# Modules and Abstract Data Types

COS 326
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# The Reality of Development

We rarely know the *right* algorithms or the *right* data structures when we start a design project.

- When implementing a search engine, what data structures and algorithms should you use to build the index? To build the query evaluator?

Reality is that *we often have to go back and change our code*, once we've built a prototype.

- Often, we don't even know what the *user wants* (requirements) until they see a prototype.
- Often, we don't know where the *performance problems* are until we can run the software on realistic test cases.
- Sometimes we just want to change the design -- come up with *simpler* algorithms, architecture later in the design process

# Engineering for Change

Given that we know the software will change, how can we write the code so that doing the changes will be easier?

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The primary trick: use *data and algorithm abstraction*.

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Given that we know the software will change, how can we write the code so that doing the changes will be easier?

The primary trick: use *data and algorithm abstraction*.

- *Don't* code in terms of *concrete representations* that the language provides.
- *Do* code with *high-level abstractions* in mind that fit the problem domain.
- Implement the abstractions using a *well-defined interface*.
- Swap in *different implementations* for the abstractions.
- *Parallelize* the development process.

# Example

Goal: Implement a query engine.

Requirements: Need a scalable *dictionary* (a.k.a. index)

- maps words to *set* of URLs for the pages on which words appear.
- want the index so that we can efficiently satisfy queries
  - e.g., all links to pages that contain "Dave" and "Jill".

Wrong way to think about this:

- Aha! A *list* of pairs of a word and a *list* of URLs.
- We can look up "Dave" and "Jill" in the *list* to get back a *list* of URLs.

# Example

```
type query =
  Word of string
| And of query * query
| Or of query * query ;;

type index = (string * (url list)) list ;;

let rec eval(q:query)(h:index) : url list =
  match q with
  | Word x ->
      let (_,urls) = List.find (fun (w,urls) -> w = x) h in
      urls
  | And (q1,q2) ->
      merge_lists (eval q1 h) (eval q2 h)
  | Or (q1,q2) ->
      (eval q1 h) @ (eval q2 h)
```

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merge expects to be passed sorted lists.

# Example

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   | Or (q1,q2) ->
        (eval q1 h) @ (eval q2 h)
```

merge expects to be passed sorted lists.

Oops!

# Example

```
type query =
  Word of string
| And of query * query
| Or of query * query

type index = string (url list) hashtable ;;

let rec eval(q:query)(h:index) : url list =
  match q with
  | Word x ->
      let i = hash_string x in
      let l = Array.get h [i] in
      let urls = assoc_list_find l x in
      urls
  | And (q1,q2) -> ...
  | Or (q1,q2) -> ...
```

I find out there's a better hash-table implementation

# A Better Way

```
type query =
  Word of string
| And of query * query
| Or of query * query ;;

type index = string url_set dictionary ;;

let rec eval(q:query)(d:index) : url_set =
  match q with
   | Word x -> Dict.lookup d x
   | And (q1,q2) -> Set.intersect (eval q1 h) (eval q2 h)
   | Or (q1,q2) -> Set.union (eval q1 h) (eval q2 h)
```

# A Better Way

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```

The problem domain talked about an abstract type of *dictionaries* and *sets of URLs*.

# A Better Way

```
type query =
  Word of string
| And of query * query
| Or of query * query ;;

