



Procedural Modeling

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Procedural Modeling

Goal:

- Describe 3D models algorithmically

Best for models resulting from ...

- Repeating processes
- Self-similar processes
- Random processes

Advantages:

- Automatic generation
- Concise representation
- Parameterized classes of models

Procedural Modeling

Sweeps

Fractals

Grammars

Probabilistic models

Probabilistic grammars

Procedural Modeling

Sweeps ←

Fractals

Grammars

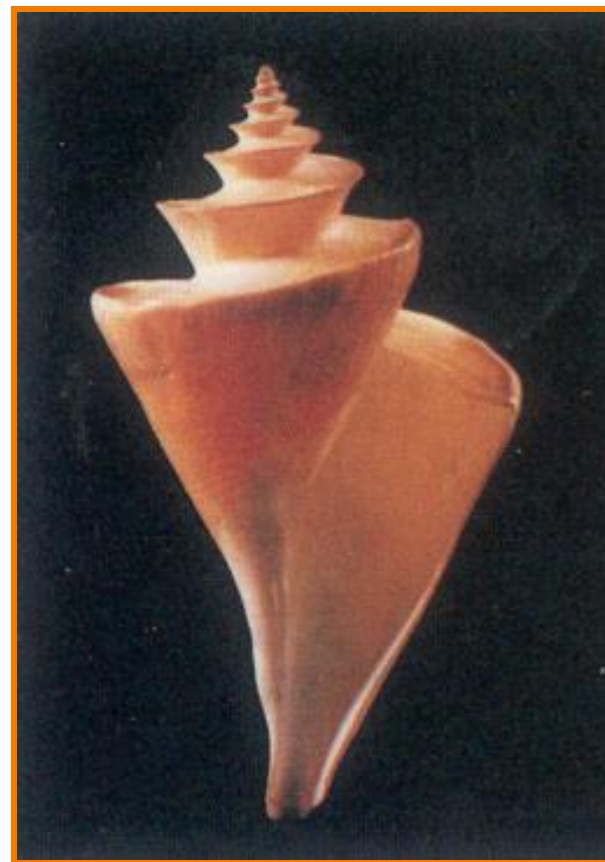
Probabilistic models

Probabilistic grammars

Example: Seashells

Create 3D polygonal surface models of seashells

**“Modeling Seashells,”
Deborah Fowler, Hans Meinhardt,
and Przemyslaw Prusinkiewicz,
Computer Graphics (SIGGRAPH 92),
Chicago, Illinois, July, 1992, p 379-387.**



Fowler et al. Figure 7

Example: Seashells

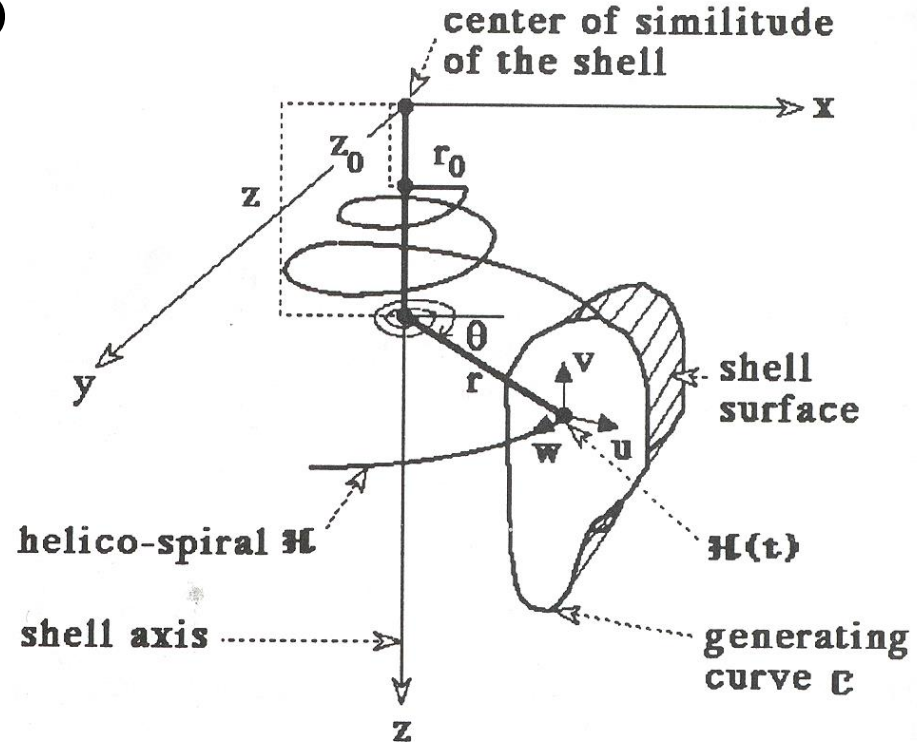
Sweep generating curve around helico-spiral axis

Helico-spiral definition:

$$\Theta_{i+1} = \Theta_i + \Delta\Theta$$

$$r_{i+1} = r_i \lambda_r$$

$$z_{i+1} = z_i \lambda_z$$



Example: Seashells

Connect adjacent points to form polygonal mesh

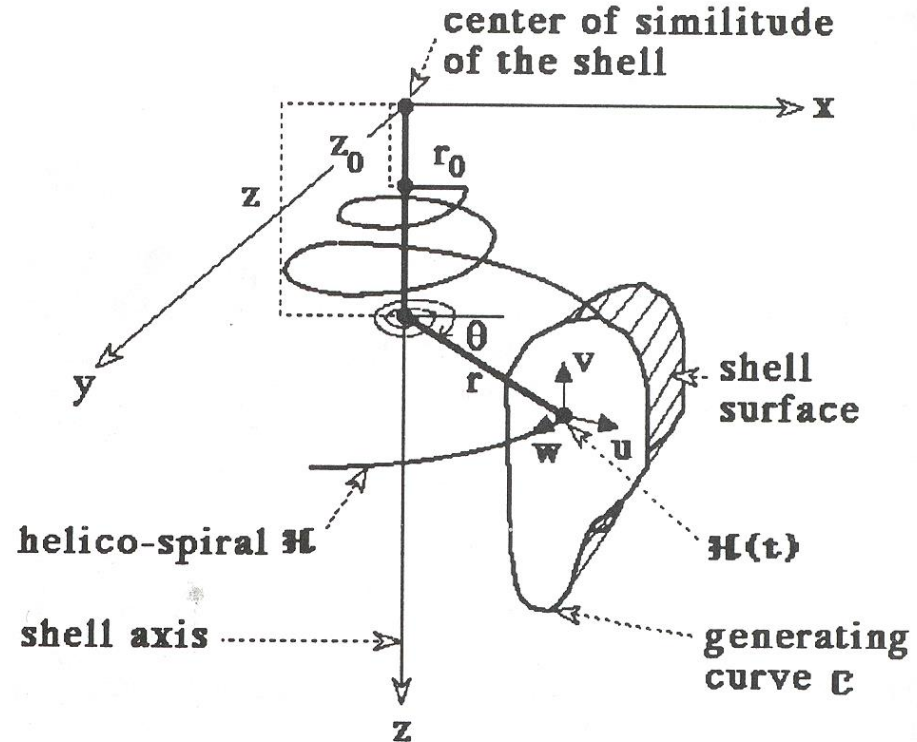


Fowler et al. Figure 6

Example: Seashells

Model is parameterized:

