Simple Data

COS 326
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What is the single most important mathematical concept ever developed in human history?

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An answer: The mathematical variable

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(runner up: natural numbers/induction)

Why is the mathematical variable so important?

The mathematician says:

“Let x be some integer, we define a polynomial over x ...”

Why is the mathematical variable so important?

The mathematician says:

"Let x be some integer, we define a polynomial over x ..."

What is going on here? The mathematician has separated a *definition* (of x) from its *use* (in the polynomial).

This is the most primitive kind of *abstraction* (x is *some* integer)

Abstraction is the key to controlling complexity and without it, modern mathematics, science, and computation would not exist.

It allows *reuse* of ideas, theorems ... functions and programs!

OCAML BASICS:
LET DECLARATIONS

Abstraction

- Good programmers identify repeated patterns in their code and factor out the repetition into meaningful components
- In O'Caml, the most basic technique for factoring your code is to use let expressions
- Instead of writing this expression:

```
(2 + 3) * (2 + 3)
```

Abstraction & Abbreviation

- Good programmers identify repeated patterns in their code and factor out the repetition into meaning components
- In O'Caml, the most basic technique for factoring your code is to use let expressions
- Instead of writing this expression:

```
(2 + 3) * (2 + 3)
```

- We write this one:

```
let x = 2 + 3 in
x * x
```

A Few More Let Expressions

```
let x = 2 in
let squared = x * x in
let cubed = x * squared in
squared * cubed
```

A Few More Let Expressions

```
let x = 2 in
let squared = x * x in
let cubed = x * squared in
squared * cubed
```

```
let a = "a" in
let b = "b" in
let as = a ^ a ^ a in
let bs = b ^ b ^ b in
as ^ bs
```

Abstraction & Abbreviation

- Two kinds of let:

```
if tuesday() then
     let x = 2 + 3 in
     x + x
else
     0
```

```
let x = 2 + 3

let y = x + 17 / x
```

let … **in** … is an *expression* that can appear inside any other *expression*

The scope of x does not extend outside the enclosing “in”

let … without “in” is a top-level *declaration*

Variables x and y may be exported; used by other modules

(Don’t need ;; if another let comes next; do need it if the next top-level declaration is an expression)

Binding Variables to Values

- Each OCaml variable is *bound* to 1 value
- *The value to which a variable is bound to never changes*!

```
let x = 3

let add_three (y:int) : int = y + x
```

Binding Variables to Values

- Each OCaml variable is *bound* to 1 value
- *The value to which a variable is bound to never changes*!

```
let x = 3

let add_three (y:int) : int = y + x
```

It does not matter what I write next. add_three will always add 3!

Binding Variables to Values

- Each OCaml variable is bound to 1 value
- *The value a variable is bound to never changes*!

a distinct variable that "happens to be spelled the same"

```
let x = 3

let add_three (y:int) : int = y + x

let x = 4

let add_four (y:int) : int = y + x
```

Binding Variables to Values

- Since the 2 variables (both happened to be named x) are actually different, unconnected things, we can rename them

rename x to zzz if you want to, replacing its uses

```
let x = 3

let add_three (y:int) : int = y + x

let zzz = 4

let add_four (y:int) : int = y + zzz

let add_seven (y:int) : int =
  add_three (add_four y)
```

Binding Variables to Values

- Each OCaml variable is bound to 1 value
- OCaml is a statically scoped (or lexically scoped) language

we can use add_three without worrying about the second definition of x

```
let x = 3

let add_three (y:int) : int = y + x

let x = 4

let add_four (y:int) : int = y + x

let add_seven (y:int) : int =
  add_three (add_four y)
```

How do let expressions operate?

```
let x = 2 + 1 in x * x
```

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```
let x = 2 + 1 in x * x
```

-->

```
let x = 3 in x * x
```

How do let expressions operate?

```
let x = 2 + 1 in x * x
```

-->

```
let x = 3 in x * x
```

substitute
3 for x

-->

```
3 * 3
```

How do let expressions operate?

```
let x = 2 + 1 in x * x
```

-->

```
let x = 3 in x * x
```

substitute
3 for x

-->

```
        3 * 3
```

-->

```
        9
```

How do let expressions operate?