type index = string url_set dictionar

let rec eval(q:query)(d:index) : url_s
  match q with
  | Word x -> Dict.lookup d x
  | And (q1,q2) -> Set.intersect (eval q1 h) (eval q2 h)
  | Or (q1,q2) -> Set.union (eval q1 h) (eval q2 h)
```

The problem domain talked about an abstract type of <u>*dictionaries*</u> and <u>*sets*</u> *of URLs.*

Once we've written the client, we know what operations we need on these abstract types.

# A Better Way

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type query =
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```

The problem domain talked about an abstract type of *__dictionaries__* and *__sets of URLs__*.

Once we've written the client, we know what operations we need on these abstract types.

Later on, when we find out linked lists aren't so good for sets, we can replace them with balanced trees.

So we can define an interface, and send a pal off to implement the *abstract types* dictionary and set.

# Abstract Data Types

Barbara Liskov
Assistant Professor, MIT
1973

Invented CLU language
that enforced data abstraction

Barbara Liskov
Professor, MIT
Turing Award 2008

"For contributions to practical and theoretical foundations of programming language and system design, especially related to data abstraction, fault tolerance, and distributed computing."

# The Abstraction Barrier

*Rule of thumb*: *Use the language to enforce the abstraction barrier*.

- *Murphy's law for unenforced data abstraction*: What is not enforced, will be broken, some time down the line, by a client
- this is what modules, signatures and structures are for
  - reveal little information about *how* something is implemented
  - provide maximum flexibility for change moving forward.
  - pays off down the line

Like all design rules, we must be able to recognize when the barrier is causing more trouble than it's worth and abandon it.

- may want to reveal more information for debugging purposes
  - eg: conversion to string so you can print things out

# Language-enforced Abstraction

***Rule of thumb*: *Use the language to enforce an abstraction.***

- **Murphy's law for unenforced data abstraction**:
  - What is not enforced, will be broken at some point, by a client
- **This is what module systems are for!**
  - reveal little information about *how* something is implemented
  - provide maximum flexibility for change moving forward.
  - pays off down the line
- **Like all design rules, break it when necessary**
  - recognize when a barrier is causing more trouble than it's worth
- **ML has a particularly great module system**

# Building Abstract Types in OCaml

Use OCaml modules to build new abstract data types!

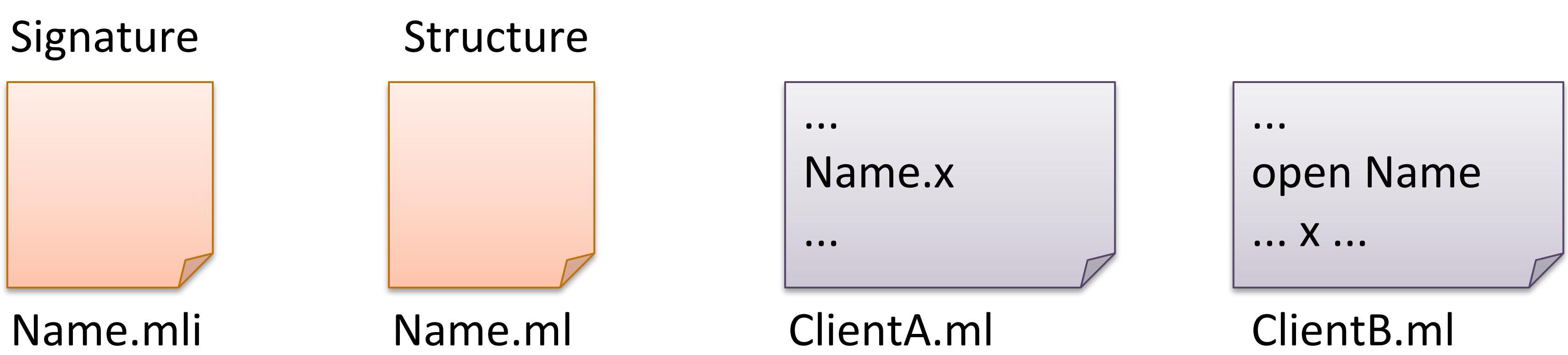
- ***signature***: an interface.
  - specifies the abstract type(s) without specifying their implementation
  - specifies the set of operations on the abstract types
- ***structure***: an implementation.
  - a collection of type and value definitions
  - notion of an implementation matching or satisfying an interface
    - gives rise to a notion of sub-typing
- ***functor***: a parameterized module
  - really, a function from modules to modules
  - allows us to factor out and re-use modules

# Simple Modules

OCaml Convention:

- file Name.ml is a *structure* implementing a module named Name
- file Name.mli is a *signature* for the module named Name
  - if there is no file Name.mli, OCaml infers the default signature
- Other modules, like ClientA or ClientB can:
  - use *dot notation* to refer to contents of Name. eg: Name.val
  - open Set: get access to all elements of Name
    - opening a module puts lots of names in your namespace

    *Open modules with discretion!*



# At first glance: OCaml modules = C modules?

**C has:**

- .h files (signatures) similar to .mli files?
- .c files (structures) similar to .ml files?

**But ML also has:**

- tighter control over type abstraction
  - define abstract, transparent or translucent types in signatures
    - *i.e.,* give none, all or some of the type information to clients
- more structure
  - modules can be defined within modules
  - *i.e.,* signatures and structures can be defined inside files
- more reuse
  - multiple modules can satisfy the same interface
  - the same module can satisfy multiple interfaces
  - modules take other modules as arguments (functors)
- fancy features: dynamic, first class modules!

# At first glance: OCaml modules = C modules?

**C has:**

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- .c files (structures) similar to .ml files?

**But ML also has:**

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  - define ab nspa atures
    - *i.e.*, give to clients
- more
  - modules
  - *i.e.*, sig es
- more reuse
  - multiple modul an sa e sa rface
  - the same module can sat multiple i faces
  - modules take other modu s as arguments (functors)
- fancy features: dynamic, first class modules!

**ML = Winning!**

# Example Signature

```
module type INT_STACK =
  sig
    type t
    val empty : unit -> t
    val push  : int -> t -> t
    val is_empty : t -> bool
    val pop : t -> t
    val top : t -> int option
  end
```

# Example Signature

```
module type INT_STACK =
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    type t
    val empty : unit -> t
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    val is_empty : t -> bool
    val pop : t -> t
    val top : t -> int option
  end
```

*convention:* when the module is about 1 data type, use **t** as the name of the type.

clients refer to **Stack.t**

# Example Signature

```
module type INT_STACK =
  sig
    type t
    val empty : unit -> t
    val push  : int -> t -> t
    val is_empty : t -> bool
    val pop : t -> t
    val top : t -> int option
  end
```

empty and push are abstract ***constructors***: functions that build our abstract type.

# Example Signature

```
module type INT_STACK =
  sig
    type t
    val empty : unit -> t
    val push  : int -> t -> t
    val is_empty : t -> bool
    val pop : t -> t
    val top : t -> int
  end
```

is_empty is an ***observer*** – useful for determining properties of the ADT.

# Example Signature

```
module type INT_STACK =
  sig
    type t
    val empty : unit -> t
    val push  : int -> t -> t
    val is_empty : t -> bool
    val pop : t -> t
    val top : t -> int option
  end
```

pop is sometimes called a ***mutator*** (though it doesn't really change the input)

# Example Signature

```
module type INT_STACK =
  sig
    type t
    val empty : unit -> t
    val push  : int -> t -> t
    val is_empty : t -> bool
    val pop : t -> t
    val top : t -> int option
  end
```

top is also an *observer*, in this functional setting since it doesn't change the stack.

# Put comments in your signature!

```
module type INT_STACK =
  sig
    type t
    (* create an empty stack *)
    val empty : unit -> t

    (* push an element on the top of the stack *)
    val push  : int -> t -> t

    (* returns true iff the stack is empty *)
    val is_empty : t -> bool

    (* pops top element off the stack;
       returns empty stack if the stack is empty *)
    val pop : t -> t

    (* returns the top element of the stack; returns
        None if the stack is empty *)
    val top : t -> int option
  end
```

# Signature Comments

- Signature comments are for clients of the module
  - explain what each function should do
    - how it manipulates abstract values (stacks)
  - ***not*** how it manipulates concrete values
  - don't reveal implementation details that should be hidden behind the abstraction

- Don't copy signature comments into your structures
  - your comments will get out of date in one place or the other
  - an extension of the general rule: don't copy code

- Place implementation comments inside your structure
  - comments about implementation invariants hidden from client
  - comments about helper functions

# Example Structure

```
module ListIntStack : INT_STACK =
  struct
    type t = int list
    let empty () : stack = []
    let push (i:int) (s:stack) : stack = i::s
    let is_empty (s:stack) =
      match s with
       | [] -> true
       | _::_ -> false
    let pop (s:stack) : stack =
      match s with
       | [] -> []
       | _::t -> t
    let top (s:stack) : int option =
      match s with
       | [] -> None
       | h::_ -> Some h
  end
```

# Example Structure