- Helico-spiral: $z_0, \lambda_z, r_0, \lambda_r, N_\theta, \Delta\theta$
- Generating curve: shape, N_c, λ_c



Example: Seashells

Generate different shells by varying parameters



Different helico-spirals

Example: Seashells

Generate different shells by varying parameters

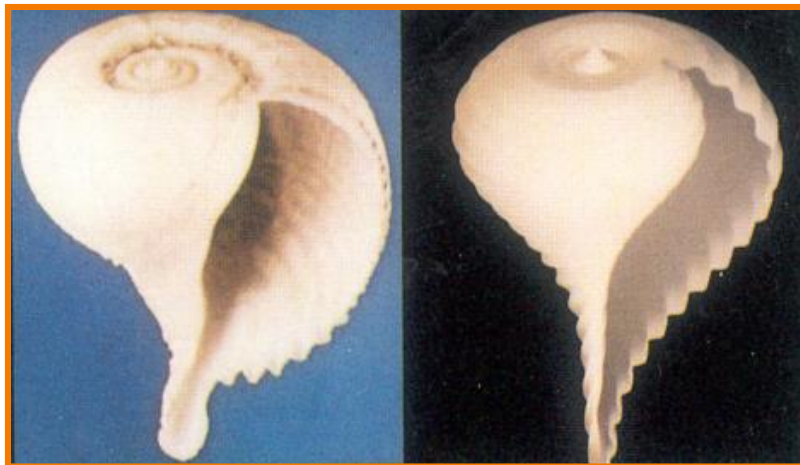


Different generating curves

Example: Seashells



Generate many interesting shells
with a simple procedural model!



Fowler et al. Figures 4,5,7

Procedural Modeling

Sweeps

Fractals ←

Grammars

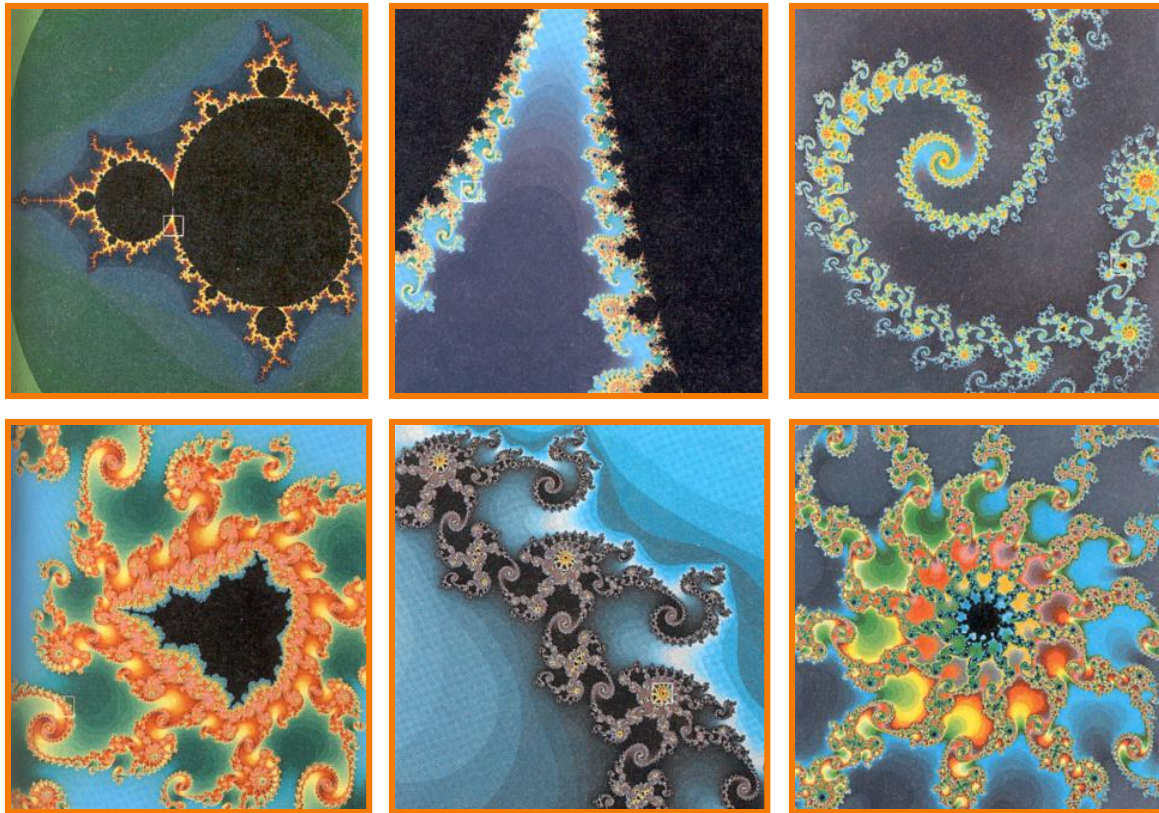
Probabilistic models

Probabilistic grammars

Fractals

Defining property:

- Self-similar with infinite resolution



Mandelbrot Set

Fractals

Useful for describing natural 3D phenomenon

- Terrain
- Plants
- Clouds
- Water
- Feathers
- Fur
- etc.



H&B Figure 10.80

Fractal Generation

Deterministically self-similar fractals

- Parts are scaled copies of original

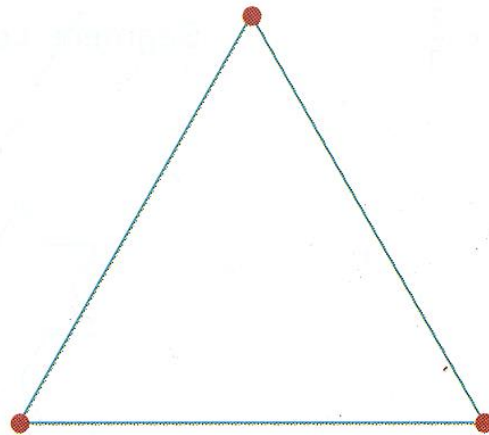
Statistically self-similar fractals

- Parts have same statistical properties as original

Deterministic Fractal Generation

General procedure:

- Initiator: start with a shape
- Generator: replace subparts with scaled copy of original



