COMPUTER SCIENCE SEDGEWICK/WAYNE

COMPUTER SCIENCE An Interdisciplinary Approach

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Prologue: A Simple Machine

COMPUTER SCIENCE SEDGEWICK/WAYNE PART I: PROGRAMMING IN JAVA

Prologue: A Simple Machine

• Brief introduction

- Secure communication with a one-time pad
- Linear feedback shift registers
- Implications

CS.0.A.Prologue.Introduction

What is this course about?

A broad introduction to computer science.

Goals

- Demystify computer systems.
- Empower you to exploit available technology.
- Build awareness of substantial intellectual underpinnings.

Topics

- Programming in Java.
- Design and architecture of computers.
- Theory of computation.
- Applications in science and engineering.

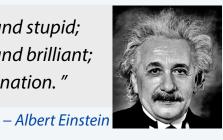
and art, music, finance, and many other fields.

" Science is everything we understand well enough to explain to a computer."



– Don Knuth

" Computers are incredibly fast, accurate, and stupid; humans are incredibly slow, inaccurate, and brilliant; together they are powerful beyond imagination."



COMPUTER SCIENCE S E D G E W I C K / W A Y N E PART I: PROGRAMMING IN JAVA

Image sources

http://pixabay.com/en/network-media-binary-computer-65923/
http://commons.wikimedia.org/wiki/File:KnuthAtOpenContentAlliance.jpg
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CS.O.A.Prologue.Introduction

COMPUTER SCIENCE SEDGEWICK/WAYNE PART I: PROGRAMMING IN JAVA

Prologue: A Simple Machine

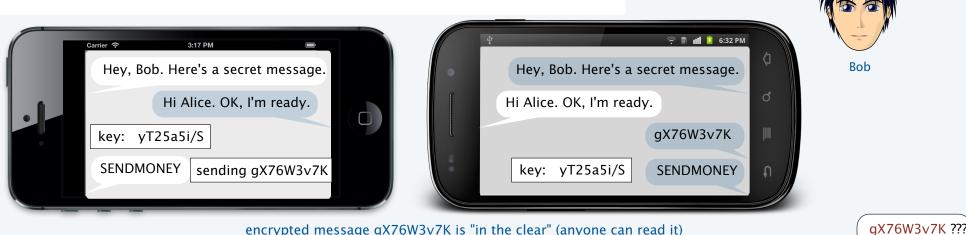
- Brief introduction
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CS.0.B.Prologue.OneTimePad

Sending a secret message with a cryptographic key

Alice wants to send a secret message to Bob.

- Sometime in the past, they exchange a cryptographic key.
- Alice uses the key to encrypt the message.
- Bob uses the *same* key to decrypt the message.



encrypted message gX76W3v7K is "in the clear" (anyone can read it)

Critical point: Without the key, Eve cannot understand the message.

Q. How does the system work?



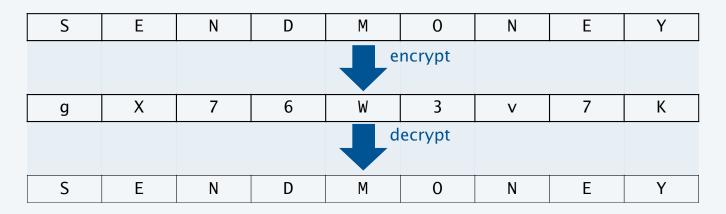
Alice

"use yT25a5i/S if I ever send you an encrypted message"

"OK'

Encrypt/decrypt methods

Goal. Design a method to encrypt and decrypt data.



Example 1. Enigma encryption machine [German code, WWII]

- Broken by Turing bombe (one of the first uses of a computer).
- Broken code helped win Battle of Atlantic by providing U-boat locations.

Example 2. One-time pad [details to follow]

Example 3. Linear feedback shift register [later this lecture]



A digital world

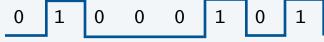
A *bit* is a basic unit of information.

- Two possible values (0 or 1).
- Easy to represent in the physical world (on or off).

In modern computing and communications systems, we represent *everything* as a sequence of bits.

- Text [details to follow in this lecture]
- Numbers
- Sound [details to follow in this course]
- Pictures [details to follow in this course]
- ...
- Programs [profound implications, stay tuned].





 $01000101_2 = 69_{10}$

Bottom line. If we can send and receive bits, we can send and receive *anything*.

well, not cars or cats (yet)

Encoding text as a sequence of bits

Base64 encoding of character strings

- A simple method for representing text.
- 64 different symbols allowed: A-Z, a-z, 0-9, +, /.
- 6 bits to represent each symbol.
- ASCII and Unicode methods used on your computer are similar.

