

COMPUTER SCIENCE
SEDGEWICK/WAYNE
PART II: ALGORIthms, MACHINES, and theory

## 17. A Computing Machine

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## A TOY computing machine

TOY is an imaginary machine similar to

- Ancient computers.
- Today's smartphone processors.
- Countless other devices designed and built over the past 50 years.


Reasons to study TOY

Prepare to learn about computer architecture

- How does your computer's processor work?
-What are its basic components?
- How do they interact?

Learn about machine-language programming.

- How do Java programs relate to your computer?

- Intellectual challenge of a new programming regime.
- Still necessary in some modern applications.
multimedia, computer games, embedded devices, scientific computing,...

Learn fundamental abstractions that have informed processor design for decades.

## Bits and words

Everything in TOY is encoded with a sequence of bits (value 0 or 1)

- Why? Easy to represent two states (on and off) in real world.
- Bits are organized in 16 -bit sequences called words.


More convenient for humans: hexadecimal notation (base 16)

- 4 hex digits in each word
- Convert to and from binary 4 bits at a time.



## Memory

Holds data and instructions

- 256 words
- 16 bits in each word
- Connected to registers
- Words are addressable

Use hexadecimal for addresses

- Number words from 00 to FF
- Think in hexadecimal
- Use array notation
- Example: M[2A] = C024


Table of 256 words completely specifies contents of memory.

Components of TOY machine

## - Memory

- Registers
- Arithmetic and logic unit (ALU)
- Program counter (PC)
- Instruction register (IR)


## Arithmetic and logic unit (ALU)

## ALU

- TOY's computational engine
- A calculator, not a compute
- Hardware that implements all data-type operations
- How? Stay tuned for computer architecture lectures



## Registers

Registers

- 16 words, addressable in hex from 0 to $F$ (use names R[0] through R[F])
- Scratch space for calculations and data movement.
- Connected to memory and ALU
- By convention, R[0] is always $0 . \longleftarrow$ often simplifies code (stay tuned In our code, we often also keep 0001 in R[1].
Q. Why not just connect memory directly to ALU?
A. Too many different memory names (addresses).
Q. Why not just connect memory locations to one another?
A. Too many different connections.

| Registers |  |
| :---: | :---: |
| R[0] 0 | ] 0000 |
| R[1] 0 | ] 0001 |
| $\mathrm{R}[2] \mathrm{F}$ | ] FFFE |
| R[3] 1 | ] 1 CAB |
| R[4] 0 | ] 0001 |
| $\mathrm{R}[5] 0$ | ] 0000 |
| $\mathrm{R}[6] \mathrm{F}$ | ] FACE |
| R[7] 0 | ] 000 |
| R [8] F | F 001 |
| R [9] 0 | ] 000 |
| R[A] 0 | ] 0005 |
| $\mathrm{R}[\mathrm{B}] 0$ | ] 0008 |
| $\mathrm{R}[\mathrm{C}] 0$ | ] 0000 |
| R[D] 0 | ] 0000 |
| R[E] 0 | ] 0000 |
| R[F] 0 | ] 0000 |

R[0] 0000

R[3] 1 CAB
 R[5] 00000 R[6] FACE
 R[9] 0000 R[A] 0005 R[B] 00008 $R[C] 0000$ $\left.\begin{array}{llll}\mathrm{R}[\mathrm{D}] & 0 & 0 & 0\end{array}\right]$ R[F] 0000

## Program counter and instruction register

TOY operates by executing a sequence of instructions.

Critical abstractions in making this happen

- Instruction Register (IR). Instruction being executed


Fetch-increment-execute cycle

- Fetch: Get instruction from memory into IR.
- Increment: Update PC to point to next instruction.
- Execute: Move data to or from memory, change PC or perform calculations, as specified by $\mathbb{R}$.


## The state of the machine

Contents of memory, registers, and PC at a particular time

- Provide a record of what a program has done.
- Completely determines what the machine will do.

ALU and IR hold intermediate states of computation

| IR |  |
| :--- | :--- | :--- |
| Memory |  |
|  |  |
|  |  |
|  |  |

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Image sources
http://pixabay
http://en.wikipedia.org/wiki/Marchant_calculator\#/media/File:Marchant_-_Odhner_clone_1950.png http://en.wikipedia.org/wiki/Marchant_calculator\#/media/File:SCM_Marchant_calculator.jpg http:///commons.wikimedia.org/wiki/File: Cal culator_casio. jpg http://commons.wikimedia.riki/File Abacus_5.jpg

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## 17. A Computing Machine

- Overview


## - Data types

- Instructions
- Operating the machine
- Machine language programming

CS.17.B.MachineI.Types



TOY data type
A data type is a set of values and a set of operations on those values. TOY's data type is 16 -bit two's complement integers.

