

COS 326 Functional Programming: An elegant weapon for the modern age

David Walker Princeton University



Alonzo Church, 1903-1995 Princeton Professor, 1929-1967 In 1936, Alonzo Church invented the lambda calculus. He called it a logic, but it was a language of pure functions -- the world's first programming language.

He said:

"There may, indeed, be other applications of the system than its use as a logic."



Greatest technological understatement of the 20th century?

He said:

Alonzo Church, 1903-1995 Princeton Professor, 1929-1967 "There may, indeed, be other applications of the system than its use as a logic."



Alonzo Church 1934 -- developed lambda calculus



Programming Languages



Alan Turing (PhD Princeton 1938)
1936 -- developed Turing machines



Computers

Optional reading: **The Birth of Computer Science at Princeton in the 1930s** by Andrew W. Appel, 2012. http://press.princeton.edu/chapters/s9780.pdf

A few designers of functional programming languages



Alonzo Church: λ-calculus, 1934



John McCarthy (PhD Princeton 1951) LISP, 1958



Guy Steele & Gerry Sussman: Scheme, 1975

A few designers of functional programming languages



Alonzo Church: λ-calculus, 1934



Robin Milner ML, 1978



Appel & MacQueen: SML/NJ, 1988

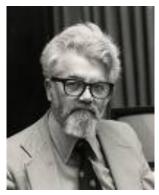


Xavier Leroy: Ocaml, 1990's

They were younger than they appear...