S

F

	bits	symbols
Base64	6	64
ASCII	8	256
Unicode	16	65,536+

000000 A	001000 I	010000 Q	011000 Y	100000 g	101000 o	110000 w	111000 4
000001 B	001001 J	010001 R	011001 Z	100001 h	101001 p	110001 x	111001 5
000010 C	001010 K	010010 S	011010 a	100010 i	101010 q	110010 y	111010 6
000011 D	001011 L	010011 T	011011 b	100011 j	101011 r	110011 z	111011 7
000100 E	001100 M	010100 U	011100 c	100100 k	101100 s	110100 0	111100 8
000101 F	001101 N	010101 V	011101 d	100101]	101101 t	110101 1	111101 9
000110 G	001110 0	010110 W	011110 e	100110 m	101110 u	110110 2	111110 +
000111 H	001111 P	010111 X	011111 f	100111 n	101111 v	110111 3	111111 /
	-	-		-	-	-	

D

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Example:

SENDMONEY	01001000	0100001101	000011	001100	001110	001101	000100	011000

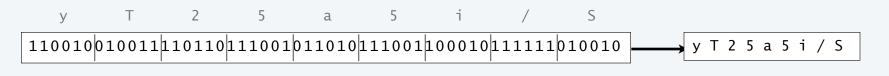
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9

One-Time Pads

What is a one-time pad?

- A *cryptographic key* known only to the sender and receiver.
- Good choice: A *random* sequence of bits (stay tuned).
- Security depends on each sequence being used only once.



000000 A	001000 I	010000 Q	011000 Y	100000 g	101000 o	110000 w	111000 4
000001 B	001001 J	010001 R	011001 Z	100001 h	101001 p	110001 ×	111001 5
000010 C	001010 K	010010 S	011010 a	100010 i	101010 q	110010 y	111010 6
000011 D	001011 L	010011 T	011011 b	100011 j	101011 r	110011 z	111011 7
000100 E	001100 M	010100 U	011100 c	100100 k	101100 s	110100 0	111100 8
000101 F	001101 N	010101 V	011101 d	100101]	101101 t	110101 1	111101 9
000110 G	001110 0	010110 W	011110 e	100110 m	101110 u	110110 2	111110 +
000111 H	001111 P	010111 X	011111 f	100111 n	101111 v	110111 3	111111 /

more convenient than bits for initial exchange

Note: Any sequence of bits can be decoded into a sequence of characters.

Encryption with a one-time pad

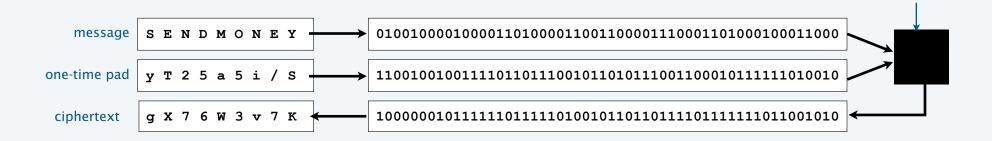
Preparation

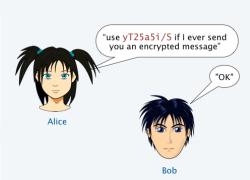
- Create a "random" sequence of bits (a one-time pad).
- Send one-time pad to intended recipient through a secure channel.

Encryption

- Encode text as a sequence of *N* bits.
- Use the first *N* bits of the pad. <---- important point: need to have as many bits in the pad as there are in the message.
- Compute a new sequence of *N* bits from the message and the pad.
- Decode result to get a sequence of characters.

Result: A ciphertext (encrypted message).





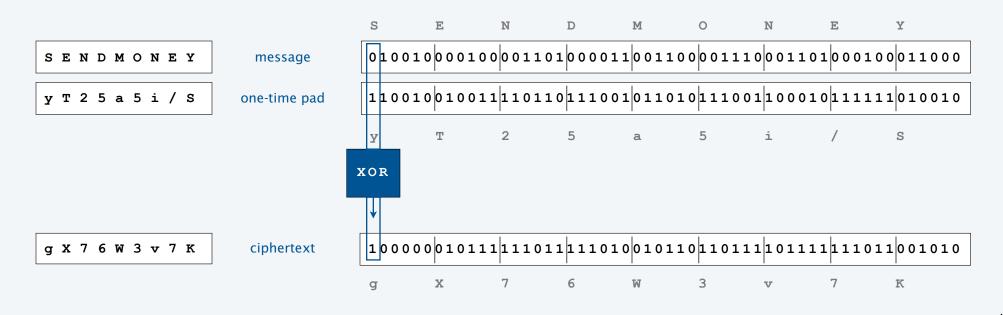
a simple machine

A (very) simple machine for encryption

To compute a ciphertext from a message and a one-time pad

- Encode message and pad in binary.
- Each ciphertext bit is the *bitwise exclusive or* of corresponding bits in message and pad.

Def. The bitwise exclusive or of two bits is 1 if they differ, 0 if they are the same.