```
module ListIntStack : INT_STACK =
  struct
    type t = int list
    let empty () : stack = []
    let push (i:int) (s:stack) = i:
    let is_empty (s:stack) =
      match s with
       | [] -> true
       | _::_ -> false
    let pop (s:stack) : stack =
      match s with
       | [] -> []
       | _::t -> t
    let top (s:stack) : int option =
      match s with
       | [] -> None
       | h::_ -> Some h
  end
```

Inside the module, we know the ***concrete type*** used to implement the abstract type.

# Example Structure

```
module ListIntStack : INT_STACK =
  struct
    type t = int list
    let empty () : stack = []
    let push (i:int) (s:stack) = i
    let is_empty (s:stack) =
      match s with
       | [] -> true
       | _::_ -> false
    let pop (s:stack) : stack =
      match s with
       | [] -> []
       | _::t -> t
    let top (s:stack) : int option =
      match s with
       | [] -> None
       | h::_ -> Some h
  end
```

But by giving the module the INT_STACK interface, which does not reveal how stacks are being represented, we prevent code outside the module from knowing stacks are lists.

# An Example Client

```
module ListIntStack : INT_STACK =
  struct
    …
  end

let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let  i = ListIntStack.top s2
```

# An Example Client

```
module ListIntStack : INT_STACK =
  struct
    …
  end

let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let  i = ListIntStack.top s2
```

```
s0 : ListIntStack.stack
s1 : ListIntStack.stack
s2 : ListIntStack.stack
```

# An Example Client

```
module ListIntStack : INT_STACK =
  struct
    …
  end

let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let  i = ListIntStack.top s2
         (* i : option int = Some 4 *)
```

# An Example Client

```
module ListIntStack : INT_STACK =
  struct
    …
  end

let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let  i = ListIntStack.top s2
         (* i : option int = Some 4 *)
let  j = ListIntStack.top (ListIntStack.pop s2)
         (* j : option int = Some 3 *)
```

# An Example Client

```
module ListIntStack : INT_STACK =
  struct
    …
  end

let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let  i = ListIntStack.top s2
         (* i : option int = Some 4 *)
let  j = ListIntStack.top (ListIntStack.pop s2)
         (* j : option int = Some 3 *)
open ListIntStack
```

# An Example Client

```
module ListIntStack : INT_STACK =
  struct
    …
  end

let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let  i = ListIntStack.top s2
         (* i : option int = Some 4 *)
let  j = ListIntStack.top (ListIntStack.pop s2)
         (* j : option int = Some 3 *)
open ListIntStack
let  k = top (pop (pop s2))
         (* k : option int = None *)
```

# An Example Client

```
module type INT_STACK =
  sig
    type t
    val push  : int -> t -> t
    …

module ListIntStack : INT_STACK

let s2 = ListIntStack.push 4 s1
…
let l  = List.rev s2
Error: This expression has type stack but an
expression was expected of type 'a list.
```

Notice that the client is not allowed to know that the stack is a list.

# Example Structure

```
module ListIntStack (* : INT_STACK *) =
  struct
    type t = int list
    let empty () : stack = []
    let push (i:int) (s:stack) = i::s
    let is_empty (s:stack) =
      match s with
       | [ ] -> true
       | _::_ -> false
    exception EmptyStack
    let pop (s:stack) =
      match s with
       | [] -> []
       | _::t -> t
    let top (s:stack) =
      match s with
       | [] -> None
       | h::_ -> Some h
  end
```

Note that when you are debugging, you may want to comment out the signature ascription so that you can access the contents of the module.

# The Client without the Signature

```
module ListIntStack (* : INT_STACK *) =
  struct
    …
  end

let s = ListIntStack.empty()
let s1 = ListIntStack.push 3 s
let s2 = ListIntStack.push 4 s1

…
let  l = List.rev s2
         (* l : int list = [3; 4] *)
```

If we don't seal the module with a signature, the client can know that stacks are lists.

# Example Structure

```
module ListIntStack : INT_STACK =
  struct
    type stack = int list
    let empty () : stack = []
    let push (i:int) (s:stack) = i
    let is_empty (s:stack) =
      match s with
       | [ ] -> true
       | _::_ -> false
    exception EmptyStack
    let pop (s:stack) =
      match s with
       | [] -> []
       | _::t -> t
    let top (s:stack) =
      match s with
       | [] -> None
       | h::_ -> Some h
  end
```

When you put the signature on here, you are restricting client access to the information in the signature (which does *not* reveal that stack = int list.) So clients can *only* use the stack operations on a stack value (not list operations.)

# Example Structure

```
module type INT_STACK =
  sig
    type stack
    ...

    val inspect : stack -> int list
    val run_unit_tests : unit -> unit

  end


module ListIntStack : INT_STACK =
  struct
    type stack = int list
    ...

    let inspect (s:stack) : int list = s
    let run_unit_tests () : unit = ...
  end
```

Another technique:

Add testing components to your signature.

Another option we will see: have 2 signatures, one for testing and one for the rest of the code)

# “CORNER CASES”

# Interface design

```
module type INT_STACK =
  sig
    type t
    (* create an empty stack *)
    val empty : unit -> t

    (* push an element on the top of the stack *)
    val push  : int -> t -> t

    (* returns true iff the stack is empty *)
    val is_empty : t -> bool

    (* pops top element off the stack;
       returns empty stack if the stack is empty *)
    val pop : stack -> stack

    (* returns the top element of the stack; returns
        None if the stack is empty *)
    val top : t -> int option
  end
```

Isn't that
a bit bogus?

# Design choices

```
sig
 type t
 (* pops top element;
    returns empty if empty
  *)
  val pop : t -> t
end
```

```
sig
 type t
 (* pops top element;
    returns arbitrary stack
  if empty *)
  val pop : t -> t
end
```

```
sig
 type t
 (* pops top element;
    returns option *)
  val pop: t -> t option
end
```

```
sig
 type t
 exception EmptyStack
 (* pops top element;
    raises EmptyStack if empty
  *)
  val pop : t -> t
end
```

# Design choices

For some functions,

there are some input values

outside the *domain*

of the function.