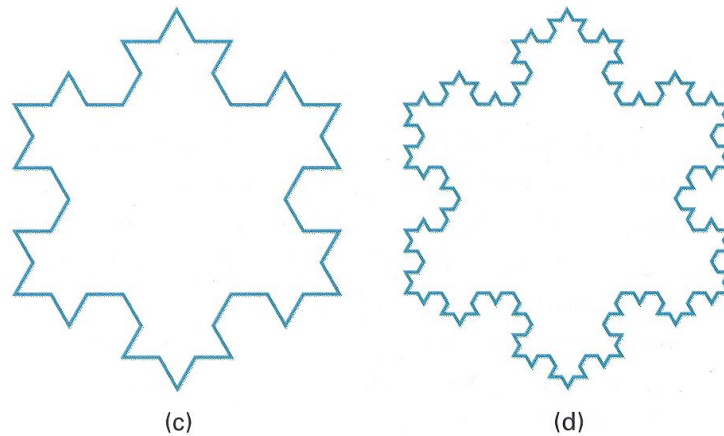
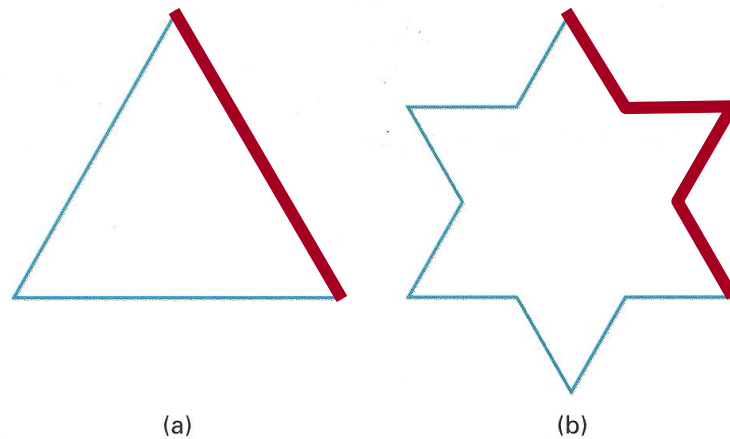
Initiator



Generator

Deterministic Fractal Generation

Apply generator repeatedly

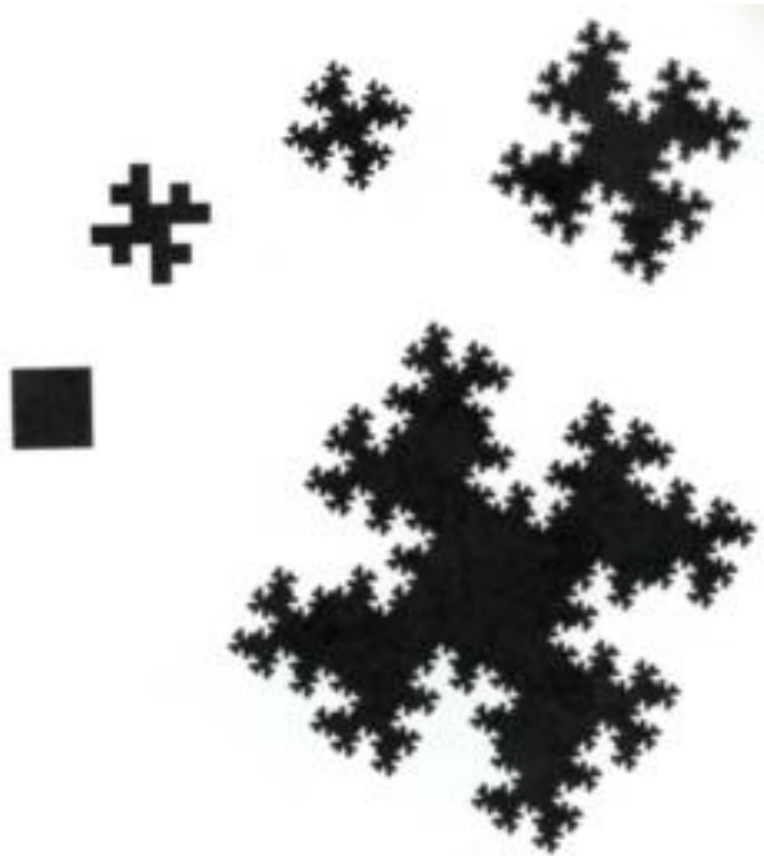


Koch Curve

H&B Figure 10.69

Deterministic Fractal Generation

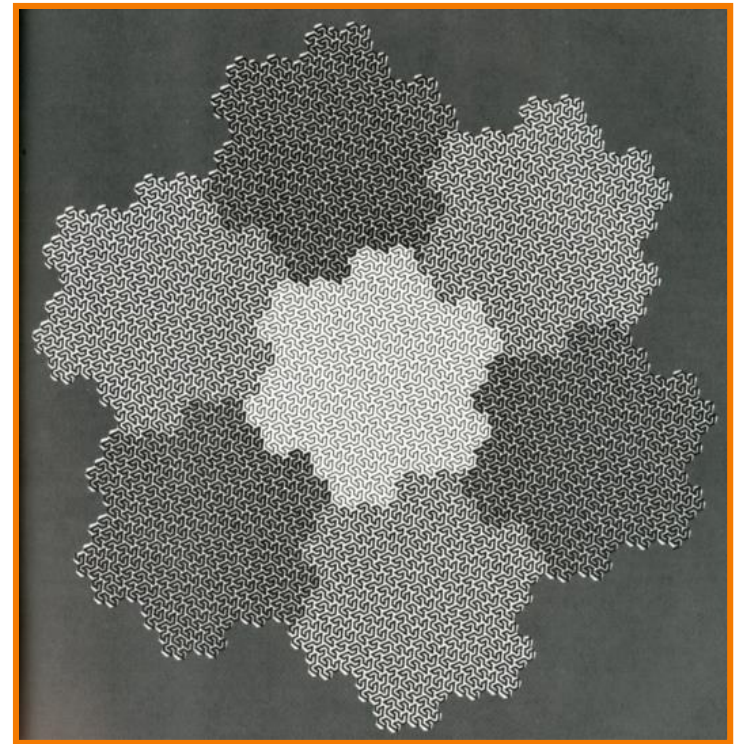
Useful for creating interesting shapes!