Pop quiz on bitwise XOR encryption

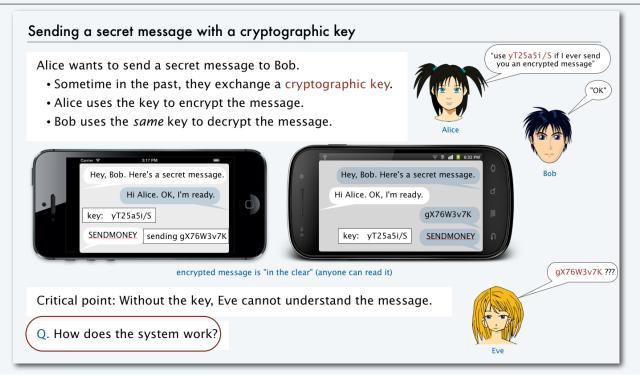
Q. Encrypt the message E A S Y with the pad 0 1 2 3.

Pop quiz on bitwise XOR encryption

Q. Encrypt the message E A S Y with the pad 0 1 2 3.

000000	A	001000	I 01000	0 Q	011000	Y :	100000	g 1	01000	0	110000 w	111000 4	
000001	В	001001	01000	1 R	011001	Ζ 1	100001	h 1	01001	р	110001 x	111001 5	
000010	С	001010	< 01001	0 <mark>S</mark>	011010	a	100010	i 1	01010	q	110010 y	111010 6	
000011	D	001011	01001	1 T	011011	b 1	100011	j 1	01011	r	110011 z	111011 7	get coding table
000100	E	001100	01010	0 U	011100	C	100100	k 1	01100	s	110100 <mark>0</mark>	111100 8	
000101	F	001101	01010	1 V	011101	d 1	100101	1 1	01101	t	110101 1	111101 9	
000110	G	001110	01011	0 W	011110	e 1	100110	m 1	01110	u	110110 2	111110 +	
000111	Н	001111	P 01011	1 X	011111	f	100111	n 1	01111	v	110111 3	111111 /	
			Е		А		S		Y				
		0	00100	00	0000	01	0010	01	1000)		encod	e message
			0		1		2		3			enc	ode pad
		1	10100	11	0101	11	0110	11	0111	-		enco	oue pau
		1	10000	11	0101	10	0100	10	1111	-		XOR t	o encrypt
			W		1		k		V			de	ecode
					_				-				

Decryption with a one-time pad



A. Alice's device uses a "bitwise exclusive or" machine to encrypt the message.

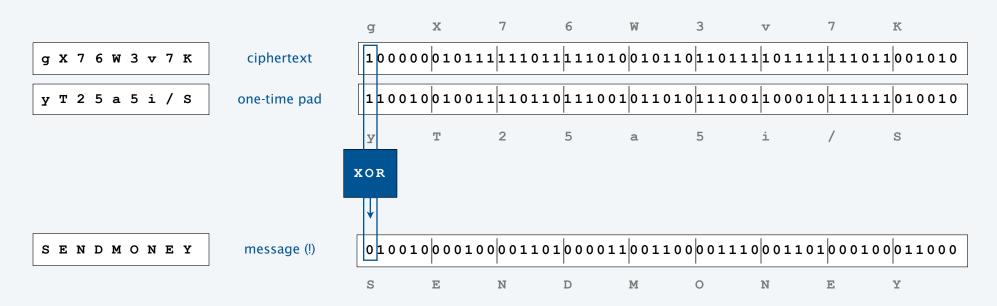
Q. What kind of machine does Bob's device use to *decrypt* the message?

A. The same one (!!)

A (very) simple machine for encryption and decryption

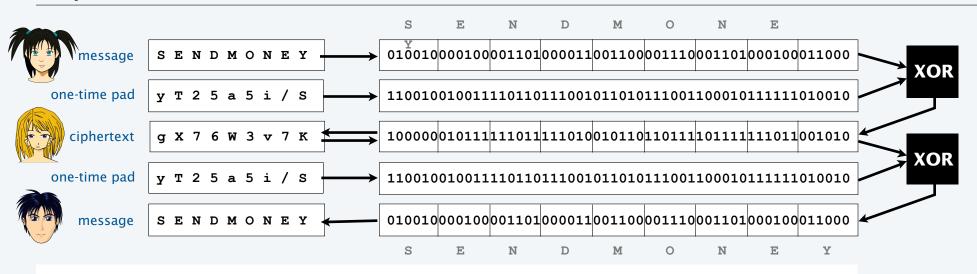
To compute a *message* from a *ciphertext* and a one-time pad

- Use binary encoding of ciphertext and pad.
- Each message bit is the *bitwise exclusive or* of corresponding bits in ciphertext and pad.



1 if they differ; 0 if they are the same

Why does it work?