Alonzo Church: λ -calculus, 1934 Photo ~1960



John McCarthy (PhD Princeton 1951) LISP, 1958 Photo ~1975



Guy Steele & Gerry Sussman: Scheme, 1975 Photo ~1995 Photo ~1995



Robin Milner ML, 1978

Luca Cardelli Edinburgh ML, 1981 Photo ~1995





Appel & MacQueen: SML/NJ, 1988 Photo ~2000 Photo 2005

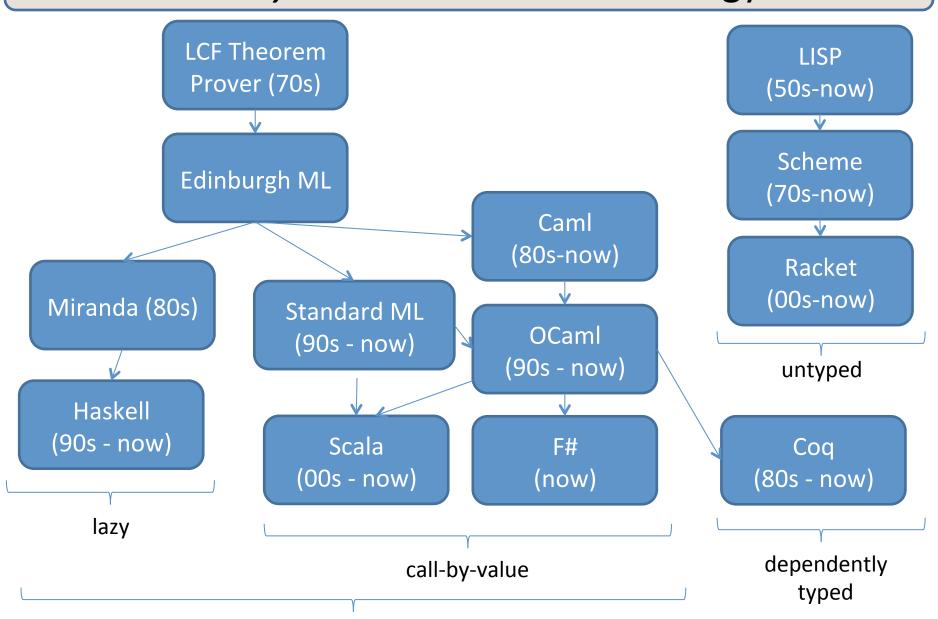


Xavier Leroy: Ocaml, 1990's

Photo ~200

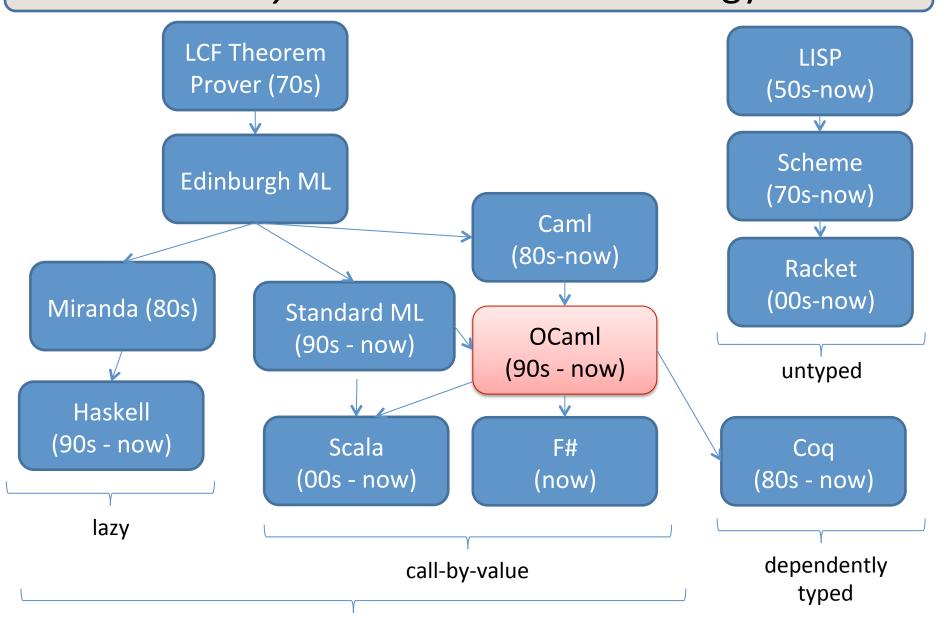
Photo ~2005

Vastly Abbreviated FP Geneology



typed, polymorphic

Vastly Abbreviated FP Geneology



typed, polymorphic

But Why Functional Programming Now?

- Functional programming will introduce you to new ways to think about and structure your programs:
 - new reasoning principles
 - new abstractions
 - new design patterns
 - new algorithms
 - elegant code
- Technology trends point to increasing parallelism:
 - multicore, gpu, data center
 - functional programming techniques such as map-reduce provide a plausible way forward for many applications

Functional Languages: Who's using them?

map-reduce in their data centers





Scala for correctness, maintainability, flexibility



Microsoft* Be what's next.

F# in Visual Studio

mathematicians

Erlang for concurrency, Haskell for managing PHP

Coq (re)proof of 4-color theorem



Haskell to synthesize hardware



www.artima.com/scalazine/articles/twitter_on_scala.html
gregosuri.com/how-facebook-uses-erlang-for-real-time-chat
www.janestcapital.com/technology/ocaml.php
msdn.microsoft.com/en-us/fsharp/cc742182
labs.google.com/papers/mapreduce.html
www.haskell.org/haskellwiki/Haskell in industry

Haskell for specifying equity derivatives

Functional Languages: Join the crowd

- Elements of functional programming are showing up all over
 - F# in Microsoft Visual Studio
 - Scala combines ML (a functional language) with Objects
 - runs on the JVM
 - C# includes "delegates"
 - delegates == functions
 - Python includes "lambdas"
 - lambdas == more functions
 - Javascript
 - find tutorials online about using functional programming techniques to write more elegant code
 - C++ libraries for map-reduce
 - enabled functional parallelism at Google
 - Java has generics and GC

– ...

COURSE LOGISTICS

Course Staff

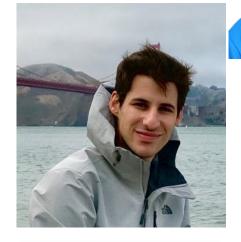


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Resources

- Web:
 - http://www.cs.princeton.edu/~cos326
- Lecture schedule and readings:
 - \$(coursehome)/lectures.php
- Assignments:
 - \$(coursehome)/assignments.php
- Precepts
 - useful if you want to do well on exams and homeworks
- Install OCaml: \$(coursehome)/resources.php