```
sig
 type t
  (* pops top element;
     returns arbitrary stack
   if empty *)
  val pop : t -> t
end
```

It's reasonable to say the function returns an arbitrary result on those inputs.

When proving things about the program, there's an extra proof obligation:  Prove that the input is in the domain of the function

# Design choices

For some functions, there are some input values outside the *domain* of the function.

```
sig
 type t
 (* pops top element;
    crashes the program
    if empty *)
  val pop : t -> t
end
```

This is *not completely crazy*. One might still be able to guarantee that the input is always in the domain of the function.

It's what the C language does, for example.

# Design choices

For some functions, there are some input values outside the *domain* of the function.

```
sig
 type t
 (* pops top element;
    crashes the program
    if empty *)
  val pop : t -> t
end
```

This is not *completely* crazy.  One might still be able to guarantee that the input is always in the domain of the function.

It's what the C language does, for example.

But it's ***almost completely*** crazy.  This is the biggest source of security vulnerabilities ever.  It's why the hackers can drive your car, steal your money, read your e-mails, …

# Design choices

```
sig
 type stack
 (* pops top element;
    returns empty if empty
  *)
  val pop : stack -> stack
end
```

```
sig
 type stack
 (* pops top element;
     returns arbitrary stack
   if empty *)
  val pop : stack -> stack
end
```

It's also reasonable to say the function returns a *specified, convenient,* result on those inputs.
This is pretty much the same thing, in practice.

# Design choices

For some functions,
there are some
input values
outside the *domain*
of the function.

That's what exceptions are for!
Raise an exception for values
not in the domain.

```
sig
 type stack
 exception EmptyStack
  (* pops top element;
     raises EmptyStack if empty
   *)
  val pop : stack -> stack
end
```

# Careful with exceptions, though!

let const (x: ‘a) (y: ‘b) : ‘a = x

Claim: for all expressions *e*, const 7 *e* == *7*.

Is it true?

```
sig
 type stack
 exception EmptyStack
 (* pops top element;
    raises EmptyStack if empty
   *)
  val pop : stack -> stack
end
```

# Careful with exceptions, though!

let const (x: ‘a) (y: ‘b) : ‘a = x

Claim: for all expressions *e*, const 7 *e* == *7*.

Is it true?

```
sig
 type stack
 exception EmptyStack
  (* pops top element;
     raises EmptyStack if empty
   *)
  val pop : stack -> stack
end
```

***No!***

const 7 (pop (empty())) ≠ 7

To reason about expressions,

you must prove the exception

will not be raised in the particular case (i.e., *input in domain*).

# Design choices

Finally, you can just use option types in the obvious way.

```
sig
 type stack
 (* pops top element;
    returns option *)
  val pop:
       stack -> stack option
end
```

# Design choices

```
sig
  type stack
  (* pops top element;
     returns empty if empty
   *)
   val pop : stack -> stack
end
```

```
sig
  type stack
  (* pops top element;
     returns arbitrary stack
   if empty *)
   val pop : stack -> stack
end
```

All of these are reasonable design choices!

```
sig
  type stack
  (* pops top element;
     returns option *)
   val pop:
        stack -> stack option
end
```

```
  type stack
  exception EmptyStack
  (* pops top element;
     raises EmptyStack if empty
   *)
   val pop : stack -> stack
end
```

# ANOTHER EXAMPLE

# Polymorphic Queues

```
module type QUEUE =
  sig
    type 'a queue
    val empty : unit -> 'a queue
    val enqueue : 'a -> 'a queue -> 'a queue
    val is_empty : 'a queue -> bool
    exception EmptyQueue
    val dequeue : 'a queue -> 'a queue
    val front : 'a queue -> 'a
  end
```

# Polymorphic Queues

```
module type QUEUE =
  sig
    type ‘a queue
    val empty : unit -> ‘a queue
    val enqueue : ‘a -> ‘a queue -> ‘a queue
    val is_empty : ‘a queue -> bool
    exception EmptyQueue
    val dequeue : ‘a queue ->         queue
    val front : ‘a queue -> ‘a
  end
```

These queues are re-usable for different element types.

Here's an exception that client code might want to catch

# One Implementation

```
module AppendListQueue : QUEUE =
  struct
    type 'a queue = 'a list
    let empty() = []
    let enqueue(x:'a)(q:'a queue) : 'a queue = q @ [x]
    let is_empty(q:'a queue) =
     match q with
     | [] -> true
     | _::_ -> false

    ...



end
```

# One Implementation

```
module AppendListQueue : QUEUE =
  struct
    type 'a queue = 'a list
    let empty() = []
    let enqueue(x:'a)(q:'a queue) : 'a queue = q @ [x]
    let is_empty(q:'a queue) = ...

    exception EmptyQueue
    let deq(q:'a queue) : ('a * 'a queue) =
      match q with
      | [] -> raise EmptyQueue
      | h::t -> (h,t)
    let dequeue(q:'a queue) : 'a queue = snd (deq q)
    let front(q:'a queue) : 'a = fst (deq q)
end
```

# One Implementation

```
module AppendListQueue : QUEUE =
  struct
    type 'a queue = 'a list
    let empty() = []
    let enqueue(x:'a)(q:'a queue) : 'a
    let is_empty(q:'a queue) = ...

    exception EmptyQueue
    let deq(q:'a queue) : ('a * 'a queue) =
      match q with
      | [] -> raise EmptyQueue
      | h::t -> (h,t)
    let dequeue(q:'a queue) : 'a que
    let front(q:'a queue) : 'a = fst (de
end
```

Notice deq is a helper function that doesn't show up in the signature.

You can't use it outside the module.

# One Implementation

```
module AppendListQueue : QUEUE =
  struct
    type 'a queue = 'a list
    let empty() = []
    let enqueue(x:'a)(q:'a queue) : 'a queue = q @ [x]
    let is_empty(q:'a queue) = ...

    exception EmptyQueue
    let deq(q:'a queue) : ('a *
      match q with
      | [] -> raise EmptyQueue
      | h::t -> (h,t)
    let dequeue(q:'a queue) : 'a queue = snd (deq q)
    let front(q:'a queue) : 'a = fst (deq q)
end
```

enqueue takes time proportional to the length of the queue ☹

Dequeue runs in constant time ☺

# An Alternative Implementation