Two kinds of operations

- Arithmetic.

- Bitwise.

All other types of data must be implemented with software

- 32-bit and 64-bit integers.
-32-bit and 64-bit floating point values
- Characters and strings.
- ...

All values are represented in 16 -bit words.


## TOY data type (original design): Unsigned integers

Values. 0 to $2^{16}-1$, encoded in binary (or, equivalently, hex).

Operations

- Add
- Subtract.
- Test if 0 .

[^0]\[

$$
\begin{array}{lllllllllllllllll}
\text { binary } & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1
\end{array}
$$
\]

$$
\begin{array}{ccccc}
\text { hex } & 1 & 8 & \text { E } & 7 \\
& 1 \times 16^{3} & +8 \times 16^{2} & +14 \times 16 & +7 \\
& 4096 & +2048 & +224 & +7
\end{array}
$$

Example. 18E7 + 18E7 = 31CE

$$
\begin{aligned}
& \quad 0
\end{aligned} \begin{array}{ll|l|l|l|l|l|l|l|l|l|l|l} 
& 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 \\
+ & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{array} 00
$$

## TOY data type (better design): two's complement

| Values. - $2^{15}$ to $2^{15}-1$, encoded in 16 -bit two's complement. | decimal | hex | binary |
| :---: | :---: | :---: | :---: |
|  | +32,767 | 7FFF | 011111111111111 |
| Operations. includes negative integers! | +32,766 | 7FFE | 011111111111110 |
| - Add. | +32,765 | 7FFD | 0111111111111101 |
| - Subtract. | $\ldots$ |  |  |
| - Test if positive, negative, or 0 . | +3 | 0003 | 0000000000000011 |
|  | +2 | 0002 | 0000000000000010 |
| 16 bit two's complement <br> - 16-bit binary representation of x for positive x . <br> - 16 -bit binary representation of $2^{16}-\|x\|$ for negative $x$. | +1 | 0001 | 0000000000000001 |
|  | 0 | 0000 | 000000000000000 |
|  | -1 | FFFF | 1111111111111111 |
|  | -2 | FFFE | 1111111111111110 |
| Useful properties <br> - Leading bit (bit 15) signifies sign. <br> - 0000000000000000 represents zero. <br> - Add/subtract is the same as for unsigned. | -3 | FFFD | 1111111111111101 |
|  | ... |  |  |
|  | -32,766 | 8002 | 1000000000000010 |
|  | -32,767 | 8001 | 100000000000001 |
|  | -32,768 | 8000 | 1000000000000000 |

## Two's complement: conversion

To convert from decimal to two's complement

- If greater than $+32,767$ or less than $-32,768$ report error.
- Convert to 16 -bit binary.
- If not negative, done.
- If negative, flip all bits and add 1

To convert from two's complement to decima

- If sign bit is 1 , flip all bits and add 1 and output minus sign.
- Convert to decimal


## To add/subtract

- Use same rules as for unsigned binary
- (Still) ignore overflow.

| Examples |  |  |
| :---: | :---: | :---: |
| +1310 | 0000000000001101 | 000D |
| -1310 | 1111111111110011 | 1 FFF3 |
| +25610 | 0000000100000000 | 0100 |
| -25610 | 1111111100000000 | FF00 |
| Examples |  |  |
| 0001 | 0000000000000001 | 110 |
| FFFF | 1111111111111111 | $-110$ |
| FFOD | 1111111100001101 | -24310 |
| 00F3 | 0000000011110011 | +24310 |
| Example |  |  |
| $-25610$ | 1111111100000000 | FF00 |
| +1310 | +0000000000001011 | +000D |
| $=-24310$ | $=1111111100001101$ | =FFOD |



Operations

- Bitwise AND.
- Bitwise XOR.
- Shift left.
- Shift right.
$0 \begin{array}{llllllllllllllllllllll}x & y & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\end{array}$
AND $0 \begin{array}{lllllllllllllllllll}0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0\end{array}$
$=\begin{array}{lllllllllllllllllll}0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1\end{array}$
$\begin{array}{lllllllllllllllllll}0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & x \times R y\end{array}$
XOR $\begin{array}{llllllllllllllllllll}1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1\end{array}$