```
let x = 2 + 1 in x * x
```

-->

```
let x = 3 in x * x
```

substitute
3 for x

-->

```
3 * 3
```

-->

```
9
```

Note: I write
e1 --> e2
when e1 evaluates
to e2 in one step

Did you see what I did there?

Did you see what I did there?

I defined the language in terms of itself:

let x = 2 in x + 3 --> 2 + 3

I'm trying to train you to think at a high level of abstraction.

I didn't have to mention low-level abstractions like assembly code or registers or memory layout

Another Example

```
let x = 2 in
let y = x + x in
y * x
```

Another Example

```
let x = 2 in
let y = x + x in
y * x
```

substitute
2 for x

-->

```
let y = 2 + 2 in
y * 2
```

Another Example

```
let x = 2 in
let y = x + x in
y * x
```

substitute
2 for x

-->

```
let y = 2 + 2 in
y * 2
```

-->

```
let y = 4     in
y * 2
```

Another Example

```
let x = 2 in
let y = x + x in
y * x
```

substitute
2 for x

-->

```
let y = 2 + 2 in
y * 2
```

-->

```
let y = 4       in
y * 2
```

substitute
4 for y

-->

```
4 * 2
```

Another Example

```
let x = 2 in
let y = x + x in
y * x
```

substitute 2 for x

-->

```
let y = 2 + 2 in
y * 2
```

-->

```
let y = 4     in
y * 2
```

substitute 4 for y

-->

```
4 * 2
```

-->

```
8
```

Moral: Let operates by *substituting* computed values for variables

What would happen in an imperative language?

C program:

```
x = 2;
x += x;
return x*2;
```

substitute
2 for x

-->

```
x += 2   ???
return x*2;
```

substituting computed values for variables

This principle works in functional languages, not so well in imperative languages

OCAML BASICS:
TYPE CHECKING AGAIN

Back to Let Expressions ... Typing

x granted type of e1 for use in e2

```
let x = e1 in

e2
```

overall expression takes on the type of e2

Back to Let Expressions ... Typing

x granted type of e1 for use in e2

```
let x = e1 in

e2
```

overall expression takes on the type of e2

x has type int for use inside the let body

```
let x = 3 + 4 in

string_of_int x
```

overall expression has type string

OCAML BASICS: FUNCTIONS

Defining functions

```
let add_one (x:int) : int = 1 + x
```

Defining functions

let keyword

```
let add_one (x:int) : int = 1 + x
```

function name

argument name

type of argument

type of result

expression that computes value produced by function

Note: recursive functions with begin with "**let rec**"

Defining functions

- Nonrecursive functions:

```
let add_one (x:int) : int = 1 + x

let add_two (x:int) : int = add_one (add_one x)
```

definition of add_one must come before use

Defining functions

- Nonrecursive functions:

```
let add_one (x:int) : int = 1 + x

let add_two (x:int) : int = add_one (add_one x)
```

- With a local definition:

local function definition hidden from clients

```
let add_two' (x:int) : int =
   let add_one x = 1 + x in
   add_one (add_one x)
```

I left off the types. O'Caml figures them out

Good style: types on top-level definitions

Types for Functions

Some functions:

```
let add_one (x:int) : int = 1 + x

let add_two (x:int) : int = add_one (add_one x)

let add (x:int) (y:int) : int = x + y
```

function with two arguments

Types for functions:

```
add_one : int -> int

add_two : int -> int

add : int -> int -> int
```

Rule for type-checking functions

General Rule:

If a function f : T1 -> T2
and an argument e : T1
then f e : T2

Example:

```
add_one : int -> int

3 + 4 : int

add_one (3 + 4) : int
```

Rule for type-checking functions

- Recall the type of add:

Definition:

```
let add (x:int) (y:int) : int =
  x + y
```

Type:

```
add : int -> int -> int
```

Rule for type-checking functions

- Recall the type of add:

Definition:

```
let add (x:int) (y:int) : int =
  x + y
```

Type:

```
add : int -> int -> int
```

Same as:

```
add : int -> (int -> int)
```

Rule for type-checking functions

General Rule:

If a function f : T1 -> T2
and an argument e : T1
then f e : T2

A -> B -> C

same as:

A -> (B -> C)

Example:

```
add : int -> int -> int

3 + 4 : int

add (3 + 4) : ???
```

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Example:

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add : int -> (int -> int)

3 + 4 : int

add (3 + 4) :
```

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Example:

```
add : int -> (int -> int)

3 + 4 : int

add (3 + 4) : int -> int
```

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General Rule:

If a function f : T1 -> T2
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same as:

A -> (B -> C)

Example:

```
add : int -> int -> int

3 + 4 : int

add (3 + 4) : int -> int

(add (3 + 4)) 7 : int
```

Rule for type-checking functions

General Rule:

If a function f : T1 -> T2
and an argument e : T1
then f e : T2

A -> B -> C

same as:

A -> (B -> C)

Example:

```
add : int -> int -> int

3 + 4 : int

add (3 + 4) : int -> int

add (3 + 4) 7 : int
```

Rule for type-checking functions

Example:

```
let munge (b:bool) (x:int) : ?? =
  if not b then
    string_of_int x
  else
    "hello"
;;

let y = 17;;
```

```
munge (y > 17) : ??

munge true (f (munge false 3)) : ??
  f : ??

munge true (g munge) : ??
  g : ??
```

Rule for type-checking functions

Example:

```
let munge (b:bool) (x:int) : ?? =
  if not b then
    string_of_int x
  else
    "hello"
;;

let y = 17;;
```

```
munge (y > 17) : ??

munge true (f (munge false 3)) : ??
  f : string -> int

munge true (g munge) : ??
  g : (bool -> int -> string) -> int
```

One key thing to remember

- If you have a function f with a type like this:

A -> B -> C -> D -> E -> F

- Then each time you add an argument, you can get the type of the result by knocking off the first type in the series

f a1 : B -> C -> D -> E -> F (if a1 : A)

f a1 a2 : C -> D -> E -> F (if a2 : B)

f a1 a2 a3 : D -> E -> F (if a3 : C)

f a1 a2 a3 a4 a5 : F (if a4 : D and a5 : E)

OUR FIRST* COMPLEX DATA STRUCTURE! THE TUPLE

* it is really our second complex data structure since functions are data structures too!

Tuples

- A tuple is a fixed, finite, ordered collection of values
- Some examples with their types:

```
(1, 2)                           : int * int

("hello", 7 + 3, true)           : string * int * bool

('a', ("hello", "goodbye"))      : char * (string * string)
```

Tuples

- To use a tuple, we extract its components
- General case:

```
let (id1, id2, …, idn) = e1 in e2
```

- An example:

```
let (x,y) = (2,4) in x + x + y
```

Tuples

- To use a tuple, we extract its components
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```

- An example:

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let (x,y) = (2,4) in x + x + y
-->  2 + 2 + 4
```

substitute!

Tuples

- To use a tuple, we extract its components
- General case:

```
let (id1, id2, …, idn) = e1 in e2
```

- An example:

```
let (x,y) = (2,4) in x + x + y
-->  2 + 2 + 4
-->  8
```

Rules for Typing Tuples

if e1 : t1 and e2 : t2
then (e1, e2) : t1 * t2

Rules for Typing Tuples

if e1 : t1 and e2 : t2
then (e1, e2) : t1 * t2

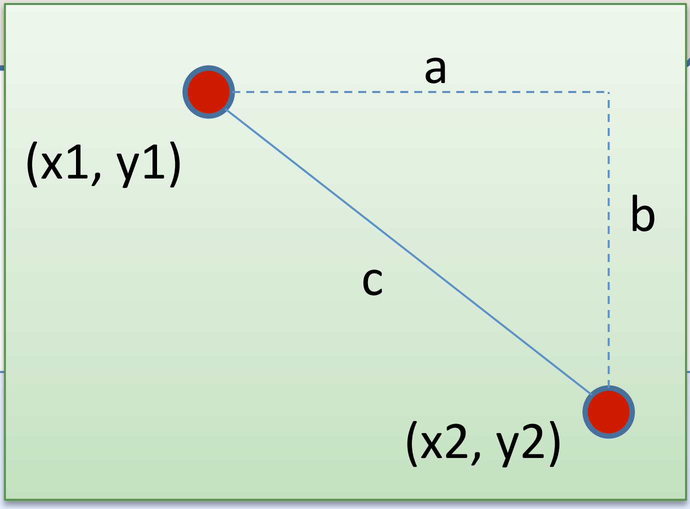
if e1 : t1 * t2 then
x1 : t1 and x2 : t2
inside the expression e2