```
module DoubleListQueue : QUEUE =
  struct
    type ‘a queue = {front:’a list; rear:’a list}

    ...

end
```

# In Pictures

abstraction

a, b, c, d, e

implementation

`{front=[a; b];rear=[e; d; c]}`

- **let** q0 = empty {front=[];rear=[]}
- **let** q1 = enqueue 3 q0 {front=[];rear=[3]}
- **let** q2 = enqueue 4 q1 {front=[];rear=[4;3]}
- **let** q3 = enqueue 5 q2 {front=[];rear=[5;4;3]}
- **let** q4 = dequeue q3 {front=[4;5];rear=[]}
- **let** q5 = dequeue q4 {front=[5];rear=[]}
- **let** q6 = enqueue 6 q5 {front=[5];rear=[6]}
- **let** q7 = enqueue 7 q6 {front=[5];rear=[7;6]}

# An Alternative Implementation

```
module DoubleListQueue : QUEUE =
  struct
    type 'a queue = {front:'a list;
                     rear:'a list}
    let empty() = {front=[]; rear=[]}

    let enqueue x q = {front=q.front; rear=x::q.rear}

    let is_empty q =
      match q.front, q.rear with
      | [], [] -> true
      | _, _ -> false

    ...
end
```

enqueue runs in constant time ☺

# An Alternative Implementation

```
module DoubleListQueue : QUEUE =
  struct
    type `a queue = {front:'a list;
                     rear:'a list}
    exception EmptyQueue

    let deq (q:'a queue) : `a * `a queue =
      match q.front with
      | h::t -> (h, {front=t; rear=q.rear})
      | [] -> match List.rev q.rear with
              | h::t -> (h, {front=t; rear=[]})
              | [] -> raise EmptyQueue

    let dequeue (q:'a queue) : `a queue = snd(deq q)
    let front (q:'a queue) : `a = fst(deq q)
  end
```

dequeue runs in *amortized* constant time ☺

# How would we design an abstraction?

Think:

- what data do you want?
  - define some types for your data
- what  operations on that data do you want?
  - define some types for your operations

Write some test cases:

- example data, operations

From this, we can derive a signature:

- list the types
- list the operations with their types
- don’t forget to provide enough operations that you can debug!

Then we can build an implementation:

- when prototyping, build the simplest thing you can.
- later, we can swap in a more efficient implementation.
- (assuming we respect the abstraction barrier.)

# Common Interfaces

The stack and queue interfaces are quite similar:

```
module type STACK =
  sig
    type 'a stack
    val empty : unit -> 'a stack
    val push  : 'a -> 'a stack -> 'a stack
    val is_empty : 'a stack -> bool
    exception EmptyStack
    val pop
    val top
  end
```

```
module type QUEUE =
  sig
    type 'a queue
    val empty : unit -> 'a queue
    val enqueue : 'a -> 'a queue -> 'a queue
    val is_empty : 'a queue -> bool
    exception EmptyQueue
    val dequeue : 'a queue -> 'a queue
    val front : 'a queue -> 'a
  end
```

# Common Interfaces

The stack and queue interfaces are *quite* similar:

```
module type STACK =
  sig
    type 'a t
    val empty : unit -> 'a t
    val insert : 'a -> 'a t -> 'a t
    val is_empty : 'a t -> bool
    exception Empty
    val rem
    val fir
  end
```

```
module type QUEUE =
  sig
    type 'a t
    val empty : unit -> 'a t
    val insert : 'a -> 'a t -> 'a t
    val is_empty : 'a t -> bool
    exception Empty
    val remove : 'a t -> 'a t
    val first : 'a t -> 'a
  end
```

# It's a good idea to factor out patterns

Stacks and Queues share common features.

Both can be considered "containers"

Create a reuseable container interface!

```
module type CONTAINER =
  sig
    type 'a t
    val empty : unit -> 'a t
    val insert : 'a -> 'a t -> 'a t
    val is_empty : 'a t -> bool
    exception Empty
    val remove : 'a t -> 'a t
    val first : 'a t -> 'a
  end
```

# It’s a good idea to factor out patterns

Stacks and Queues sh

Both can be considere

Create a reuseable co

```
module type STACK =
  sig
    type 'a t
    val empty : unit -> 'a t
    val insert : 'a -> 'a t -> 'a t
    val is_empty : 'a t -> bool
    exception Empty
    val remove : 'a t -> 'a t
    val first : 'a t -> 'a
  end
```

```
module type CONTAI
  sig
    type 'a t
    val empty : unit -> 'a t
    val insert : 'a -> 'a t -> 'a t
    val is_empty : 'a t -> bool
    exception Empty
    val remove : 'a t -> 'a t
    val first : 'a t -> 'a
  end
```

*more than quite similar! exactly the same*

# It's a good idea to factor out patterns

Stacks and Queues sh

Both can be considere

Create a reuseable co

```
module type STACK =
```

```
module type QUEUE =
  sig
    type 'a t
    val empty : unit -> 'a t
    val insert : 'a -> 'a t -> 'a t
    val is_empty : 'a t -> bool
    exception Empty
    val remove : 'a t -> 'a t
    val first : 'a t -> 'a
  end
```

```
module type CONTAI
  sig
    type 'a t
    val empty : unit -> 'a t
    val insert : 'a -> 'a t -> 'a t
    val is_empty : 'a t -> bool
    exception Empty
    val remove : 'a t -> 'a t
    val first : 'a t -> 'a
  end
```

*still exactly the same!*

# It's a good idea to factor out patterns

```
module type CONTAINER = sig ... end

module Queue : CONTAINER = struct ... end
module Stack : CONTAINER = struct ... end
```