Mandelbrot Figure X

Deterministic Fractal Generation

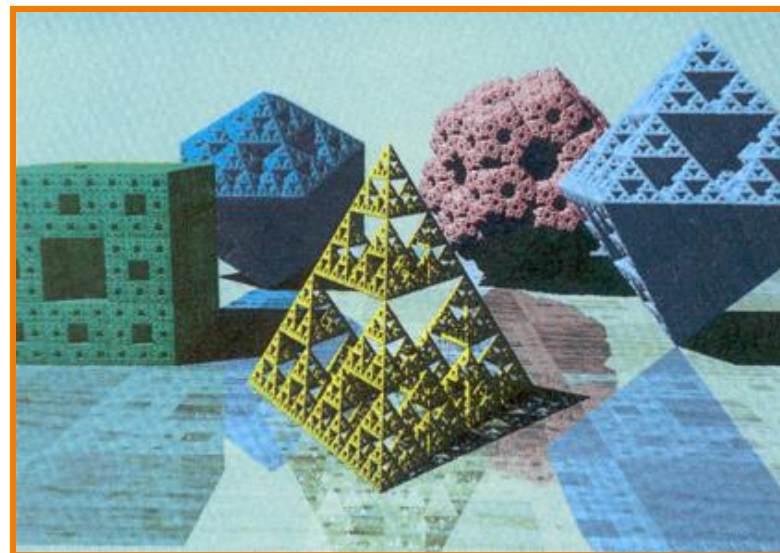
Useful for creating interesting shapes!



Mandelbrot Figure 46

Deterministic Fractal Generation

Useful for creating interesting shapes!



Fractal Generation

Deterministically self-similar fractals

- Parts are scaled copies of original

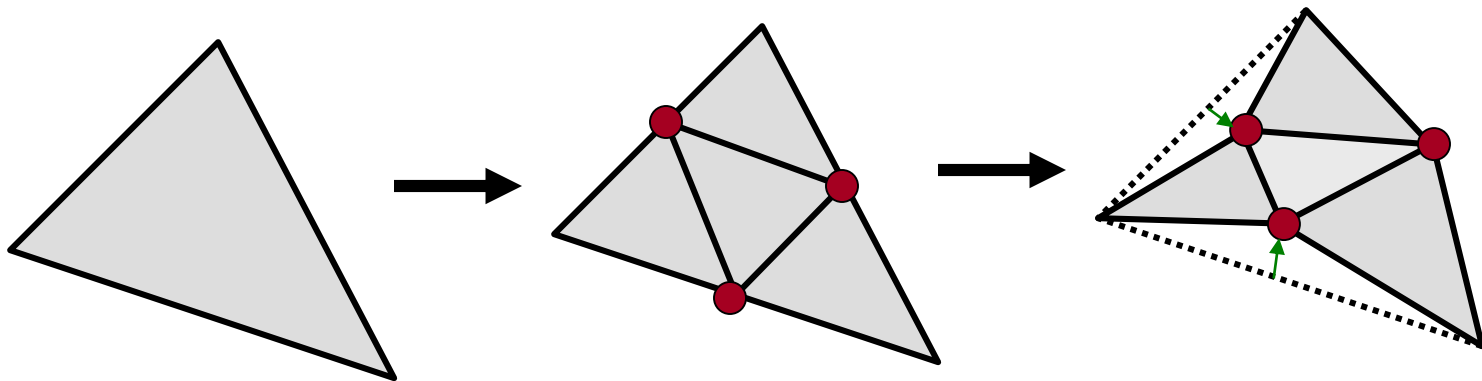
Statistically self-similar fractals

- Parts have same statistical properties as original

Statistical Fractal Generation

General procedure:

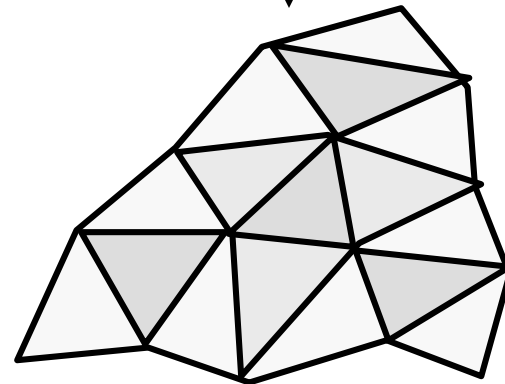
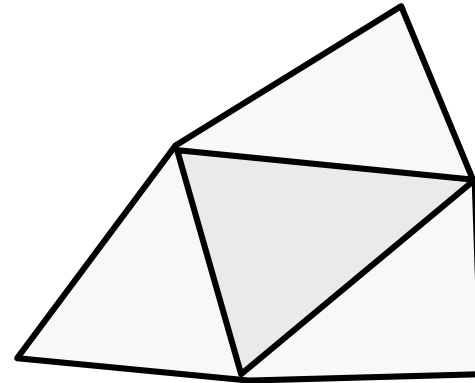
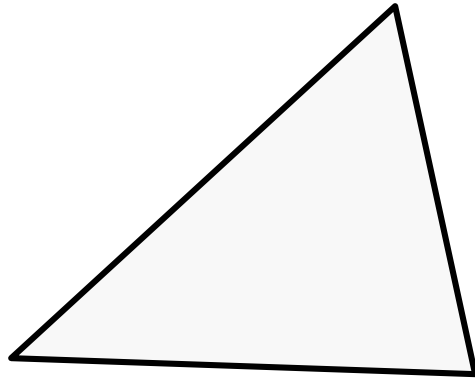
- Initiator: start with a shape
- Generator: replace subparts with a self-similar **random** pattern