$\begin{array}{llllllllllllllll}0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1\end{array}$
Shift right $3,0000000^{1-1} 00 \rightarrow 001$
fill with 0s

Special note: Shift left/right operations also implement multiply/divide by powers of 2 for integers.

$$
\text { shift right fills with } 1 \text { s if leading bit is } 1
$$



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TOY instructions

| ANY 16-bit (4 hex digit) value defines a TOY instruction. |  |  |  | opcode | instruction |
| :---: | :---: | :---: | :---: | :---: | :---: |
| First hex digit specifies which instruction. |  |  |  | 1 | add |
|  |  |  |  | 2 | subtract |
| Each instruction changes machine state in a well-defined way. |  |  |  | 3 | bitwise and |
|  |  |  |  | 5 | shift left |
|  |  |  |  | 6 | shift right |
| category | opcodes | implements | changes | 7 | load address |
| operations | 123456 | data-type operations | registers | 8 | load |
|  |  |  |  | 9 | store |
| data movement | 789 A B | data moves between registers and memory | registers, memory | A | load indirect |
|  |  |  |  | B | store indirect |
|  |  |  |  | C | branch if zero |
| flow of control | OCDEF | conditionals, loops, and functions | PC | D | branch if positive |
|  |  |  |  | E | jump register |
|  |  |  |  | F | jump and link |

## Encoding instructions

ANY 16-bit (4 hex digit) value defines a TOY instruction.
Two different instruction formats

- Type RR: Opcode and 3 registers.

- Type A: Opcode, 1 register, and 1 memory address.


Examples
1 CAB Add $\mathrm{R}[\mathrm{A}]$ to $\mathrm{R}[\mathrm{B}]$ and put result in $\mathrm{R}[\mathrm{C}]$.
8A15 Load into R[A] data from M[15].

| opcode |  | instruction |
| :---: | :---: | :---: |
| 0 | RR | halt |
| 1 | RR | add |
| 2 | RR | subtract |
| 3 | RR | bitwise and |
| 4 | RR | bitwise xor |
| 5 | RR | shift left |
| 6 | RR | shift right |
| 7 | A | load address |
| 8 | A | load |
| 9 | A | store |
| A | RR | load indirect |
| B | RR | store indirect |
| C | A | branch if zero |
| D | A | branch if positive |
| E | RR | jump register |
| F | A | jump and link |

## A TOY program

## Add two integers

- Load operands from memory into registers.
- Add the registers.
- Put result in memory.


## Load into $\mathrm{R}[\mathrm{A}]$ data from M[15]

Load into $R[B]$ data from $M[16]$
Add $R[A]$ and $R[B]$ and put result into $R[C]$
Store R[C] into M[17]
Halt
Q. How can you tell whether a word is an instruction?
A. If the PC has its address, it is an instruction!
$\uparrow$
$u$
0
0

Memor

## Registers

A 0008
0005
000 D
$\mathrm{R}[\mathrm{A}] \leftarrow \mathrm{M}[15$
$R[B] \leftarrow M[16]$
$R[C] \leftarrow R[A]+R[B]$
$M[17] \leftarrow R[C]$
halt

Same program with different data
Add two integers

- Load operands from memory into registers.
- Add the registers.
- Put result in memory

Load into R[A] data from M[15]
Load into R[B] data from M[16] Add $\mathrm{R}[\mathrm{A}]$ and $\mathrm{R}[\mathrm{B}]$ and put result into $\mathrm{R}[\mathrm{C}]$ Store R[C] into M[17]

Halt
Q. How can you tell whether a word is data ?
A. If it is added to another word, it is data !
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Outside the box

User interface

- Switches.
- Lights.
- Control Buttons

First step: Turn on the machine!


## Loading a program into memory

To load an instruction

- Set 8 memory address switches.
- Set 16 data switches to instruction encoding.
- Press LOAD to load instruction from switches into addressed memory word.



## Loading instructions into memory

To load an instruction

- Set 8 memory address switches.
- Set 16 data switches to instruction encoding.
- Press LOAD to load instruction from switches into addressed memory word.


10: 8A15
5 11: $8 B 16$ 2. 1CAB 13: 9 Cl 7 14:0000

## Loading instructions into memory

To load an instruction

- Set 8 memory address switches.
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## Loading instructions into memory

To load an instruction

- Set 8 memory address switches.
- Set 16 data switches to instruction encoding.
- Press LOAD to load instruction from switches into addressed memory word.