```
module DepthFirstSearch : SEARCHER =
  struct
    type to_do : Graph.node Queue.t

  end
```

```
module BreadthFirstSearch : SEARCHER =
  struct
    type to_do : Graph.node Stack.t

  end
```

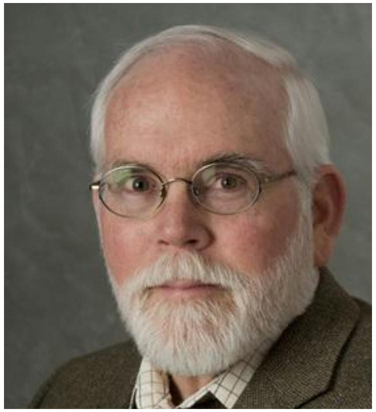
Still repeated code!

Breadth-first and depth-first search code is the same!

Just use different containers!

Need parameterized modules!

# FUNCTORS



David MacQueen
Bell Laboratories 1983-2001
U. of Chicago 2001-2012

Designer of ML module system,
functors,
sharing constraints, etc.

# Matrices

Suppose I ask you to write a generic package for matrices.

- e.g., matrix addition, matrix multiplication

The package should be *parameterized* by the element type.

- We may want to use ints or floats or complex numbers or binary values or … for the elements.
- And the elements still have a collection of operations on them:
  - addition, multiplication, zero element, etc.

What we'll see:

- RING: a signature to describe the type (and necessary operations) for matix elements
- MATRIX: a signature to describe the available operations on matrices
- DenseMatrix: a functor that will generate a MATRIX with a specific RING as an element type

# Ring Signature

```
module type RING =
  sig
    type t
    val zero : t
    val one  : t
    val add  : t -> t -> t
    val mul  : t -> t -> t
  end
```

# Some Rings

```
module IntRing =
  struct
    type t = int
    let zero = 0
    let one = 1
    let add x y= x + y
    let mul x y = x * y
  end
```

```
module FloatRing =
  struct
    type t = float
    let zero = 0.0
    let one = 1.0
    let add = (+.)
    let mul = ( *. )
  end
```

```
module BoolRing =
  struct
    type t = bool
    let zero = false
    let one = true
    let add x y= x || y
    let mul x y = x && y
  end
```

# Matrix Signature

```
module type MATRIX =
  sig
    type elt
    type matrix
    val matrix_of_list : elt list list -> matrix
    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
   end
```

# The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct

  ...

end
```

# The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct

   ...

end
```

Argument R must be a RING

Result must be a MATRIX

Specify Result.elt = R.t

# The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct

  ...

end

module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

Use DenseMatrix like
it is a function from
modules to modules

# The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct

  ...

end

module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

redacted

```
module type MATRIX =
  sig
    type elt
    type matrix

    val matrix_of_list :
          elt list list -> matrix

    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
  end
```

abstract = unknown!

nonexistent

# The DenseMatrix Functor

**module** DenseMatrix (R:RING) : (MATRIX ~~with type elt = R.t~~) =

**struct**

**end**

redacted

If the "with" clause is redacted then IntMatrix.elt is abstract -- we could never build a matrix because we could never generate an elt

```
module type MATRIX =
  sig
    type elt
    type matrix

    val matrix_of_list :
          elt list list -> matrix

    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
  end
```

abstract = unknown!

nonexistent

```
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

# The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct

  ...

end
```

sharing constraint

```
module type MATRIX =
  sig
    type elt = int
    type matrix

    val matrix_of_list :
          elt list list -> matrix

    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
  end
```

known to be int when R.t = int like when R = IntRing

list of list of ints

```
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

# The DenseMatrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct

end

module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

sharing constraint

The "with" clause makes IntMatrix.elt equal to int -- we can build a matrix from any int list list

```
module type MATRIX =
  sig
    type elt = int
    type matrix

    val matrix_of_list :
          elt list list -> matrix

    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
  end
```

known to be int when R.t = int like when R = IntRing

list of list of ints

# Matrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct
  type elt = R.t
  type matrix = (elt list) list
  let matrix_of_list rows = rows
  let add m1 m2 =
    List.map (fun (r1,r2) ->
                    List.map (fun (e1,e2) -> R.add e1 e2))
                      (List.combine r1 r2))
       (List.combine m1 m2)
  let mul m1 m2 = (* good exercise *)
end

module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

Satisfies the sharing constraint

# ANONYMOUS STRUCTURES

# Another Example

```
module type UNSIGNED_BIGNUM =
sig
  type ubignum
  val fromInt : int -> ubignum
  val toInt : ubignum -> int
  val plus : ubignum -> ubignum -> ubignum
  val minus : ubignum -> ubignum -> ubignum
  val times : ubignum -> ubignum -> ubignum
  …
end
```

# An Implementation

```
module My_UBignum_1000 : UNSIGNED_BIGNUM =
struct
  let base = 1000

  type ubignum = int list

  let toInt(b:ubignum):int = …

  let plus(b1:ubignum)(b2:ubignum):ubignum = …

  let minus(b1:ubignum)(b2:ubignum):ubignum = …

  let times(b1:ubignum)(b2:ubignum):ubignum = …
  …
end
```

What if we want to change the base? Binary? Hex? 2^32? 2^64?

# Another Functor Example

```
module type BASE =
sig
  val base : int
end

module UbignumGenerator(Base:BASE) : UNSIGNED_BIGNUM =
struct
  type ubignum = int list
  let toInt(b:ubignum):int =
    List.fold_left (fun a c -> c*Base.base + a) 0 b   …
end

module Ubignum_10 =
  UbignumGenerator(struct let base = 10 end)

module Ubignum_2 =
  UbignumGenerator(struct let base = 2 end)
```

Anonymous structures

# SIGNATURE SUBTYPING

# Subtyping

- A module matches any interface as long as it provides *at least* the definitions (of the right type) specified in the interface.

- But as we saw earlier, the module can have more stuff.
  - e.g., the deq function in the Queue modules

- Basic principle of subtyping for modules:
  - wherever you are expecting a module with signature S, you can use a module with signature S', as long as all of the stuff in S appears in S'.
  - That is, S' is a bigger interface.

# Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    type t
    val zero : t
    val one  : t
    val add  : t -> t -> t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

# Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    type t
    val zero : t
    val one  : t
    val add  : t -> t -> t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

RING is a sub-type of GROUP.

# Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    type t
    val zero : t
    val one  : t
    val add  : t -> t -> t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

There are *more* modules matching the GROUP interface than the RING one.

# Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    type t
    val zero : t
    val one  : t
    val add  : t -> t -> t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

Any module expecting a GROUP can be passed a RING.

# Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    include GROUP
    val one  : t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

The **include** primitive is like cutting-and-pasting the signature's content here.