```
let (x1,x2) = e1 in

e2
```

overall expression
takes on the type of e2

Distance between two points



$$c^2 = a^2 + b^2$$

Problem:

- A point is represented as a pair of floating point values.
- Write a function that takes in two points as arguments and returns the distance between them as a floating point number

Writing Functions Over Typed Data

Steps to writing functions over typed data:

1. Write down the function and argument names
2. Write down argument and result types
3. Write down some examples (in a comment)

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5. Build new output values
    - *the result type suggests how you do it*

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   - your code should be elegant and easy to read

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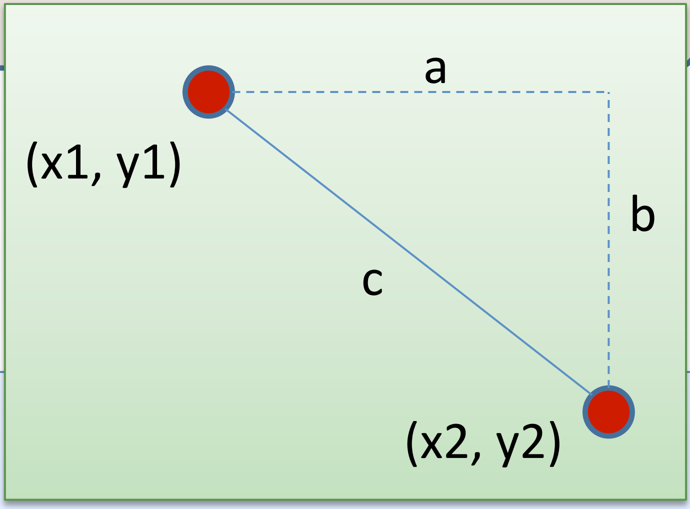
    - your code should be elegant and easy to read

Types help structure your thinking about how to write programs.

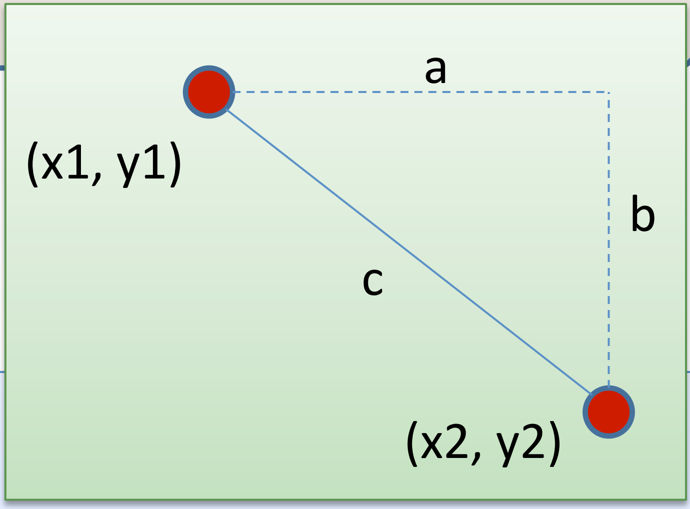
Distance between two points



a type abbreviation

```
type point = float * float
```

Distance between two points

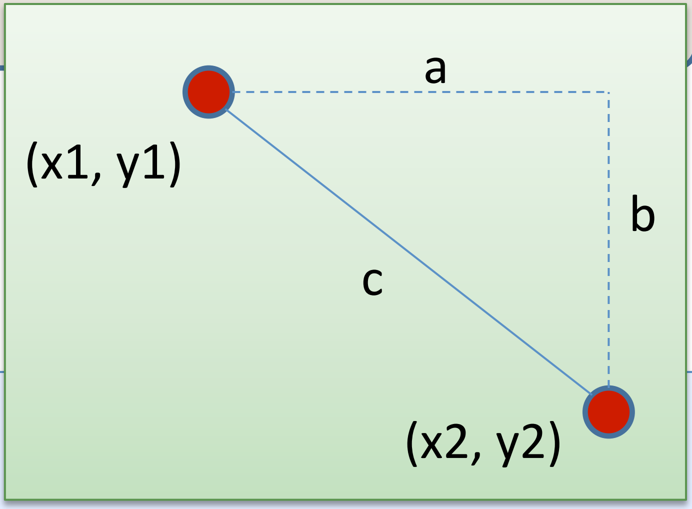