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## Loading data into memory

To load data, use the same procedure as for instructions

- Set 8 memory address switches.
- Set 16 data switches to data encoding.
- Press LOAD to load data from switches into addressed memory word.

To double check that you loaded the data correctly

- Set 8 memory address switches.
- Press LOOK to examine the addressed memory word.


11: 8 B1 6

## Loading data into memory

To load data, use the same procedure as for instructions

- Set 8 memory address switches.
- Set 16 data switches to data encoding.
- Press LOAD to load data from switches into addressed memory word.



## Running a program

To run a program, set the address switches to the address of first instruction and press RUN
[ data lights may flash, but all (and RUN light) go off when HALT instruction is reached ] To see the output, set the address switches to the address of expected result and press LOOK.


| 10: $8 A 15$ |
| :--- |
| 11: $8 B 16$ |
| 12: $1 C A B$ |
| 13: $9 C 17$ |
| 14: 0000 |

To run the program again, enter different data and press RUN again.

Switches and lights
Q. Did people really program this way?
A. Yes!



## Machine language programming

TOY instructions support the same basic programming constructs as Java.

- Primitive data types.

- Functions and libraries (see text).
- Linked structures (see text)

Linked structures (see text).

- Assignment statements.
- Conditionals and loops.
- Arrays (next lecture).
- Standard input and output (next).
any program you might want to write


## Standard input and output

An immediate problem

- We're not going to be able to address real-world problems
with just switches and lights for I/O
One solution: Paper tape.



## Conditionals and loops

To control the flow of instruction execution

- Test a register's value.
- Change the PC, depending on the value.

| opcode | instruction |
| :---: | :---: |
| C | branch if zero |
| D | branch if positive |

D branch if positive

10 DA 12 If R[A] > 0 set PC to 12 (skip 11)
2 A0A Subtract $R[A]$ from 0 ( $\mathrm{R}[0]$ ) and put result into $R[A]$
12 $\qquad$
Example: Typical while loop (assumes R[1] is 0001)

$$
10 \text { CA } 15 \text { If } R[A] \text { is } 0 \text { set PC to } 15
$$



13 2AA1 Decrement R[A] by 1 C010 Set PC to 10
15 ...

Standard input and output

Punched paper tape

- Encode 16-bit words in two 8-bit rows
- To write a word, punch a hole for each 1.
- To read a word, shine a light behind the tape and sense the holes

TOY mechanism

- Connect hardware to memory location FF.
- To write the contents of a register to stdout, store to FF.
- To read from stdin into a register, load from FF


## Flow of control and standard output example: Fibonacci numbers

$\begin{array}{llllllllllllll}\text { Register trace } & A & 1 & 1 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 89\end{array}$
$\begin{array}{llllllllllll}\text { B } & 1 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 89 & 144\end{array}$
$\begin{array}{llllllllllll}\text { C } & 2 & 8 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 89 & 144\end{array}$
$\begin{array}{llllllllllll}9 & \mathrm{~A} & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0\end{array}$
417 A $01 \quad R[A]=1 \quad a=1$;
427 B $01 \quad R[B]=1 \quad b=1$;
43894 C $R[9]=M[4 C] \quad i=N$;
44 C 94 B if (R[9] == 0) $P C=4 B$ while (i != 0) \{
459 A F F write $R[A]$ to stdout StdOut.print (a);
46 1 C A B $\quad R[C]=R[A]+R[B] \quad c=a+b$;
STDOUT
47 1 A B O $\quad \mathrm{R}[\mathrm{A}]=\mathrm{R}[\mathrm{B}]$
48 1 BCO $\mathbf{R}[B]=R[C]$
$492991 \quad R[9]=R[9]-1 \quad i=i-1$;
4A C $044 \quad \mathrm{PC}=44$
\}
4B 00000 hal
4C 000 A

TOY reference card


## Pop quiz 2 on TOY

Q. How does one flip all the bits in a TOY register?

1A75 as a TOY instruction?
1A75 as a two's complement integer value?

OFFF as a TOY instruction?
OFFF as a two's complement integer value?

8888 as a TOY instruction?
8888 as a two's complement integer value? (Answer in base 16).
Q. What does the following TOY program leave in $\mathrm{R}[2]$ ?

```
10 7 C O A R[C] = 1010
11 7101 R[1] = 1
127201 R[2] = 1
13}10122%24 R[2]=R[2]+R[2
14 2CCC 1 R[C] = R[C] - 1
15 D C 1 3 if (R[C] > 0) PC = 13
16 0000 HALT
```



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[^0]:    Warning. TOY ignores overflow.