# Groups versus Rings

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
    include GROUP
    val one  : t
    val mul  : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

That *ensures* we will be a sub-type of the included signature.

# ANOTHER EXAMPLE OF FUNCTORS

# A Bigger Example

```
module type SET =
  sig
    type elt
    type set
    val empty : set
    val is_empty : set -> bool
    val insert : elt -> set -> set
    val singleton : elt -> set
    val union : set -> set -> set
    val intersect : set -> set -> set
    val remove : elt -> set -> set
    val member : elt -> set -> bool
    val choose : set -> (elt * set) option
    val fold : (elt -> 'a -> 'a) -> 'a -> set -> 'a
  end
```

# Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
              :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

# Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
             :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

ListSet is a parameterized module – given a module argument for Elt, it generates a new module.

# Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
              :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

This is a very simple, anonymous signature (it just specifies there's some type t) for the argument to ListSet

# Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
            :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

This is the signature of the resulting module – we have a set plus the knowledge that the Set's elt type is equal to Elt.t

# Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
               :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

These are two SET modules that I created with the ListSet functor.

# Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
               :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set =
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

In this case, I'm passing in an anonymous module for Elt that defines t to be int.

# Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
              :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
...
end

module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

We know that IntListSet.elt = int.

# Our Set Implementation is a Functor:

```
module ListSet (Elt : sig type t end)
              :   (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = [
  let is_empty (s:set
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:el
...
end
```

```
module type SET =
  sig
    type elt = int
    type set
    val empty : set
    val is_empty : set -> bool
    val insert : elt -> set -> set
    ...
  end
```

equal to int
so we can actually
build a set using
insertions!

```
module IntListSet = ListSet(struct type t = int end)
module StringListSet = ListSet(struct type t = string end)
```

# Let's Write the Rest of the Functor

```
module ListSet (Elt : sig type t end)
               : (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  let empty : set = []
  let is_empty (s:set) =
    match xs with
    | [] -> true
    | _::_ -> false
  let singleton (x:elt) : set = [x]
  let insert (x:elt) (s:set) : set =
      if List.mem x s then s else x::s
  ...
end
```

# Let’s Write the Rest of the Functor

```
module ListSet (Elt : sig type t end)
              :(SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  ...
  let insert (x:elt) (s:set) : set =
      if List.mem x s then s else x::s
  let union (s1:set) (s2:set) : set = ???
end
```

# Let's Write the Rest of the Functor

```
module ListSet (Elt : sig type t end)
              : (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  ...
  let insert (x:elt) (s:set) : set =
      if List.mem x s then s else x::s
  let union (s1:set) (s2:set) : set =
      s1 @ s2
  ...
end
```

Ugh. Wastes space if s1 and s2 have duplicates. (Also, makes remove harder…)

# Let's Write the Rest of the Functor

```
module ListSet (Elt : sig type t end)
              : (SET with elt = Elt.t) =
struct
  type elt = Elt.t
  type set = elt list
  ...
  let insert (x:elt) (s:set) : set =
      if List.mem x s then s else x::s
  let union (s1:set) (s2:set) : set =
      List.fold_right insert s1 s2
  ...
end
```

Gets rid of the duplicates. Now remove can stop once it finds the element.

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      if List.mem x s then s else x::s
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      List.fold_right insert s1 s2
  ...
end
```

But List.mem and List.fold_right take time proportional to the length of the list. So union is quadratic.

Gets rid of the duplicates. Now remove can stop once it finds the element.

# Let's Write the Rest of the Functor

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      if List.mem x s then s else x::s
  let union (s1:set) (s2:set) : set =
      List.fold_right insert s1 s2
  ...
end
```

If we knew that s1 and s2 were *sorted* we could use the merge from mergesort to compute the sorted union in linear time.

# A Sorted List Set Functor

```
module type COMPARATOR = sig
  type t
  val compare : t -> t -> Order.order
end

module SortedListSet (Elt : COMPARATOR)
                  : (SET with elt = Elt.t) =
struct ...
  let rec insert (x:elt) (s:set) : set =
      match s with
      | [] -> [x]
      | h::t -> (match Elt.compare x h with
                 | Less -> x::s
                 | Eq -> s
                 | Greater -> h::(insert x t)) ...
end
```

# A Sorted List Set Functor

```
module type COMPARATOR = sig
  type t
  val compare : t -> t -> Order.order
end

module SortedListSet (Elt :
                    : (SET with e
struct ...
  let rec insert (x:elt) (s:set) :
      match s with
      | [] -> [x]
      | h::t -> (match Elt.compare x h with
                  | Less -> x::s
                  | Eq -> s
                  | Greater -> h::(insert x t)) ...
end
```

To support the sorting, I'm passing in a comparison operation to go with the element type.

# A Sorted List Set Functor

```
module SortedListSet (Elt : COMPARATOR)
                   : (SET with elt = Elt.t) =
struct ...
  let rec union (s1:set) (s2:set) : set =
      match s1, s2 with
      | [], _ -> s2
      | _, [] -> s1
      | h1::t1, h2::t2 ->
          (match Elt.compare h1 h2 with
             | Less -> h1::(union t1 s2)
             | Eq -> h1::(union t1 t2)
             | _ -> h2::(union s1 t2))
  …
end
```

# Simpler

```
module SortedListSet (Elt : COMPARATOR)
                    : (SET with elt = Elt.t) =
struct ...
  let rec union (s1:set) (s2:set) : set = ...

  let insert (x:elt) (s:set) : set = union [x] s ;;

end
```

# Another Alternative: Bit Vectors

```
module BitVectorSet (Elt : sig type t
                               val index : t -> int
                               val max : int
                           end)
                   : (SET with elt = Elt.t) =
struct
  type set = bool array
  let empty = Array.create Elt.max false
  let member x s = s.(Elt.index x)
  let union s1 s2 =
       Array.init Elt.max
         (fun i -> s1.(i) || s2.(i))
  let intersect s1 s2 =
       Array.init Elt.max
         (fun i -> s1.(i) && s2.(i))
  ...
```