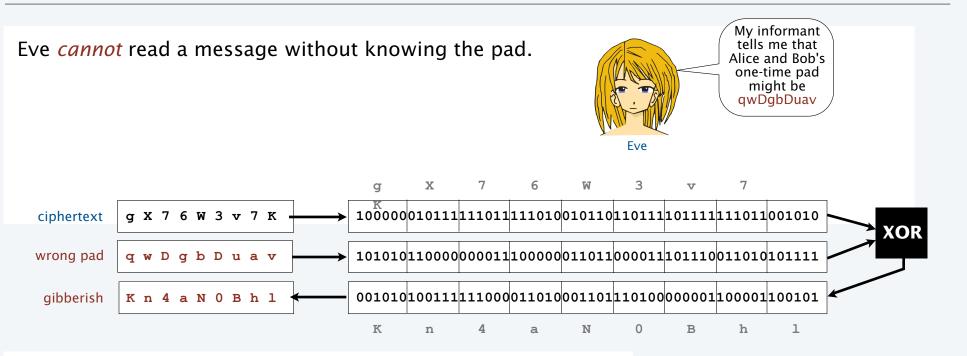


Crucial property: Decrypted message is the same as the original message. Let *m* be a bit of the message and *k* be the corresponding bit of the one-time pad. To prove: $(m \land k) \land k = m \longleftarrow$ Notation: $m \land k$ is equivalent to XOR(*m*, *k*)

Approach 1: Truth tables	т	k	m ^ k	$(m \wedge k) \wedge k$	Approach 2: Boolean algebra	$(k \wedge k) = 0$
	0	0	0	0		$m \wedge 0 = m$
	0	1	1	0		$(m \wedge k) \wedge k = m \wedge (k \wedge k)$
	1	0	1	1		$= m \wedge 0$
	1	1	0	1	\checkmark	= m 🗸

17

Decryption with the wrong pad



One-time pad is provably secure [Shannon, 1940s]

- IF each pad is used only once,
- AND the pad bits are random,
- THEN Eve cannot distinguish ciphertext from random bits.



Eve's problem with one-time pads

Eve has a computer. Why not try all possibilities?



Problem

- 54 bits, so there are 2⁵⁴ possible pad values.
- Suppose Eve could check a million values per second.
- It would still take 570+ years to check all possibilities.

Much worse problem

- There are also 2⁵⁴ possible messages.
- If Eve were to check all the pads, she'd see all the *messages*.
- No way to distinguish the real one from any other.

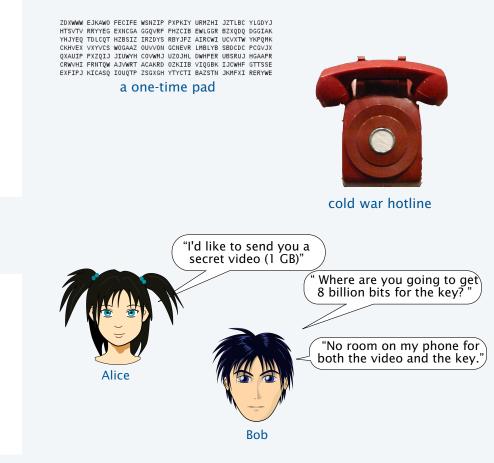
One-time pad is provably secure.

pad value	message?
ΑΑΑΑΑΑΑΑ	gX76W3∨7K
AAAAAAAB	gX76W3∨7L
AAAAAAAA	gX76W3∨7I
qwDgbDua∨	Kn4aN0Bh1
tTtpWk+1E	NEWTATTOO
yT25a5i/S	SENDMONEY
//////+	fo7FpIQE0
///////////////////////////////////////	fo7FpIQE1

Goods and bads of one-time pads

Goods.

- Very simple encryption method.
- Decrypt with the same method.
- Provably unbreakable if bits are truly random.
- Widely used in practice.



- Easily breakable if seed is re-used.
- Truly random bits are very hard to come by.
- Need separate secure channel to distribute key.
- Pad must be as long as the message.

Random bits are not so easy to find

You might look on the internet.	The rando	omness comes from atmospheric noise
RANDOM.ORG - Integer	Generator	
Home Games Numbers Lists & More Drawings Web Tools	Statistics Testimonials Learn More Login	
RANDOM.OR	G Google" Custom Seal Search	
Do you own an iPhone, iPad or iPod Touch? Check out or new app! Androi	id version coming soon.	
Random Integer Generator		
This form allows you to generate random integers. The randomness comes than the pseudo-random number algorithms typically used in computer pro		"I think I'll call it
Dent to The Testa series	-	random.org"
Part 1: The Integers		
Generate 1000 random integers (maximum 10,000).		
Each integer should have a value between 0 and 1 (both	inclusive; limits ±1,000,000,000).	
Format in 5 column(s).		
Part 2: Go!		if you truct the internet
Be patient! It may take a little while to generate your numbers		if you trust the internet.

Next: Creating a (long) sequence of "pseudo-random" bits from a (short) key.

