PRINCETON UNIVERSITY

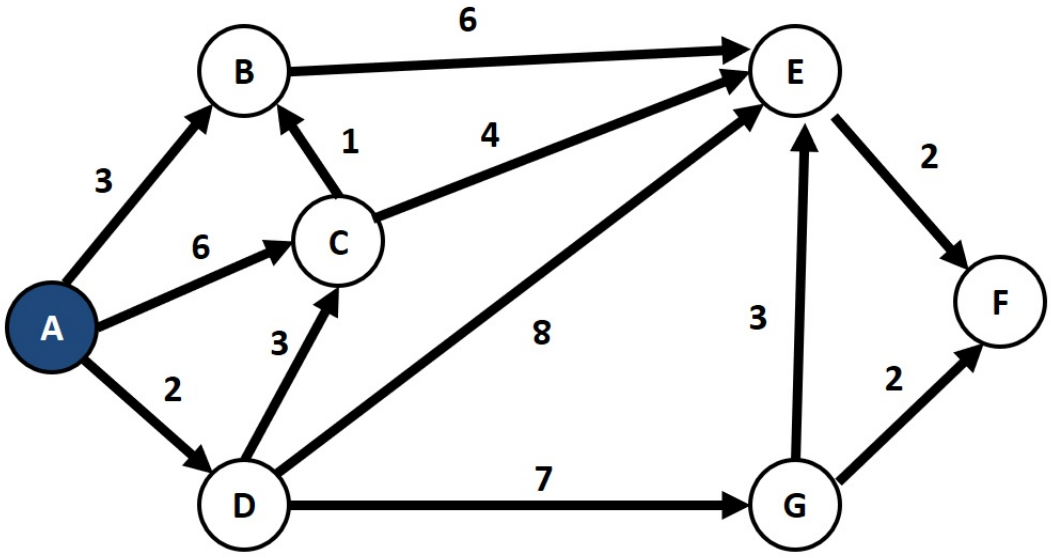
COS 226–Algorithms and Data Structures

Week 9: *Shortest Path and MST(Algs. §4.4 & videos §17.C,D,E)*

Version: November 17, 2017

Exercise 1 – Shortest Paths (Dijkstra)

A. Consider the following edge-weighted digraph.



Suppose we start with the source vertex A. Complete the table below until the first 6 vertices are removed from the priority queue.

vertex	A	B	C	D	E	F	G
distTo							
edgeTo							

B. Consider a different digraph of 8 vertices than the one given in Part A. The table below shows the shortest paths tree *edgeTo* and *distTo* immediately after vertex 4 has been relaxed.

v	distTo[]	edgeTo[]
0	∞	*null*
1	7.0	5
2	13.0	3
3	0.0	*null*
4	10.0	7
5	3.0	3
6	12.0	1
7	8.0	3

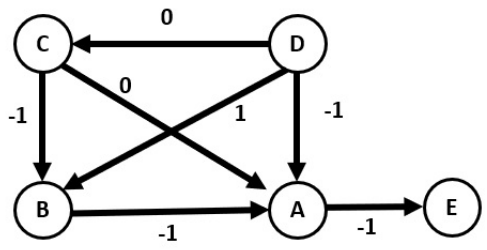
1. Based on the table above, which one is the source vertex?

2. Give the order in which the first 5 vertices were deleted from the priority queue and relaxed.

3. Is it possible to know the next vertex that will be relaxed? and why?

4. Using the *edgeTo* and *distTo* arrays, draw the shortest path tree generated from the source vertex (so far).

C. Consider the following edge-weighted digraph that has negative edge weights.



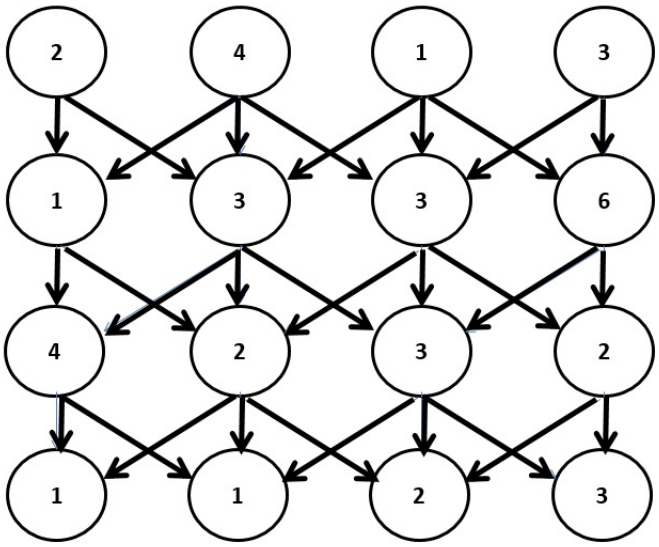
1. Show that Dijkstraś algorithm fails to find the shortest path tree starting from source vertex D.