```
type point = float * float

let distance (p1:point) (p2:point) : float =
```

write down function name
argument names and types

Distance between two points



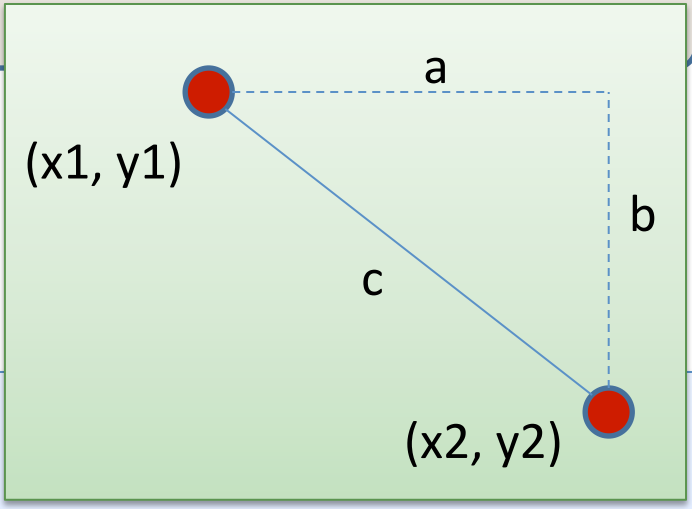
examples

```
type point = float * float


(* distance (0.0,0.0) (0.0,1.0) == 1.0
 * distance (0.0,0.0) (1.0,1.0) == sqrt(1.0 + 1.0)
 *
 * from the picture:
 * distance (x1,y1) (x2,y2) == sqrt(a^2 + b^2)
 *)


let distance (p1:point) (p2:point) : float =
```

Distance between two points



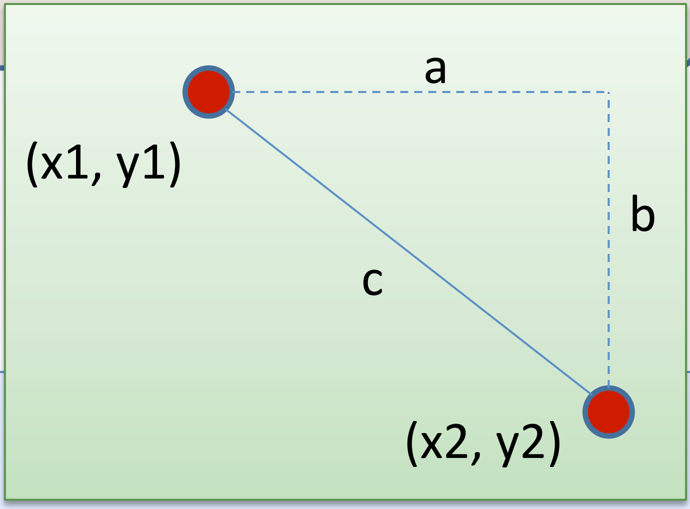

```
type point = float * float

let distance (p1:point) (p2:point) : float =

   let (x1,y1) = p1 in
   let (x2,y2) = p2 in
   ...
```

deconstruct
function inputs

Distance between two points




```
type point = float * float

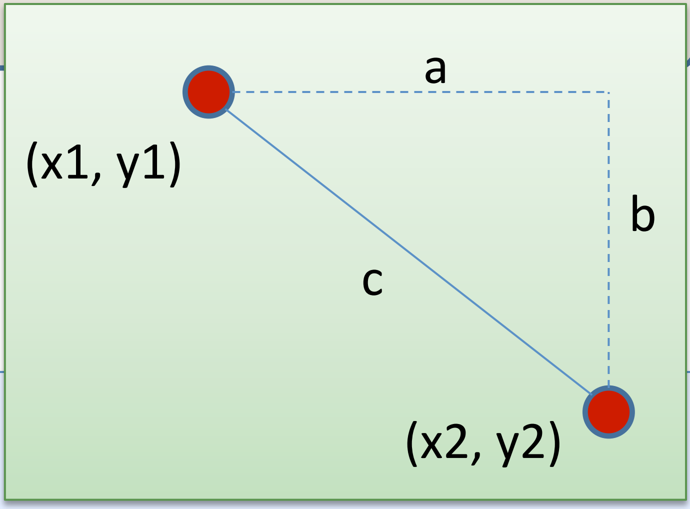
let distance (p1:point) (p2:point) : float =