Random Midpoint Displacement

Statistical Fractal Generation

Example: terrain



H&B Figure 10.83b

Statistical Fractal Generation

Useful for creating mountains



H&B Figure 10.83a

Statistical Fractal Generation

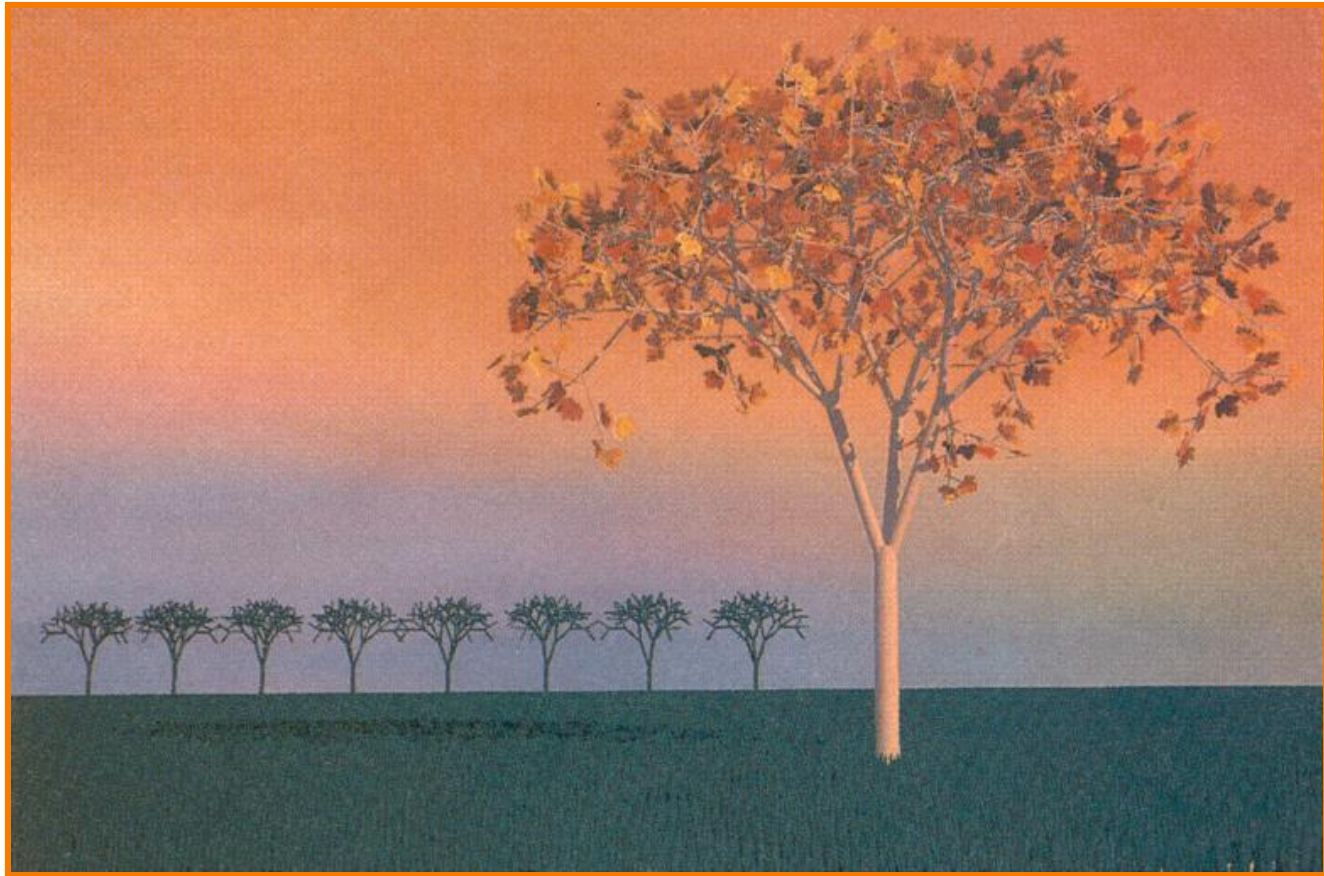
Useful for creating 3D plants



H&B Figure 10.82

Statistical Fractal Generation

Useful for creating 3D plants



H&B Figure 10.79

Procedural Modeling

Sweeps

Fractals

Grammars ←

Probabilistic models

Probabilistic grammars

Grammars

Generate description of geometric model
by applying production rules

$$\begin{array}{l} \mathbf{S} \rightarrow \mathbf{AB} \\ \mathbf{A} \rightarrow \mathbf{Ba} \mid \mathbf{a} \\ \mathbf{B} \rightarrow \mathbf{Ab} \mid \mathbf{b} \end{array}$$

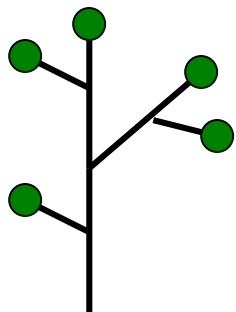
ab
bab
baab
abaab

•
•
•

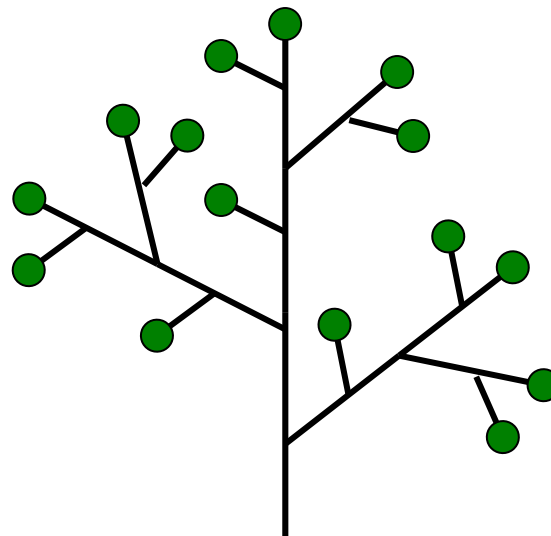
Grammars

Useful for creating plants

Tree \rightarrow **Branch Tree** | **Leaf**
Branch \rightarrow **Cylinder** | **[Tree]**



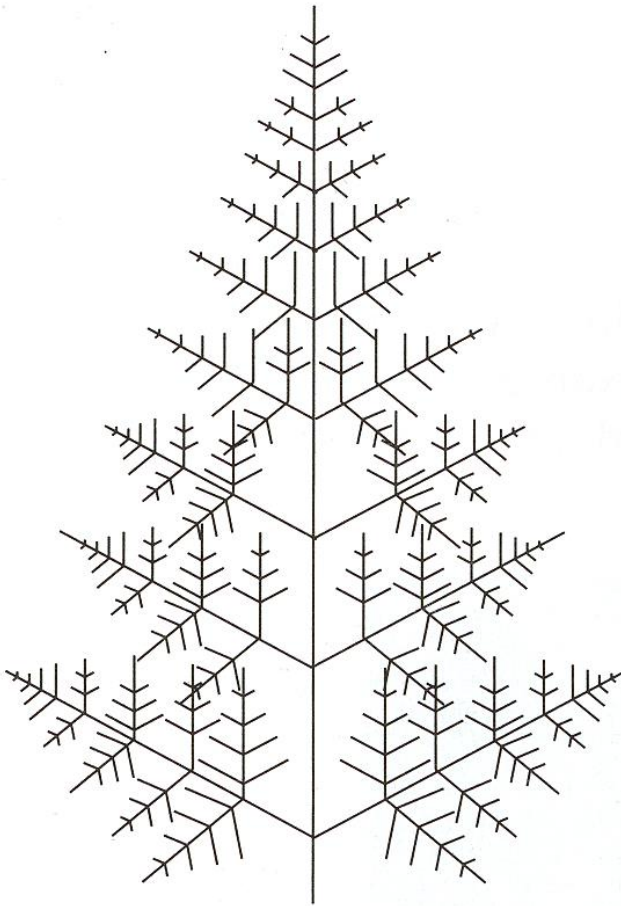
C[**CL**]**C**[**C**[**CL**][**CL**]]**C**[[**CL**][**CL**]]



C[*]**C**[*][*]

Grammars

Useful for creating plants



H&B Figure 10.77

Procedural Modeling

Sweeps

Fractals

Grammars

Probabilistic models ←

Probabilistic grammars

Probabilistic Models

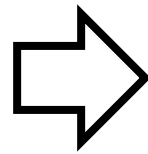


Exemplar scenes

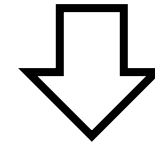
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Database of Scenes