Collaboration Policy

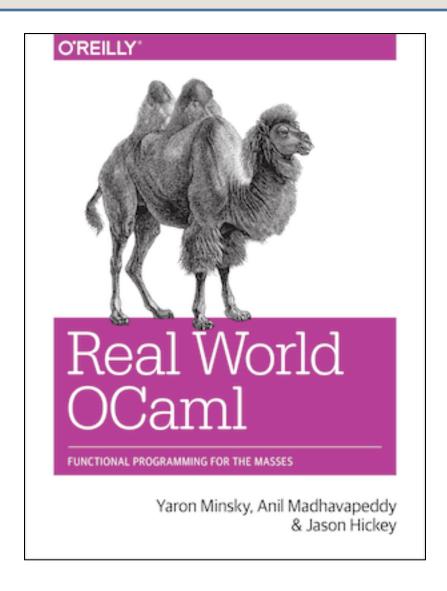
The COS 326 collaboration policy can be found here:

http://www.cs.princeton.edu/~cos326/info.php#collab

Read it in full prior to beginning the first assignment.

Please ask questions whenever anything is unclear, at any time during the course.

Course Textbook



http://realworldocaml.org/

Exams

Minterm: Wednesday of midterm week:

Wednesday, October 25

There will be a final exam, in exam period (January — Make your travel plans accordingly)

Assignment 0

Figure out how to download and install the latest version of OCaml

on your machine by the time precept begins tomorrow.

(or, how to use OCaml by ssh to Princeton University servers)

Resources Page:

http://www.cs.princeton.edu/~cos326/resources.php

Hint:

ocaml.org

Public Service Announcement

The Pen is Mighter than the Keyboard: Advantages of Longhand Over Laptop Note Taking

Pam Mueller (Princeton University)

Daniel Oppenheimer (UCLA)

Journal of Psychological Science, June 2014, vol 25, no 6

http://pss.sagepub.com/content/25/6/1159.fullkeytype=ref&siteid=sppss&ijkey=CjRAwmrlURGNw

- You learn conceptual topics better by taking notes by hand.
- Instagram and World of Warcraft distract your classmates.

A Functional Introduction

Thinking Functionally

In Java or C, you get (most) work done by changing something

```
temp = pair.x;
pair.x = pair.y;
pair.y = temp;

commands modify or change an existing data structure (like pair)
```

In ML, you get (most) work done by producing something new

```
let
(x,y) = pair
in
(y,x)

you analyze existing data (like pair)
and you produce new data (y,x)
```

This simple switch in perspective can change the way you think

about programming and problem solving.

Thinking Functionally

pure, functional code:

- outputs are everything!
- output is <u>function</u> of input
- data properties are stable
- repeatable
- parallelism apparent
- easier to test
- easier to compose

imperative code:

```
temp = pair.x;
pair.x = pair.y;
pair.y = temp;
```

- outputs are irrelevant!
- output is not function of input
- data properties change
- unrepeatable
- parallelism hidden
- harder to test
- harder to compose

Why OCaml?

Small, orthogonal core based on the lambda calculus.

- Control is based on (recursive) functions.
- Instead of for-loops, while-loops, do-loops, iterators, etc.
 - can be defined as library functions.
- Makes it easy to define semantics



- a.k.a. first-class functions or closures or lambdas.
- first-class: functions are data values like any other data value
 - like numbers, they can be stored, defined anonymously, ...
- lexically-scoped: meaning of variables determined statically.
- higher-order: functions as arguments and results
 - programs passed to programs; generated from programs

These features also found in Racket, Haskell, SML, F#, Clojure,

Why OCaml?

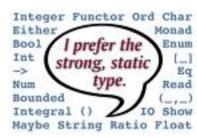
Statically typed: debugging and testing aid

- compiler catches many silly errors before you can run the code.
 - A type is worth a thousand tests (start at 6:20):
 - https://www.youtube.com/watch?v=q1Yi-WM7XqQ
- Java is also strongly, statically typed.
- Scheme, Python, Javascript, etc. are all strongly, dynamically typed type errors are discovered while the code is running.

Strongly typed: compiler enforces type abstraction.

- cannot cast an integer to a record, function, string, etc.
 - so we can utilize types as capabilities; crucial for local reasoning
- C/C++ are weakly-typed (statically typed) languages. The compiler will happily let you do something smart (more often stupid).