  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt ((x2 -. x1) *. (x2 -. x1) +.
        (y2 -. y1) *. (y2 -. y1))
```

compute function results

notice operators on floats have a "." in them

Distance between two points

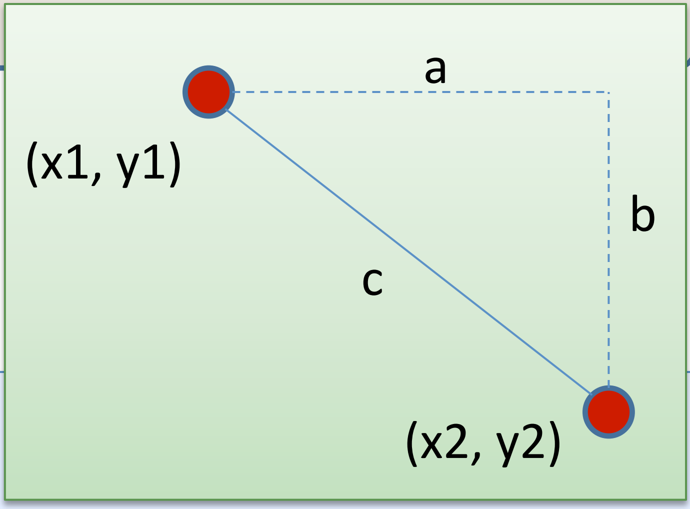



```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1)) +.
       square (y2 -. y1))
```

define helper functions to avoid repeated code

Distance between two points




```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))


let pt1 = (2.0,3.0)
let pt2 = (0.0,1.0)
let dist12 = distance pt1 pt2
```

testing

MORE TUPLES

Tuples

- Here's a tuple with 2 fields:

(4.0, 5.0) : float * float

Tuples

- Here's a tuple with 2 fields:

  (4.0, 5.0) : float * float

- Here's a tuple with 3 fields:

  (4.0, 5, "hello") : float * int * string

Tuples

- Here's a tuple with 2 fields:

(4.0, 5.0) : float * float

- Here's a tuple with 3 fields:

(4.0, 5, "hello") : float * int * string

- Here's a tuple with 4 fields:

(4.0, 5, "hello", 55) : float * int * string * int

Tuples

- Here's a tuple with 2 fields:

  (4.0, 5.0) : float * float

- Here's a tuple with 3 fields:

  (4.0, 5, "hello") : float * int * string

- Here's a tuple with 4 fields:

  (4.0, 5, "hello", 55) : float * int * string * int

- Here's a tuple with 0 fields:

  () : unit

Unit

- Unit is the tuple with zero fields!

```
() : unit
```

- the unit value is written with an pair of parens
- there are no other values with this type!

Unit

- Unit is the tuple with zero fields!

() : unit

- the unit value is written with an pair of parens
- there are no other values with this type!

- Why is the unit type and value useful?
- Every expression has a type:

(print_string "hello world\n") : ???

Unit

- Unit is the tuple with zero fields!

```
() : unit
```

- the unit value is written with an pair of parens
- there are no other values with this type!

- Why is the unit type and value useful?
- Every expression has a type:

```
(print_string "hello world\n")  :   unit
```

- Expressions executed for their *effect* return the unit value

SUMMARY:
BASIC FUNCTIONAL PROGRAMMING

Writing Functions Over Typed Data

Steps to writing functions over typed data:

1. Write down the function and argument names
2. Write down argument and result types
3. Write down some examples (in a comment)
4. Deconstruct input data structures
5. Build new output values
6. Clean up by identifying repeated patterns

For tuple types:

- when the input has type t1 * t2
  - use let (x,y) = … to deconstruct
- when the output has type t1 * t2
  - use (e1, e2) to construct

We will see this paradigm repeat itself over and over