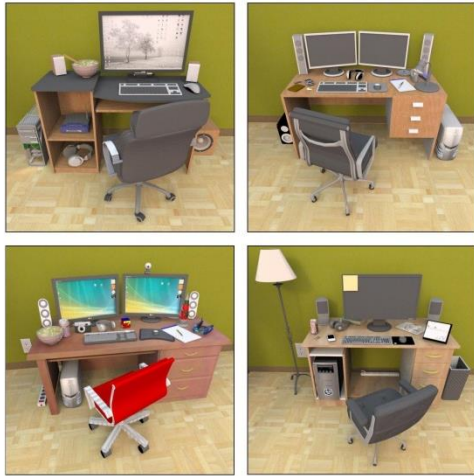


Probabilistic Model of Shape



Synthesized novel scenes

Probabilistic Models

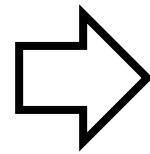


Exemplar scenes

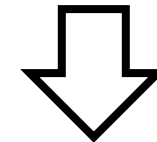
+



Database of Scenes



Probabilistic Model of Shape



Challenge

Need to learn a model with great generality from few examples



Synthesized novel scenes

Probabilistic Model of Scenes

Represent the probability of a scene S by a generative model based on category cardinalities (c), support hierarchy topology relationships (t), and spatial arrangement relationships (a)

$$P(S) = P(c, t, a) = P(a/t, c) P(t/c) P(c)$$



Exemplar scenes

Probabilistic Model of Scenes

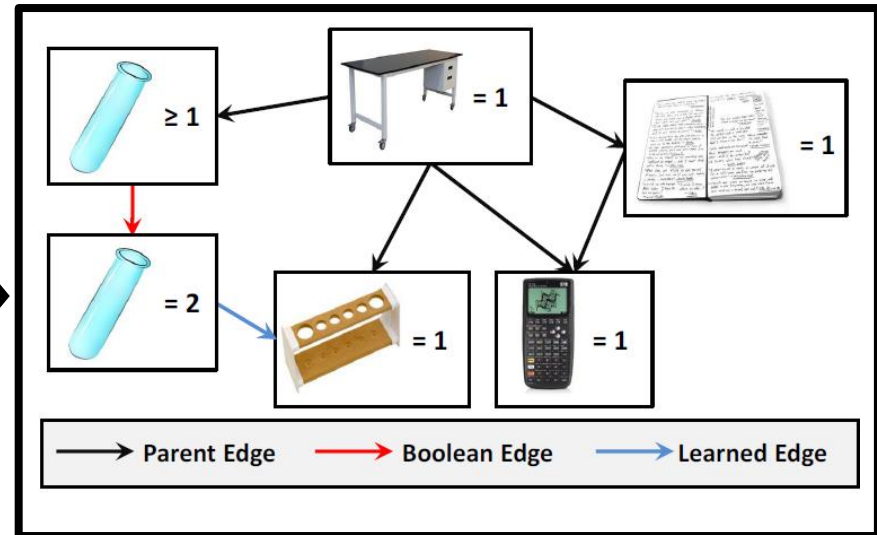
Category cardinalities: $P(c)$

- Represent with Bayesian network
- Boolean random variables (# desks > 1?)
- Add support surface constraints



Lab table	Test tube rack	Test tube	Notebook	Calculator
1	1	2	0	0
1	0	1	0	1
1	0	0	1	1

Object frequencies in target scenes
+ support constraints



Bayesian network

Probabilistic Model of Scenes

Support relationships: $P(t/c)$

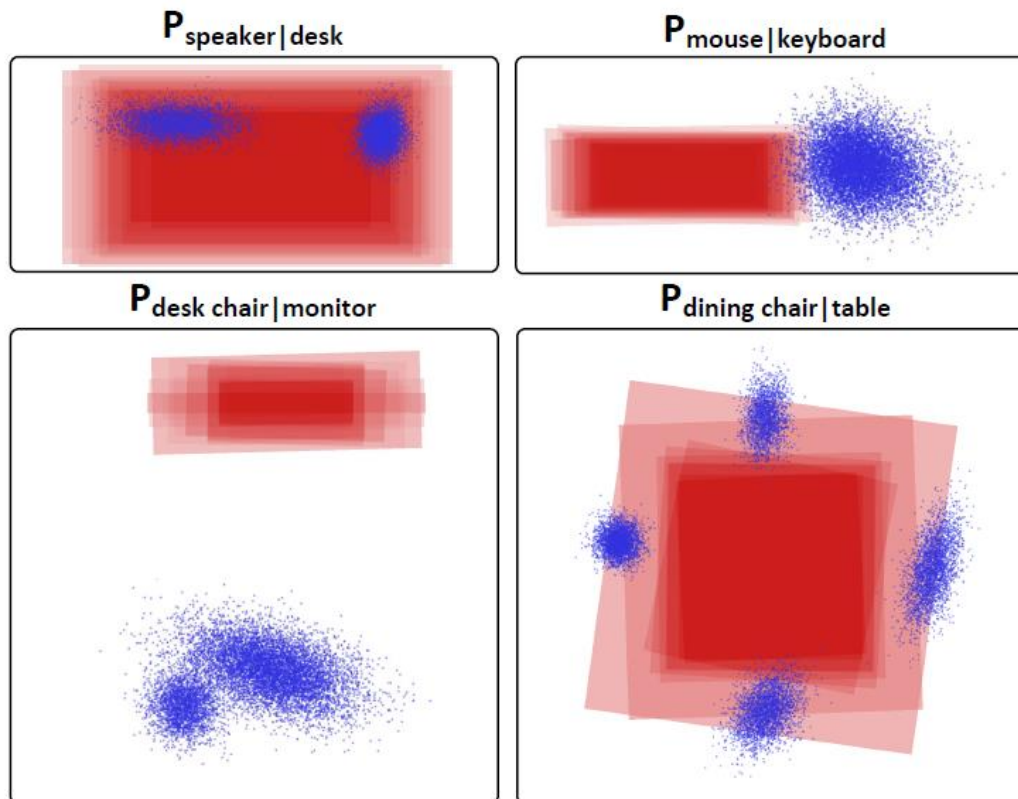
- Boolean random variables (desk supports keyboard?)
- Learn frequencies for pairs of categories
- Total probability is product over all objects in scene

$$P(t|c) = \prod_o P(C(o), C(\text{support}(o)))$$

Probabilistic Model of Scenes

Spatial arrangements: $P(a/t,c) = R(a,t,c)S(a,t,c)$

- Random variables for relative positions and orientations
- Pairwise distributions of spatial relationships