Type inference: compiler fills in types for you



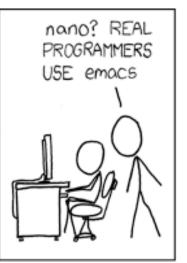
Installing, running Ocaml

- OCaml comes with compilers:
 - "ocamlc" fast bytecode compiler
 - "ocamlopt" optimizing, native code compiler
 - "ocamlbuild a nice wrapper that computes dependencies
- And an interactive, top-level shell:
 - occasionally useful for trying something out.
 - "ocaml" at the prompt.
 - but use the compiler most of the time
- And many other tools
 - e.g., debugger, dependency generator, profiler, etc.
- See the course web pages for installation pointers
 - also OCaml.org

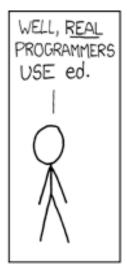
Editing Ocaml Programs

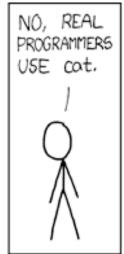
- Many options: pick your own poison
 - Emacs
 - what I'll be using in class.
 - good but not great support for OCaml.
 - I like it because it's what I'm used to
 - (extensions written in elisp a functional language!)
 - OCaml IDE
 - integrated development environment written in Ocaml.
 - haven't used it much, so can't comment.
 - Eclipse
 - I've put up a link to an Ocaml plugin
 - I haven't tried it but others recommend it
 - Sublime, atom
 - A lot of students seem to gravitate to this

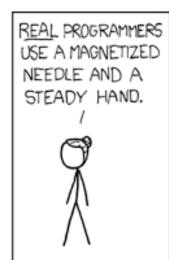
XKCD on Editors

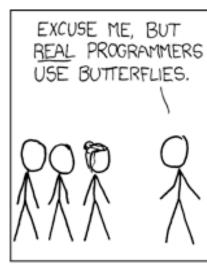












THEY OPEN THE IR
HANDS AND LET THE
DELICATE WINGS FLAP ONCE.

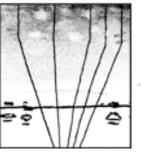
THE DISTURBANCE RIPPLES OUTWARD, CHANGING THE FLOW OF THE EDDY CURRENTS IN THE UPPER ATMOSPHERE.

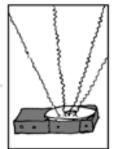


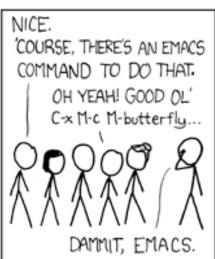


THESE CAUSE MOMENTARY POCKETS OF HIGHER-PRESSURE AIR TO FORM.

WHICH ACT AS LENSES THAT DEFLECT INCOMING COSMIC RAYS, FOCUSING THEM TO STRIKE THE DRIVE PLATTER AND FLIP THE DESIRED BIT.







AN INTRODUCTORY EXAMPLE (OR TWO)

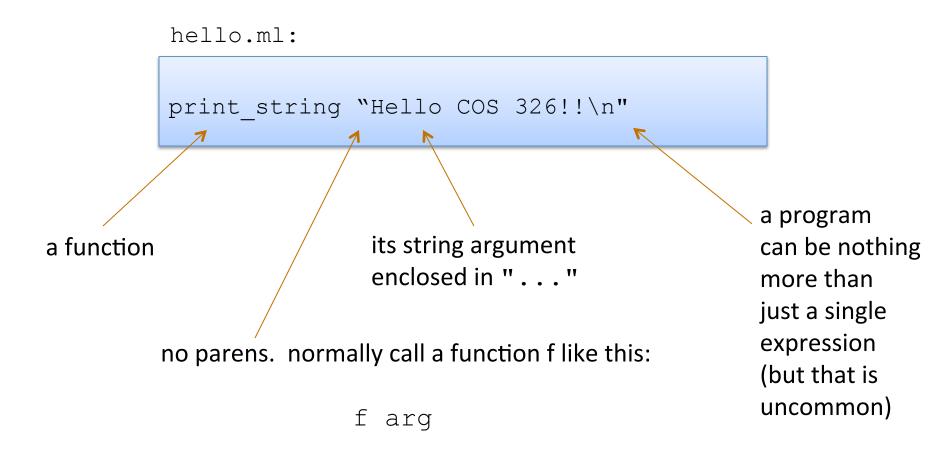
OCaml Compiler and Interpreter

Demo:

- emacs
- ml files
- writing simple programs: hello.ml, sum.ml
- simple debugging and unit tests
- ocamlc compiler

```
hello.ml:
```

```
print_string "Hello COS 326!!\n";;
```



(parens are used for grouping, precedence only when necessary)

```
hello.ml:
print string "Hello COS 326!!\n"
compiling and running hello.ml:
$ ocamlbuild hello.d.byte
 $ ./hello.d.byte
Hello COS 326!
 $
```

byte for interpreted bytecode.

(other choices .native for machine code)

.d for debugging

(other choices .p for profiled; or none)

```
hello.ml:
```

```
print_string "Hello COS 326!!\n"
```

interpreting and playing with hello.ml:

```
$ ocaml
Objective Caml Version 3.12.0
```

hello.ml:

```
print_string "Hello COS 326!!\n"
```

interpreting and playing with hello.ml:

```
$ ocaml
    Objective Caml Version 3.12.0
# 3 + 1;;
- : int = 4
#
```

A First OCaml Program

hello.ml:

```
print_string "Hello COS 326!!\n"
```

interpreting and playing with hello.ml:

```
$ ocaml
      Objective Caml Version 3.12.0
# 3 + 1;;
- : int = 4
# #use "hello.ml";;
hello cos326!!
- : unit = ()
#
```

A First OCaml Program

hello.ml:

```
print_string "Hello COS 326!!\n"
```

interpreting and playing with hello.ml:

```
$ ocaml
      Objective Caml Version 3.12.0
# 3 + 1;;
- : int = 4
# #use "hello.ml";;
hello cos326!!
- : unit = ()
# #quit;;
$
```

```
a comment
                                                (* ... *)
sumTo8.ml:
(* sum the numbers from 0 to n
   precondition: n must be a natural number
*)
let rec sumTo (n:int) : int =
  match n with
   0 -> 0
  \mid n \rightarrow n + sumTo (n-1)
let =
  print int (sumTo 8);
  print newline()
```

the name of the function being defined

```
sumTo8.ml:
(* sum the numbers from 0 to n
   precondition: n must be a natural number
*)
let rec sumTo (n:int) : int =

↑ match n with

    0 -> 0
  \mid n \rightarrow n + sumTo (n-1)
let =
  print int (sumTo 8);
  print newline()
```

the keyword "let" begins a definition; keyword "rec" indicates recursion

sumTo8.ml:

```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
*)
let rec sumTo (n:int) : int =
  match n with
   0 -> 0
                                                  result type int
  \mid n \rightarrow n + sumTo (n-1)
let =
                                                   argument
  print int (sumTo 8);
                                                   named n
  print newline()
                                                   with type int
```

deconstruct the value n using pattern matching

```
sumTo8.ml:
```

```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
*)
let rec sumTo (n:int) : int =
  match n with <
    0 -> 0
  \mid n \rightarrow n + sumTo (n-1)
let =
  print int (sumTo 8);
  print newline()
```

data to be
deconstructed
appears
between
key words
"match" and
"with"

vertical bar "|" separates the alternative patterns

```
sumTo8.ml:
(* sum the numbers from 0 to n
   precondition: n must be a natural number
*)
let rec sumTo (n:int) : int =
  match n with
    0 -> 0
    n \rightarrow n + sumTo (n-1)
  print int (sumTo 8);
  print newline()
```

deconstructed data matches one of 2 cases:

(i) the data matches the pattern 0, or (ii) the data matches the variable pattern n