2. Apply the Bellman-Ford algorithm to find the shortest path distances from D to other vertices. Use the vertices in alphabetical order when relaxing them. Show 3 passes of the algorithm

	A	B	C	D	E
Pass 1					
Pass 2					
Pass 3					

Exercise 2 – Directed acyclic graphs

Consider the directed acyclic graph below. Instead of weights on the edges, we introduce weights on the vertices. In the context of seam carver assignment, you can consider this to be graph that represents energies calculated for a small 4x4 (wxh) image.



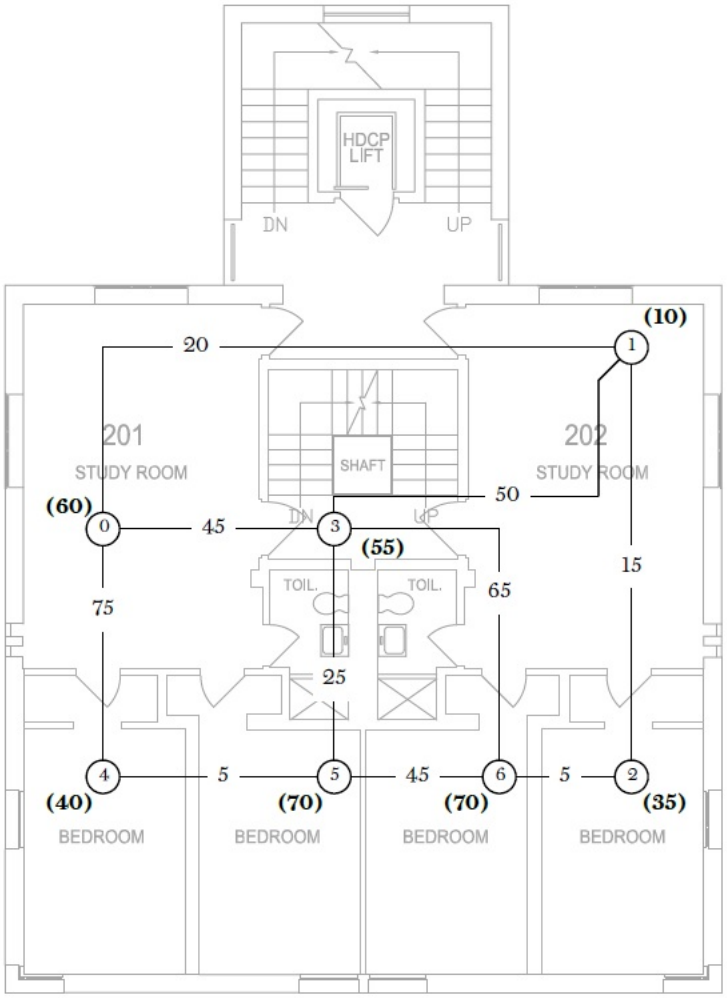
A. Find a path that minimizes the total weight from any vertex in the first row to any vertex on the bottom row in the given graph.

B. Discuss a possible algorithm that can be applied to find a path that minimizes the total weight from any vertex in the first row to any vertex on the bottom row.

C. Assume that there are n vertices in the graph and outdegree of each vertex is at most 3. What is the order of growth of your algorithm in terms of n?

Exercise 3 – Algorithm Design - Dorm Room connectivity problem (bonus)

Seven dorm rooms (numbered from 0 to 6) must be connected using a router or a hard wiring. The router cost for each room is shown in parentheses (for example, router cost for room 0 is 60). The wiring cost for connecting two rooms is shown as edge weights. (for example, wiring cost between rooms 0 and 3 is 45). Find the minimum cost to connect rooms to the internet using either a router or a hard wire between rooms.



A. Formulate the problem as a minimum spanning tree problem. To demonstrate your formulation, modify the figure above to show the MST problem that you would solve.

B. Using one of the MST algorithms, solve the problem and state the configuration (router/wire) that enable the optimal solution.