COMPUTER SCIENCE S E D G E W I C K / W A Y N E PART I: PROGRAMMING IN JAVA

Image sources

https://openclipart.org/detail/25617/astrid-graeber-adult-by-anonymous-25617 https://openclipart.org/detail/169320/girl-head-by-jza https://openclipart.org/detail/191873/manga-girl---true-svg--by-j4p4n-191873 http://commons.wikimedia.org/wiki/File:Enigma-Machine.jpg http://pixabay.com/en/binary-one-null-ball-administrator-63530/ http://commons.wikimedia.org/wiki/File:Jimmy_Carter_Library_and_Museum_99.JPG

CS.O.B.Prologue.OneTimePad

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CS.O.C.Prologue.LFSR

A pseudo-random number generator

is a *deterministic* machine that produces a long sequence of *pseudo random* bits.

Examples

Enigma. Linear feedback shift register (next). Blum-Blum-Shub generator.

[an early application of computing] [research still ongoing]





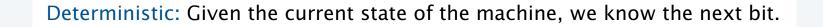
" Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin."



– John von Neumann

A pseudo-random number generator

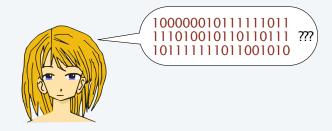
is a *deterministic* machine that produces a long sequence of *pseudo random* bits.



An absolute requirement: Alice and Bob need the same sequence.

Random: We never know the next bit.

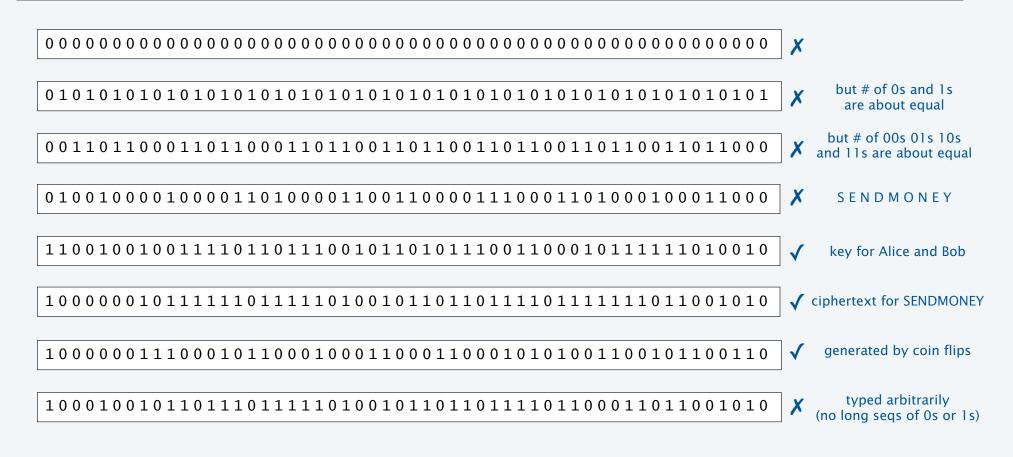
Pseudo-random: The sequence of bits appears to be random.



Appears to be random??

- A profound and elusive concept.
- For this lecture: "Has enough properties of a random sequence that Eve can't tell the difference".
- Ex. 1: No long repeats
- Ex. 2: About the same number of 0s and 1s
- Ex. 3: About the same number of 00s, 01s, 10s, and 11s.

Which of these sequences appear to be random?

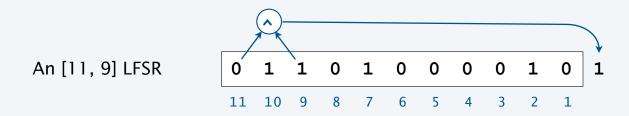


Note: Any one of them *could be* random!

Linear feedback shift register

Terminology

- Bit: 0 or 1.
- Cell: storage element that holds one bit.
- Register: sequence of cells.
- Seed: initial sequence of bits.
- Feedback: Compute XOR of two bits and put result at right.