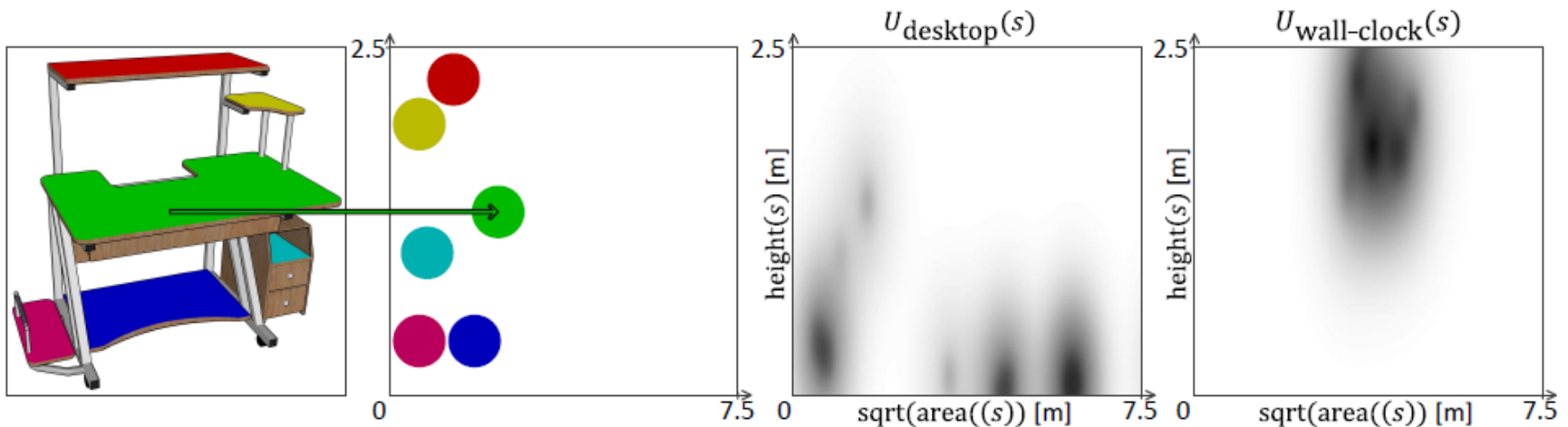


Distributions of spatial relationships for pairs of object categories

Probabilistic Model of Scenes

Spatial arrangements: $P(a/t,c)=R(a,t,c)S(a,t,c)$

- Random variables for relative positions and orientations
- Pairwise distributions of spatial relationships
- Feature distributions for positions on support surfaces



Distributions of geometric features of support surfaces

Side Note on Object Categories

Define categories of objects based on their contexts in a scene rather than basic functions

- Learned from examples by clustering of objects with similar spatial neighborhoods



Some Contextual Object Categories

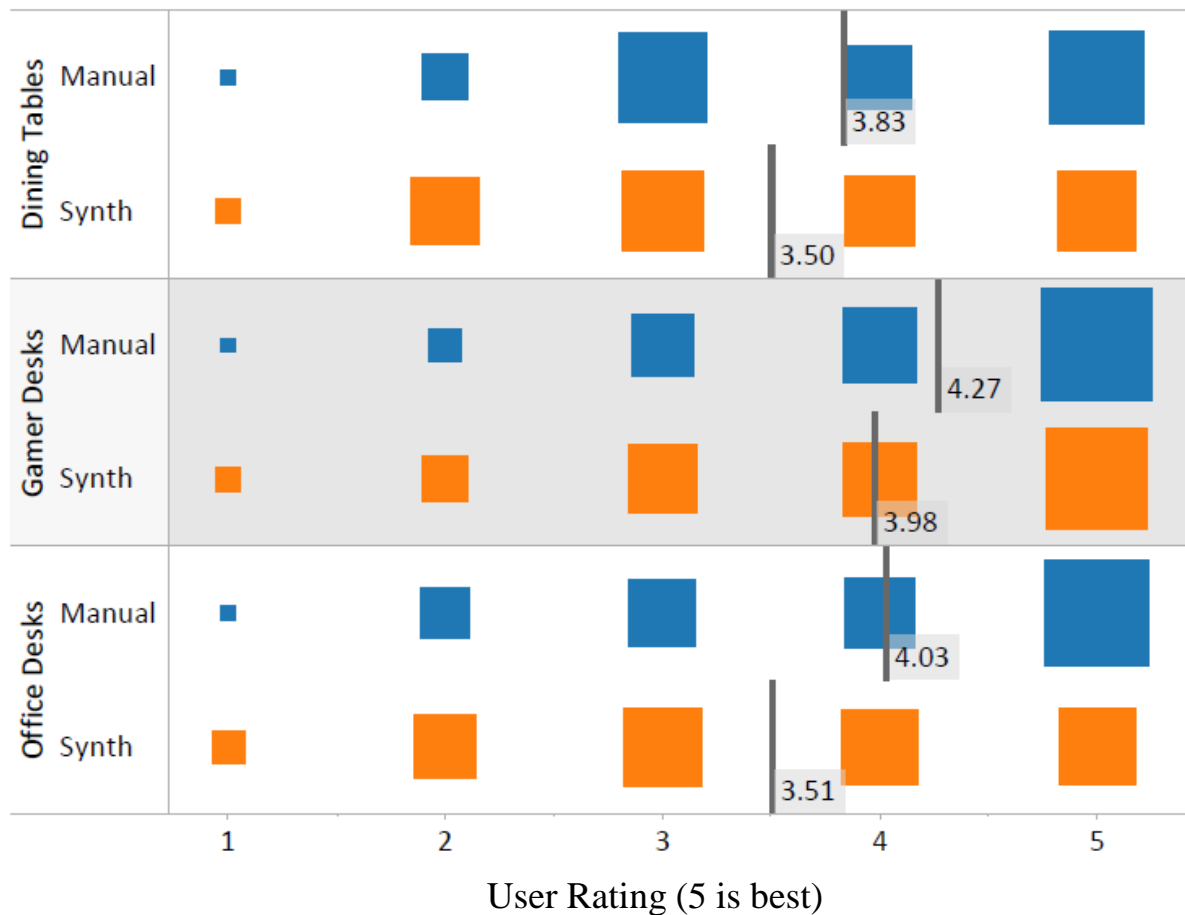
Scene Synthesis Results



Synthesized novel scenes

Scene Synthesis Results

User study suggests that people find our synthesized scenes almost as good as manually created ones