Each branch of the match statement constructs a result

```
sumTo8.ml:
```

```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
*)
let rec sumTo (n:int) : int =
  match n with
    0 -> 0
  \mid n \rightarrow n + sumTo (n-1)
let =
  print int (sumTo 8);
  print newline()
```

construct the result 0

construct
a result
using a
recursive
call to sumTo

sumTo8.ml:

```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
*)
let rec sumTo (n:int) : int =
  match n with
    0 -> 0
  \mid n \rightarrow n + sumTo (n-1)
let =
  print int (sumTo 8);
  print newline()
```

print the result of calling sumTo on 8

print a new line

OCAML BASICS: EXPRESSIONS, VALUES, SIMPLE TYPES

Terminology: Expressions, Values, Types

- Expressions are computations
 - -2+3 is a computation
- Values are the results of computations
 - 5 is a value
- Types describe collections of values and the computations that generate those values
 - int is a type
 - values of type int include
 - 0, 1, 2, 3, ..., max_int
 - -1, -2, ..., min_int

Some simple types, values, expressions

<u>Type</u> : <u>Expressions:</u>	
int -2 , 0, 42 $42 * (13 + 1)$	
float 3.14, -1., 2e12 (3.14 +. 12.0) *.	10e6
char 'a', 'b', '&' int_of_char 'a'	
string "moo", "cow" "moo" ^ "cow"	
bool true, false if true then 3 els	se 4
unit () print_int 3	

For more primitive types and functions over them, see the OCaml Reference Manual here:

http://caml.inria.fr/pub/docs/manual-ocaml/libref/Pervasives.html

Not every expression has a value

Expression:

```
42 * (13 + 1) evaluates to 588

(3.14 +. 12.0) *. 10e6 → 1514000000.

int_of_char 'a' → 97

"moo" ^ "cow" → "moocow"

if true then 3 else 4 → 3

print_int 3 → ()
```

1 + "hello" does not evaluate!

Language Definition

- There are a number of ways to define a programming language
- In this class, we will briefly investigate:
 - Syntax
 - Evaluation
 - Type checking
- Standard ML, a very close relative of OCaml, has a full definition of each of these parts and a number of proofs of correctness
 - For more on this theme, see COS 441/510
- The OCaml Manual fleshes out the syntax, evaluation and type checking rules informally

OCAML BASICS: CORE EXPRESSION SYNTAX

Core Expression Syntax

The simplest OCaml expressions e are:

- values
- id
- e₁ op e₂
- id e₁ e₂ ... e_n
- **let** id = e₁ **in** e₂
- if e₁ then e₂ else e₃
- (e)
- (e:t)

numbers, strings, bools, ...

variables (x, foo, ...)

operators (x+3, ...)

function call (foo 3 42)

local variable decl.

a conditional

a parenthesized expression

an expression with its type

A note on parentheses

In most languages, arguments are parenthesized & separated by commas:

$$f(x,y,z)$$
 sum $(3,4,5)$

In OCaml, we don't write the parentheses or the commas:

$$f x y z \qquad sum 3 4 5$$

But we do have to worry about grouping. For example,

The first one passes three arguments to f (x, y, and z)

The second passes two arguments to f (x, and the result of applying the function y to z.)

OCAML BASICS: TYPE CHECKING

Type Checking

- Every value has a type and so does every expression
- This is a concept that is familiar from Java but it becomes more important when programming in a functional language
- The type of an expression is determined by the type of its subexpressions
- We write (e:t) to say that expression e has type t. eg:

2: int "hello": string

2 + 2 : int "I say " ^ "hello" : string

- There are a set of simple rules that govern type checking
 - programs that do not follow the rules will not type check and
 O'Caml will refuse to compile them for you (the nerve!)
 - at first you may find this to be a pain ...
- But types are a great thing:
 - they help us think about how to construct our programs
 - they help us find stupid programming errors
 - they help us track down compatibility errors quickly when we edit and maintain our code
 - they allow us to enforce powerful invariants about our data structures

Example rules:

```
(1) 0: int (and similarly for any other integer constant n)
```

(2) "abc": string (and similarly for any other string constant "...")