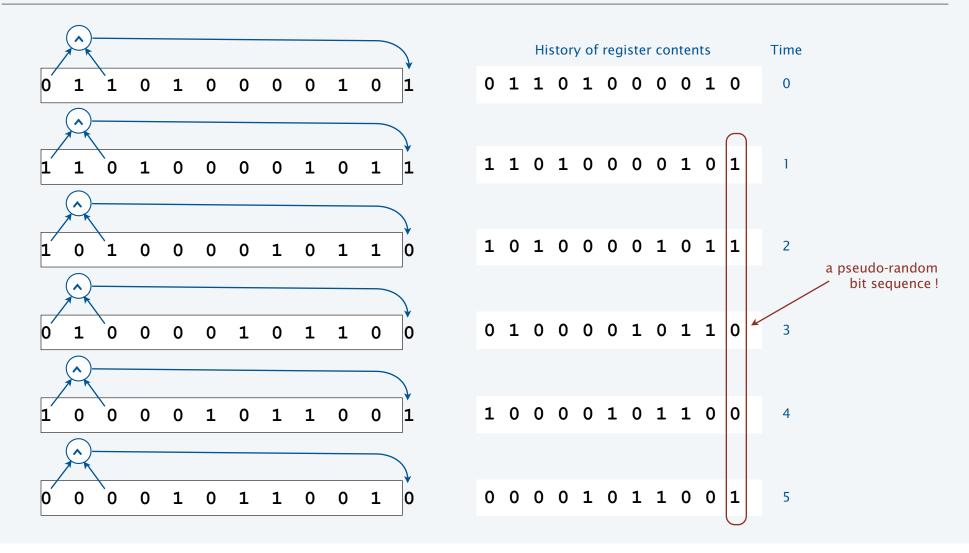


More terminology

- Tap: Bit positions used for XOR (one must be leftmost).
- [*N*, *k*] LFSR: *N*-bit register with taps at *N* and *k*.

← ____ Not all values of k give desired effect (stay tuned).

Linear feedback shift register simulation



28

A random bit sequence?

Q. Is this a random sequence?



No long repeats. 997 0s, 1003 1s. 256 00s, 254 01s, 256 10s, 257 11s.

one-time pad in our example

. . .

A. No. It is the output of an [11, 9] LFSR with seed 01101000010!

Pop quiz on LFSRs

Q. Give first 10 steps of [5, 4] LFSR with initial fill 00001.

Pop quiz on LFSRs

Q. Give first 10 steps of [5, 4] LFSR with initial fill 00001.

	2-			_	
0	0	0	0	1	0
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	0	0	1
1	0	0	0	1	1
0	0	0	1	1	0
0	0	1	1	0	0
0	1	1	0	0	1
1	1	0	0	1	0
1	0	0	1	0	1
0	0	1	0	1	0

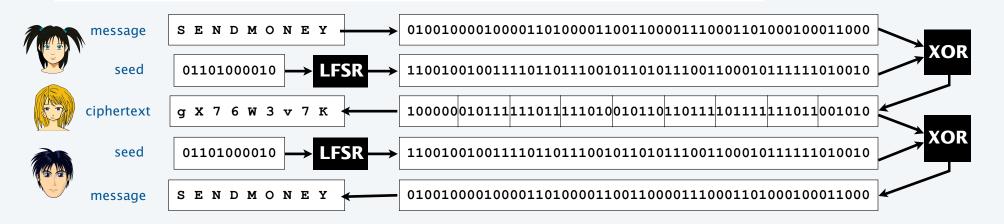
Encryption/decryption with an LFSR

Preparation

- Alice creates a book of "random" (short) seeds.
- Alice sends the book to Bob through a secure channel.

Encryption/decryption

- Alice sends Bob a description of which seed to use.
- They use the specified seed to initialize an LFSR and produce *N* bits. [and proceed in the same way as for one-time pads]



"Use the next seed in the book to decode this secret video (1 GB)"

Alice

Bob

" OK (consults book) 01101000010 "

Eve's opportunity with LFSR encryption

Without the seed, Eve cannot read the message.

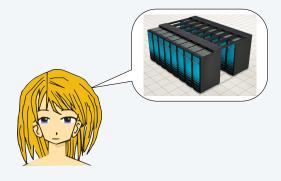
Eve has computers. Why not try all possible seeds?

- Seeds are short, messages are long.
- All seeds give a tiny fraction of all messages.
- Extremely likely that all but real seed will produce gibberish.

Good news (for Eve): This approach can work.

- Ex: 11-bit register implies 2047 possibilities.
- Extremely likely that only *one* of those is not gibberish.
- After this course, *you* could write a program to check whether any of the 2047 messages have words in the dictionary.

Bad news (for Eve): It is easy for Alice and Bob to use a much longer LFSR.



Eve

Property 1.

- Don't use all 0s as a seed!
- Fill of all 0s will not otherwise occur.



Property 1.Don't use all 0s as a seed!Fill of all 0s will not otherwise occur.	Ex. [4,3] LFSR		0	1	1
Property 2. Bitstream must eventually cycle.		001	1	0	3
• $2^{N} - 1$ nonzero fills in an <i>N</i> -bit register.		0 1 1	0	1	4
• Future output completely determined by current fill.		110	1	0	5
		101	0	1	6
		010	1	1	7
		101	1	1	8
		011	1	1	9
		1 1 1	1	0	10
		1 1 1	0	0	11
		110	0	0	12
		100	0	1	13
		000	1	0	
		001	0		15

Property 1. • Don't use all 0s as a seed!		Ph	Ì	
• Fill of all 0s will not otherwise occur.	Ex. [4,2] LFSR	0 0 1 0		0
The of all of whithout other wise occur.		0 1 0 1	1	1
		1 0 1 1	1	2
Property 2. Bitstream must eventually cycle.		0 1 1 1	1	3
 2^N – 1 nonzero fills in an N-bit register. 		1111	0	4
• Future output completely determined by current fill.		1 1 1 0	0	5
		1 1 0 0	0	6
		1 0 0 0	1	7
Property 3. Cycle length in an <i>N</i> -bit register is <i>at most</i> $2^{N} - 1$.		0001	0	8
 Could be smaller; cycle length depends on tap positions. 		0010		
 Need theory of finite groups to know good tap positions. 				