# Another Alternative:  Binary Search Trees

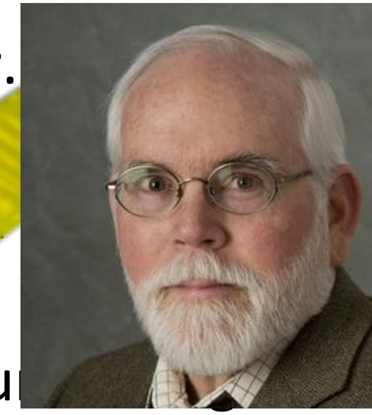
```
module BSTreeSet(Elt : sig type t
                           val compare : t -> t -> Order.order
                       end) : (SET with elt = Elt.t) =
struct
  type set = Leaf | Node of set * elt * set
  let empty() = Leaf
  let rec insert (x:elt) (s:set) : set =
      match s with
      | Leaf -> Node(Leaf,x,Leaf)
      | Node(left,e,right) ->
         (match Elt.compare x e with
              | Eq -> s
              | Less -> Node(insert x left, e, right)
              | Greater -> Node(left, e, insert x right))
  let rec member (x:elt) (s:set) : bool =
      match s with
      | Leaf -> false
      | Node(left,e,right) ->
        (match Elt.compare x e with
            | Eq -> true
            | Less -> member x left
            | Greater -> member x right)
  ... end
```

# SUMMARY

# Wrap up and Summary

- It is often tempting to break the abstraction barrier.
  - e.g., during development, you want to print out a set, so you just call a convenient function you have lying around for iterating over lists and printing them out.
- But the whole point of the barrier is to support future change in implementation.
  - e.g., moving from unsorted invariant to sorted invariant.
  - or from lists to balanced trees.
- Many languages provide ways to leak information through the abstraction barrier.
  - "good" clients should not take advantage of this.
  - but they always end up doing it.
  - so you end up having to support these leaks when you upgrade, else you'll break the clients.

# Wrap up and Summary



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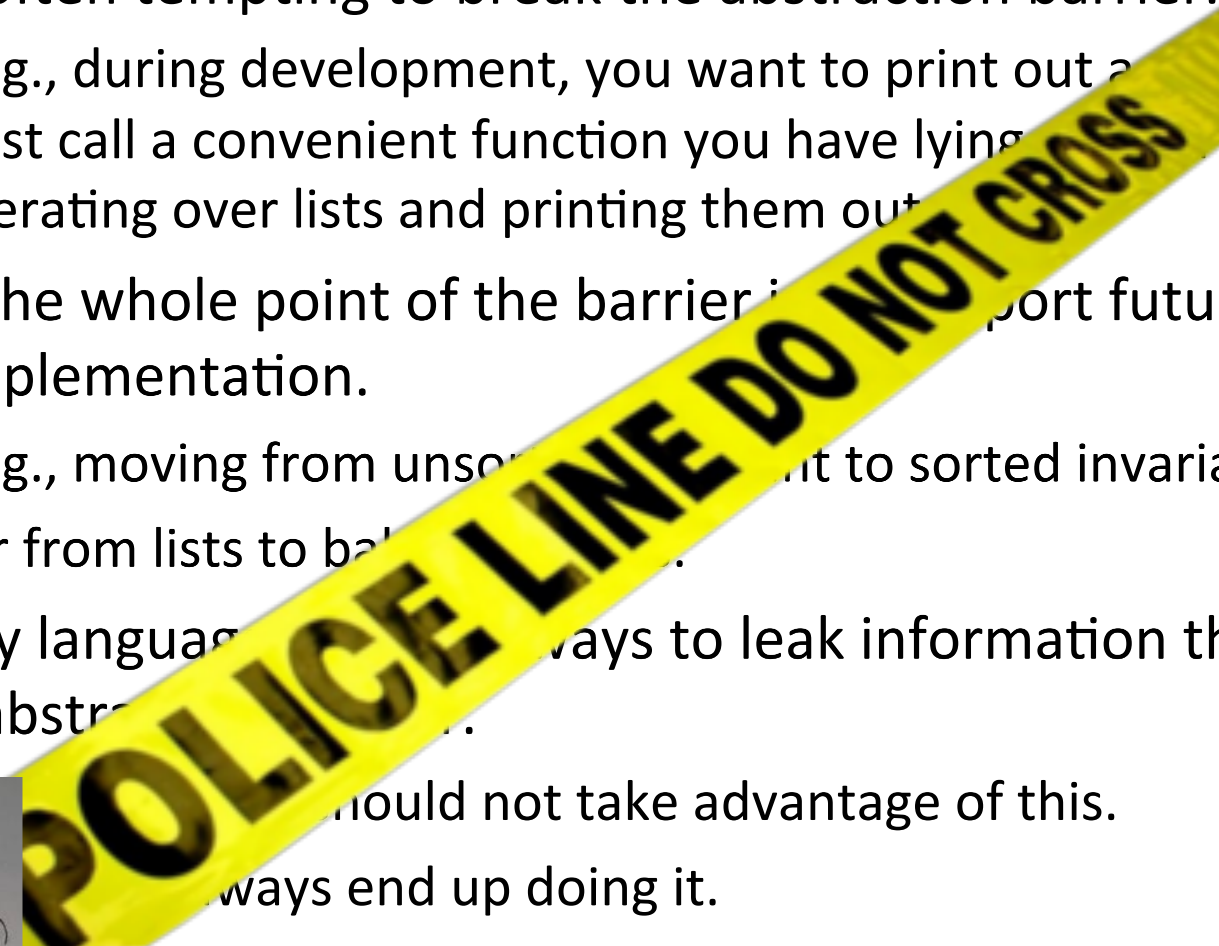


# Key Points

OCaml's linguistic mechanisms include:

- *signatures* (interfaces)
- *structures* (implementations)
- *functors* (functions from modules to modules)

We can use the module system

- provides support for *name-spaces*
- *hiding information* (types, local value definitions)
- *code reuse* (via functors, reuseable interfaces, reuseable modules)

Information hiding allows design in terms of *abstract* types and algorithms.

- think "sets" not "lists" or "arrays" or "trees"
- think "document" not "strings"
- the less you reveal, the easier it is to replace an implementation
- use linguistic mechanisms to implement information hiding
  - invariants written down as comments are easy to violate
  - use the type checker to guarantee you have strong protections in place