Example rules:

then e1 + e2: int

```
    (1) 0: int (and similarly for any other integer constant n)
    (2) "abc": string (and similarly for any other string constant "...")
    (3) if e1: int and e2: int (4) if e1: int and e2: int
```

then e1 * e2 : int

Example rules:

```
    (1) 0: int (and similarly for any other integer constant n)
    (2) "abc": string (and similarly for any other string constant "...")
    (3) if e1: int and e2: int then e1 + e2: int then e1 * e2: int
```

- (5) if e1: string and e2: string then e1 ^ e2: string
- (6) if e: int then string of int e: string

Example rules:

```
(1)
     0 : int
                        (and similarly for any other integer constant n)
     "abc" : string
                        (and similarly for any other string constant "...")
(2)
                                              if e1: int and e2: int
(3)
     if e1: int and e2: int
     then e1 + e2: int
                                              then e1 * e2 : int
(5)
                                         (6)
     if e1: string and e2: string
                                              if e:int
     then e1 ^ e2 : string
                                              then string_of_int e : string
```

```
2: int and 3: int. (By rule 1)
```

Example rules:

```
(1)
     0 : int
                        (and similarly for any other integer constant n)
     "abc" : string
                        (and similarly for any other string constant "...")
(2)
                                              if e1: int and e2: int
(3)
     if e1: int and e2: int
     then e1 + e2: int
                                              then e1 * e2 : int
(5)
                                         (6) if e: int
     if e1: string and e2: string
     then e1 ^ e2 : string
                                              then string of int e: string
```

```
2: int and 3: int. (By rule 1)
Therefore, (2 + 3): int (By rule 3)
```

Example rules:

```
(1)
     0 : int
                        (and similarly for any other integer constant n)
     "abc" : string
                        (and similarly for any other string constant "...")
(2)
                                              if e1: int and e2: int
(3)
    if e1: int and e2: int
                                         (4)
     then e1 + e2: int
                                              then e1 * e2 : int
(5)
                                         (6) if e: int
     if e1: string and e2: string
     then e1 ^ e2 : string
                                              then string of int e: string
```

```
2: int and 3: int. (By rule 1)
Therefore, (2 + 3): int (By rule 3)
5: int (By rule 1)
```

Example rules:

```
(1)
     0 : int
                       (and similarly for any other integer constant n)
    "abc" : string
                      (and similarly for
(2)
                                       FYI: This is a formal proof
    if e1: int and e2: int
(3)
                                      that the expression is well-
    then e1 + e2: int
                                                  typed!
     if e1: string and e2: string
(5)
     then e1 ^ e2 : string
                                                  anng_on_int e : string
```

```
2: int and 3: int. (By rule 1)
Therefore, (2 + 3): int (By rule 3)
5: int (By rule 1)
Therefore, (2 + 3) * 5: int (By rule 4 and our previous work)
```

Example rules:

```
(1) 0: int (and similarly for any other integer constant n)
(2) "abc": string (and similarly for any other string constant "...")
(3) if e1: int and e2: int (4) if e1: int and e2: int
```

- (5) if e1: string and e2: string then e1 ^ e2: string
- (6) if e: int

then string of int e: string

then e1 * e2 : int

Another perspective:

then e1 + e2: int

rule (4) for typing expressions says I can put any expression with type int in place of the ????



Example rules:

```
    (1) 0: int (and similarly for any other integer constant n)
    (2) "abc": string (and similarly for any other string constant "...")
    (3) if e1: int and e2: int (4) if e1: int and e2: int
```

- (5) if e1: string and e2: string then e1 ^ e2: string
- (6) if e: int then string of int e: string

then e1 * e2 : int

Another perspective:

then e1 + e2: int

rule (4) for typing expressions says I can put any expression with type int in place of the ????



Example rules:

```
    (1) 0: int (and similarly for any other integer constant n)
    (2) "abc": string (and similarly for any other string constant "...")
    (3) if e1: int and e2: int then e1 + e2: int then e1 * e2: int
```

- (5) if e1: string and e2: string then e1 ^ e2: string
- (6) if e : int then string_of_int e : string

(add_one 17) : int

Another perspective:

```
rule (4) for typing expressions says I can put any expression with type int in place of the ????
```

```
$ ocaml
Objective Caml Version 3.12.0
```

```
$ ocaml
Objective Caml Version 3.12.0
# 3 + 1;;
```

```
$ ocaml
                    Objective Caml Version 3.12.0
              3 + 1;;
               : int = 4
            #
press
return
and you
find out
the type
and the
value
```