Property 1.

- Don't use all 0s as a seed!
- Fill of all 0s will not otherwise occur.

Property 2. Bitstream must eventually cycle.

- $2^{N} 1$ nonzero fills in an *N*-bit register.
- Future output completely determined by current fill.

Property 3. Cycle length in an N-bit register is at most $2^{N} - 1$.

- Could be smaller; cycle length depends on tap positions.
- Need theory of finite groups to know good tap positions.

Bottom line.

- [11, 9] register generates 2047 bits before repeating.
- [63, 62] register generates 2⁶³ -1 bits before repeating. Definitely preferable: small cost, huge payoff.

Linear Feedback Shift Register Taps

This table lists the appropriate taps for maximum-length LFSR counters of up to 168 bits. The basic description and the table for the first 40 bits was originally published in (CELL and reprinted on page 9-24 of the 1993 and 1994 Xilinx Data Booka

Responding to repeated requests, the list is here extended to 168 bits. This information is based on unp research done by Wayne Stahnke while he was at Fairchil ductor in 1970

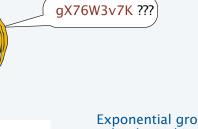
Table 3: Taps for Maximum-Length LFSR C

n	XNOR from	n	XNOR from	n	XNOR from	n	XNOR from
3	3,2	45	45,44,42,41	87	87,74	129	129,124
4	4,3	46	46,45,26,25	88	88,87,17,16	130	130,127
5	5,3	47	47,42	89	89,51	131	131,130,84,83
6	6,5	48	48,47,21,20	90	90,89,72,71	132	132,103
7	7,6	49	49,40	91	91,90,8,7	133	133,132,82,81
8	8,6,5,4	50	50,49,24,23	92	92,91,80,79	134	134,77
9	9,5	51	51,50,36,35	93	93,91	135	135,124
10	10.7	52	52,49	94	94,73	136	136,135,11,10
11	11.91	53	53,52,38,37	95	95,84	137	137,116
12		54	54,53,18,17	96	96,94,49,47	138	138,137,131,13
13	13,4,3,1	55	55,31	97	97,91	139	139,136,134,13
14	14,5,3,1	56	56,55,35,34	98	98,87	140	140,111
15	15,14	57	57,50	99	99,97,54,52	141	141,140,110,10
16	16,15,13,4	58	58,39	100	100,63	142	142,121
17	17,14	59	59,58,38,37	101	101,100,95,94	143	143,142,123,12
18	18,11	60	60,59	102	102,101,36,35	144	144,143,75,74
19	19.6.2.1	61	61,60,46,45	103	103,94	145	145,93
20	20,17	62		104	104,103,94,93	146	146,145,87,86
21	21,19	63	63, 62	105	105,89	147	147,146,110,10
22	22,21	64	05, 02	106	106,91	148	148,121
23	23,18	65	65,47	107	107,105,44,42	149	149,148,40,39
24	24,23,22,17	66	66,65,57,56	108	108,77	150	150,97
25	25,22	67	67,66,58,57	109	109,108,103,102	151	151,148
26	26,6,2,1	68	68,59	110	110,109,98,97	152	152,151,87,80
27	27,5,2,1	69	69,67,42,40	111	111,101	153	153,152
28	28,25	70	70,69,55,54	112	112,110,69,67	154	154,152,27,25
29	29,27	71	71,65	113	113,104	155	155,154,124,12
30	30,6,4,1	72	72,66,25,19	114	114,113,33,32	156	156,155,41,40
31	31,28	73	73,48	115	115,114,101,100	157	157,156,131,13
32	32,22,2,1	74	74,73,59,58	116	116,115,46,45	158	158,157,132,13
33	33,20	75	75,74,65,64	117	117,115,99,97	159	159,128
34	34,27,2,1	76	76,75,41,40	118	118,85	160	160,159,142,14
35	35,33	77	77,76,47,46	119	119,111	161	161,143
36	36,25	78	78,77,59,58	120	120,113,9,2	162	162,161,75,74
37	37,5,4,3,2,1	79	79,70	121	121,103	163	163,162,104,10
38	38,6,5,1	80	80,79,43,42	122	122,121,63,62	164	164,163,151,15
39	39,35	81	81,77	123	123,121	165	165,164,135,13
40	40,38,21,19	82	82,79,47,44	124	124,87	166	166,165,128,12
41	41,38	83	83,82,38,37	125	125,124,18,17	167	167,161
42	42,41,20,19	84	84,71	126	126,125,90,89	168	168,166,153,15
43	43,42,38,37	85	85,84,58,57	127	127,126		
44	44.43.18.17	86	86.85.74.73	128	128,126,101,99		

XILINX manual, 1990s

Eve's problem with LFSR encryption

Without the seed, Eve cannot read the message.



