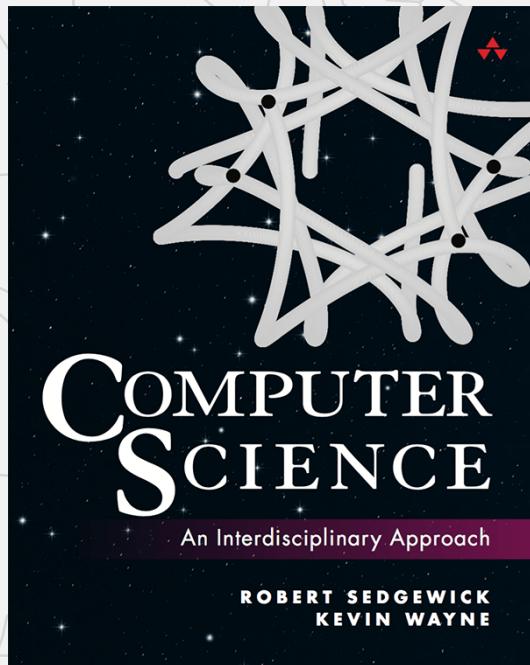
Visual bugs



wrong force loop



cut-and-paste error (x vs. y)

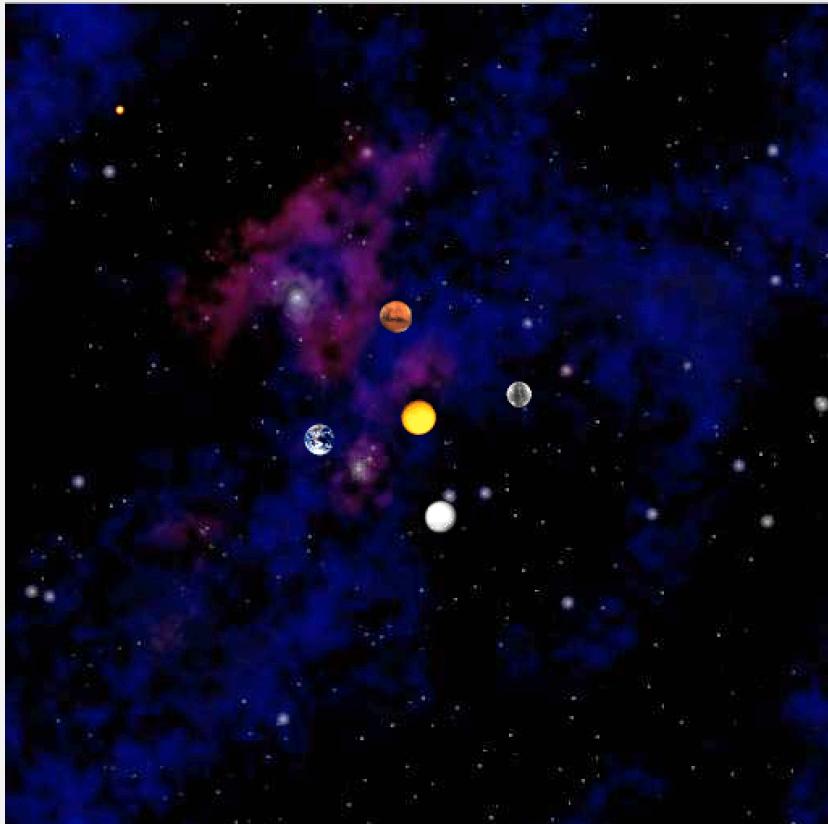


<http://introcs.cs.princeton.edu>

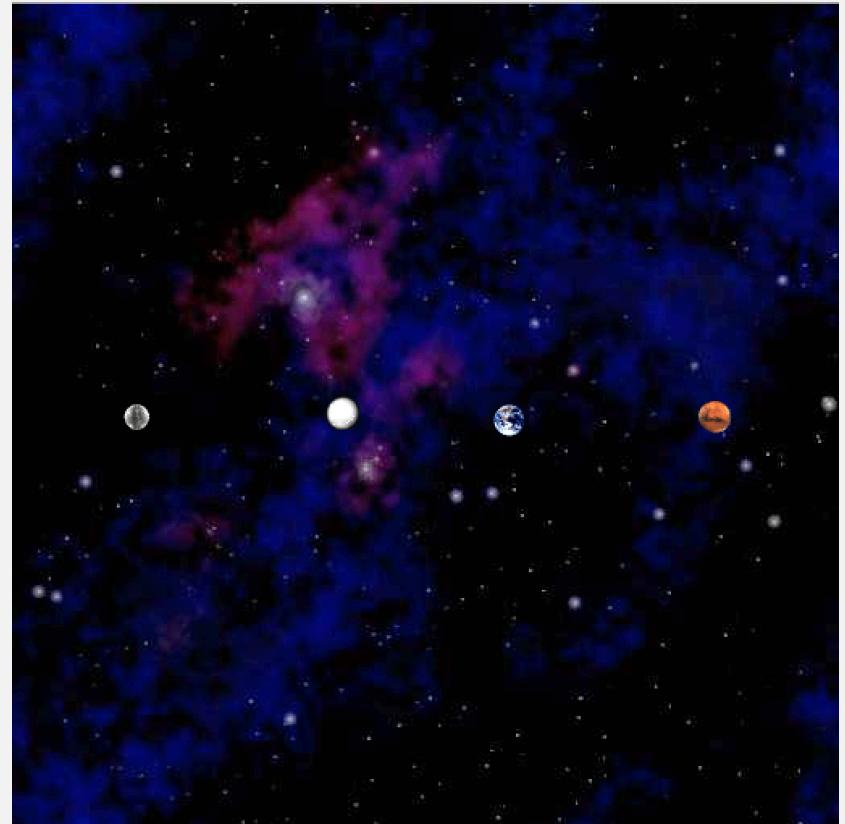
N-BODY TIPS AND TRICKS

- ▶ *overview*
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- ▶ *universes*