 You can always start up the OCaml interpreter to find out a type of a simple expression:

```
$ ocaml
      Objective Caml Version 3.12.0
# 3 + 1;;
- : int = 4
# "hello " ^ "world";;
- : string = "hello world"
# ""
```

press
return
and you
find out
the type
and the
value

```
$ ocaml
    Objective Caml Version 3.12.0
# 3 + 1;;
- : int = 4
# "hello " ^ "world";;
- : string = "hello world"
# #quit;;
$
```

Example rules:

```
(1)
     0 : int
                        (and similarly for any other integer constant n)
     "abc" : string
                        (and similarly for any other string constant "...")
(2)
(3)
     if e1: int and e2: int
                                         (4)
                                              if e1: int and e2: int
     then e1 + e2: int
                                              then e1 * e2 : int
(5)
                                         (6) if e: int
     if e1: string and e2: string
     then e1 ^ e2 : string
                                              then string of int e: string
```

Violating the rules:

```
"hello" : string (By rule 2)
1 : int (By rule 1)
1 + "hello" : ?? (NO TYPE! Rule 3 does not apply!)
```

Violating the rules:

```
# "hello" + 1;;
Error: This expression has type string but an
expression was expected of type int
```

- The type error message tells you the type that was expected and the type that it inferred for your subexpression
- By the way, this was one of the nonsensical expressions that did not evaluate to a value
- It is a good thing that this expression does not type check!

"Well typed programs do not go wrong" Robin Milner, 1978

Violating the rules:

```
# "hello" + 1;;
Error: This expression has type string but an
expression was expected of type int
```

A possible fix:

```
# "hello" ^ (string_of_int 1);;
- : string = "hello1"
```

 One of the keys to becoming a good ML programmer is to understand type error messages.

What about this expression:

```
# 3 / 0 ;;
Exception: Division_by_zero.
```

 Why doesn't the ML type checker do us the favor of telling us the expression will raise an exception?

What about this expression:

```
# 3 / 0 ;;
Exception: Division_by_zero.
```

- Why doesn't the ML type checker do us the favor of telling us the expression will raise an exception?
 - In general, detecting a divide-by-zero error requires we know that the divisor evaluates to 0.
 - In general, deciding whether the divisor evaluates to 0 requires solving the halting problem:

```
# 3 / (if turing_machine_halts m then 0 else 1);;
```

 There are type systems that will rule out divide-by-zero errors, but they require programmers supply proofs to the type checker

Isn't that cheating?

"Well typed programs do not go wrong" Robin Milner, 1978

(3 / 0) is well typed. Does it "go wrong?" Answer: No.

"Go wrong" is a technical term meaning, "have no defined semantics." Raising an exception is perfectly well defined semantics, which we can reason about, which we can handle in ML with an exception handler.

So, it's not cheating.

(Discussion: why do we make this distinction, anyway?)

Type Soundness

"Well typed programs do not go wrong"

Programming languages with this property have sound type systems. They are called safe languages.

Safe languages are generally *immune* to buffer overrun vulnerabilities, uninitialized pointer vulnerabilities, etc., etc. (but not immune to all bugs!)

Safe languages: ML, Java, Python, ...

Unsafe languages: C, C++, Pascal

Well typed programs do not go wrong



Robin Milner

Turing Award, 1991

"For three distinct and complete achievements:

- 1. LCF, the mechanization of Scott's Logic of Computable Functions, probably the first theoretically based yet practical tool for machine assisted proof construction;
- 2. ML, the first language to include polymorphic type inference together with a type-safe exception-handling mechanism;
- 3. CCS, a general theory of concurrency.

In addition, he formulated and strongly advanced full abstraction, the study of the relationship between operational and denotational semantics."

"Well typed programs do not go wrong" Robin Milner, 1978

OVERALL SUMMARY: A SHORT INTRODUCTION TO FUNCTIONAL PROGRAMMING

OCaml

OCaml is a *functional* programming language

- Java gets most work done by modifying data
- OCaml gets most work done by producing new, immutable data

OCaml is a typed programming language

- the type of an expression correctly predicts the kind of value the expression will generate when it is executed
- types help us understand and write our programs
- the type system is sound; the language is safe