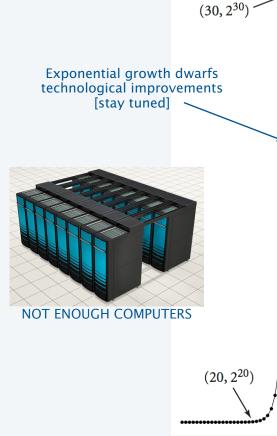
Eve has computers. Why not try all possible seeds?

- Seeds are short, messages are long.
- All seeds give a tiny fraction of all messages.
- Extremely likely that all but real seed will produce gibberish.

Bad news (for Eve): There are still way too many possibilities.

- Ex: 63-bit register implies 2⁶³ 1 possibilities.
- If Eve could check 1 million seeds per second, it would take her 2923 centuries to try them all!

Bad news (for Alice and Bob): LFSR output is not random.



> experts have cracked LFSRs

Goods and bads of LFSRs

Goods.

- Very simple encryption method.
- Decrypt with the same method.
- Scalable: 20 cells for 1 million bits; 30 cells for 1 billion bits.
- Widely used in practice. [Example: military cryptosystems.]



a commercially available LFSR

Bads.

- Easily breakable if seed is re-used.
- Still need secure key distribution.
- Experts can crack LFSR encryption.

Example.

- CSS encryption widely used for DVDs.
- Widely available DeCSS breaks it!



COMPUTER SCIENCE S E D G E W I C K / W A Y N E PART I: PROGRAMMING IN JAVA

Image sources

http://pixabay.com/en/ball-http-www-crash-administrator-216837/ http://commons.wikimedia.org/wiki/File:KnuthAtOpenContentAlliance.jpg http://commons.wikimedia.org/wiki/File:Einstein-formal_portrait-35.jpg

CS.O.C.Prologue.LFSR

COMPUTER SCIENCE SEDGEWICK/WAYNE PART I: PROGRAMMING IN JAVA

Prologue: A Simple Machine

- Brief introduction
- Secure communication with a one-time pad
- Linear feedback shift registers
- Implications

CS.0.D.Prologue.Implications

LFSRs and general-purpose computers





Important similarities.

- Both are built from simple components.
- Both scale to handle huge problems.
- Both require careful study to use effectively.

component	LFSR	computer
control	start, stop, load	same
clock		same
memory	12 bits	billions of bits
input	12 bits	bit sequence
computation	shift, XOR	+ - * /
output	pseudo-random bit sequence	any computable bit sequence

- General purpose computer can simulate *any* abstract machine.
- All general purpose computers have equivalent power (!) [stay tuned].

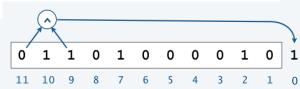
A Profound Idea

Programming. We can write a Java program to simulate the operation of any abstract machine.

- Basis for theoretical understanding of computation.
- Basis for bootstrapping real machines into existence.

Stay tuned (we cover these sorts of issues in this course).

```
public class LFSR
                        {
                            public static void main(String[] args)
                             Ł
                                 int[] a = \{ 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 1, 0 \};
                                 for (int t = 0; t < 2000; t++)
                                 {
YOU will be writing
                                    a[0] = (a[11] \land a[9]);
code like this within -
   a few weeks.
                                    System.out.print(a[0]);
                                    for (int i = 11; i > 0; i--)
                                         a[i] = a[i-1];
                                 }
                                 System.out.println();
                            }
                        }
```



% java LFSR

Profound questions

Q. What is a random number?

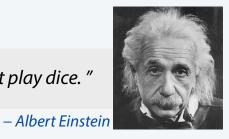
LFSRs do not produce random numbers.

- They are *deterministic*. <---- von Neumann's "state of sin": we *know* that "deterministic" is incompatible with "random"
- It is not obvious how to distinguish the bits LFSRs produce from random,
- BUT experts have figured out how to do so.
- Q. Are random processes found in nature?
 - Motion of cosmic rays or subatomic particles?
 - Mutations in DNA?



"God does not play dice."

Q. Is the natural world a (not-so-simple) deterministic machine??



COMPUTER SCIENCE S E D G E W I C K / W A Y N E PART I: PROGRAMMING IN JAVA

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CS.0.D.Prologue.Implications

COMPUTER SCIENCE SEDGEWICK/WAYNE

COMPUTER SCIENCE An Interdisciplinary Approach

ROBERT SEDGEWICK KEVIN WAYNE

http://introcs.cs.princeton.edu

Prologue: A Simple Machine