Other universes

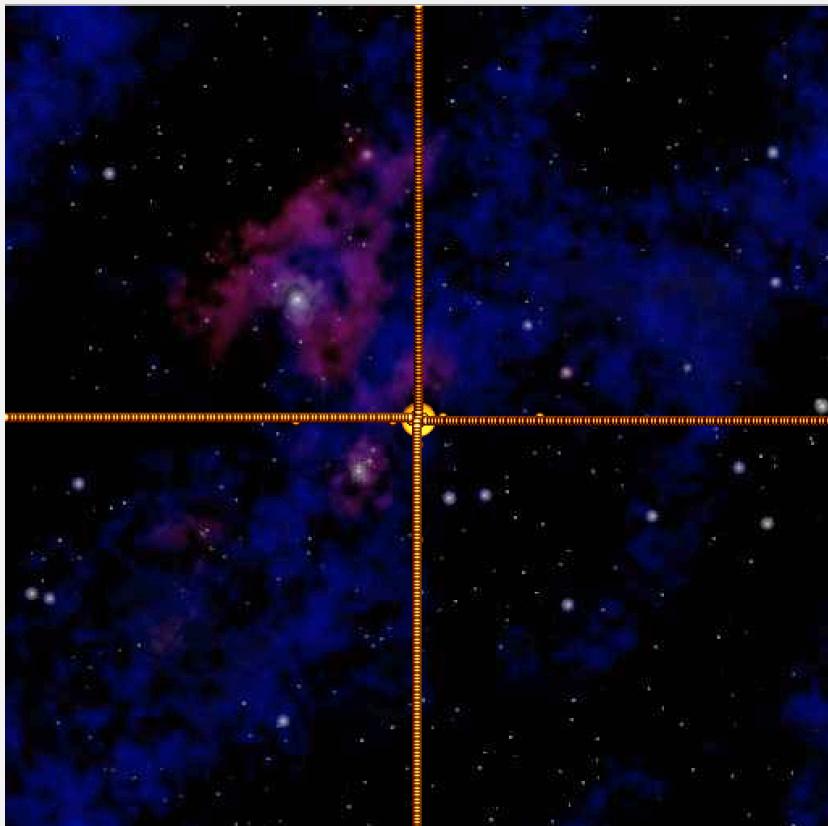


planetsparty.txt
(created by Mary Fan)

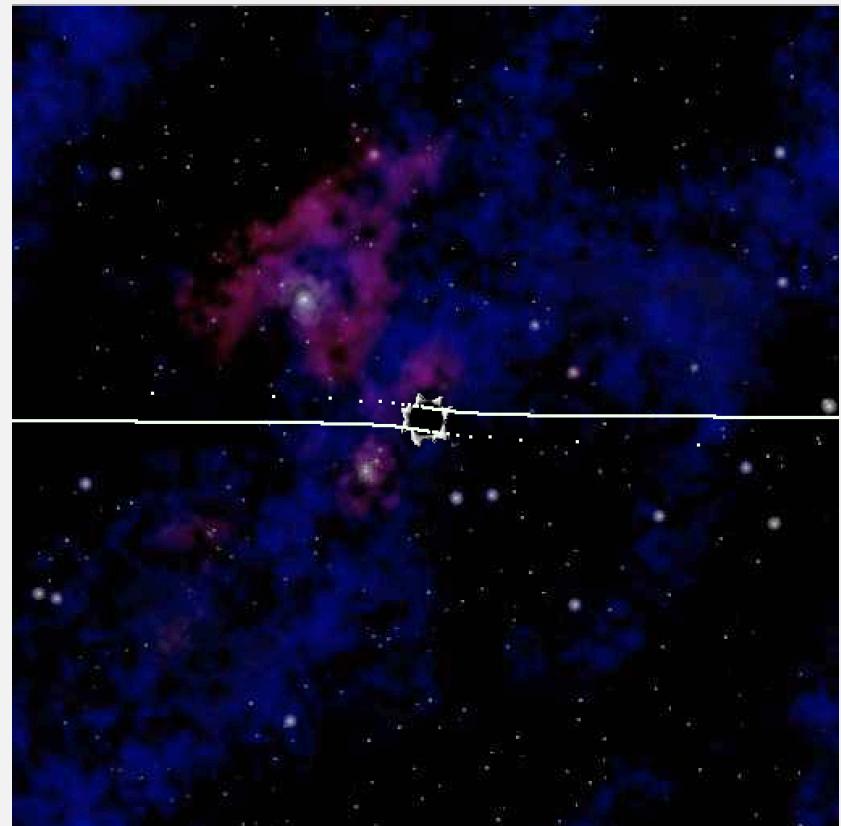


twinbinaries.txt
(David Costanzo)

Other universes



chaosblossum.txt
(created by Erik Keselica)



galaxy.txt
(created by Matt Tilghman)