Procedural Modeling

Sweeps

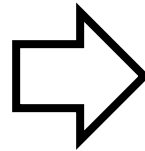
Fractals

Grammars

Probabilistic models

Probabilistic grammars ←

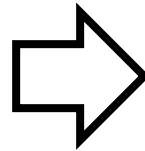
Probabilistic Grammars



Probabilistic
Model of Shape

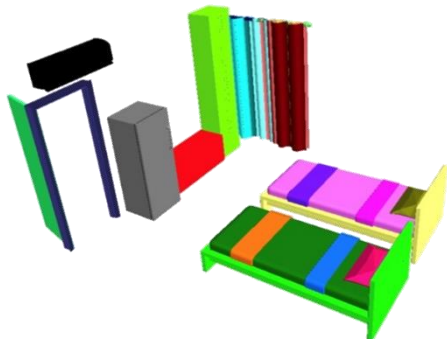
Training set of labeled scene graphs

Probabilistic Grammars



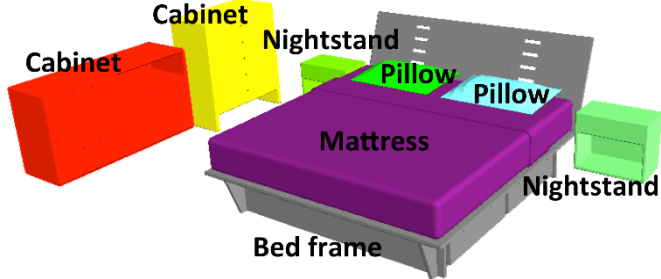
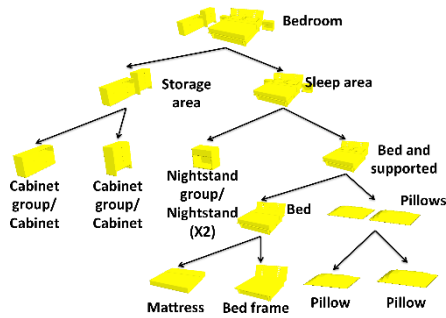
Probabilistic
Model of Shape

Training set of labeled scene graphs



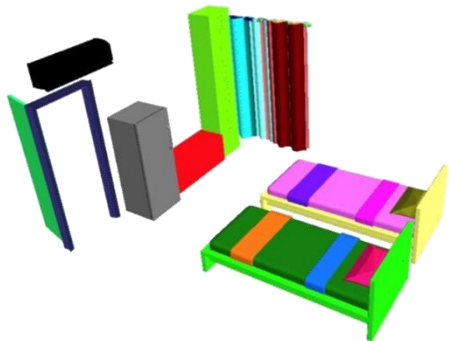
Unlabeled test scene

Probabilistic Grammars

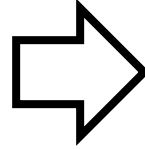


Training set of labeled scene graphs

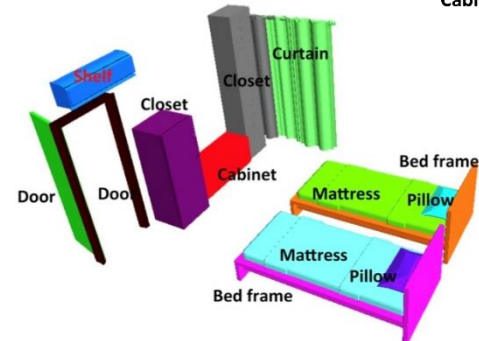
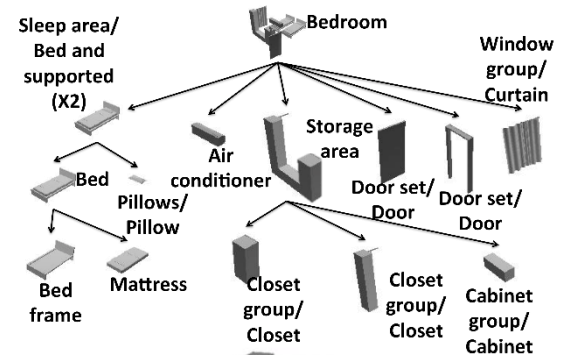
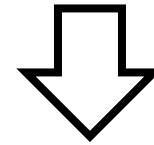
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Unlabeled test scene

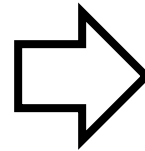
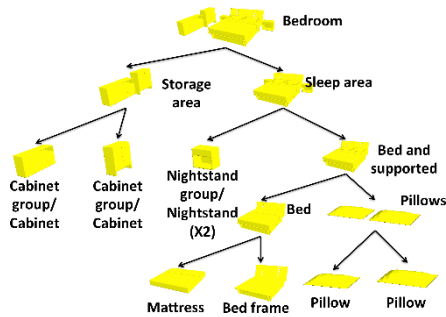


Probabilistic Model of Shape

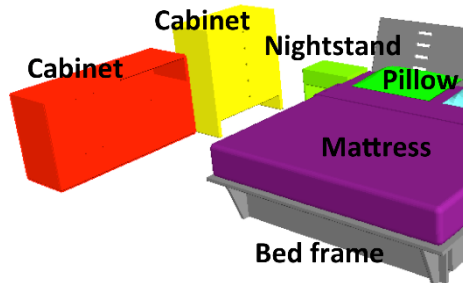
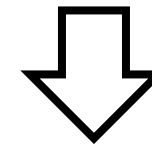


Labeled test scene graph

Probabilistic Grammars



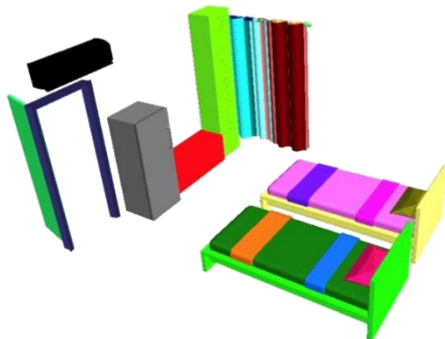
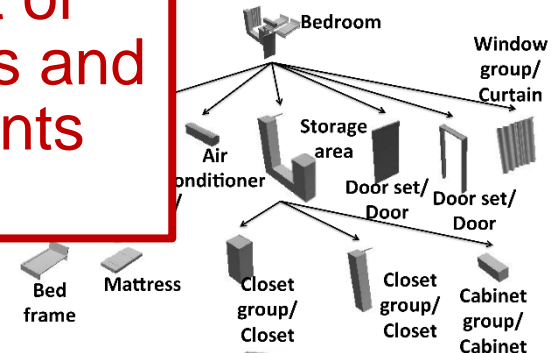
Probabilistic Model of Shape



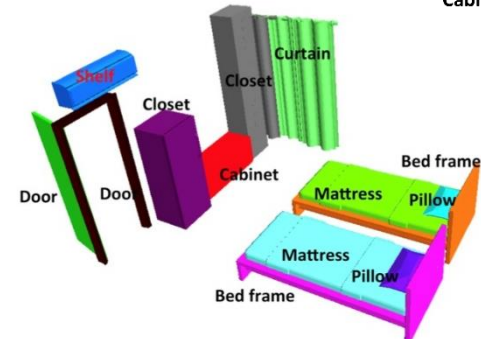
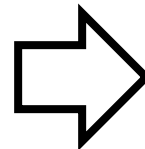
Training set of labeled



Challenge
Scenes have a lot of variability in the types and spatial arrangements of objects



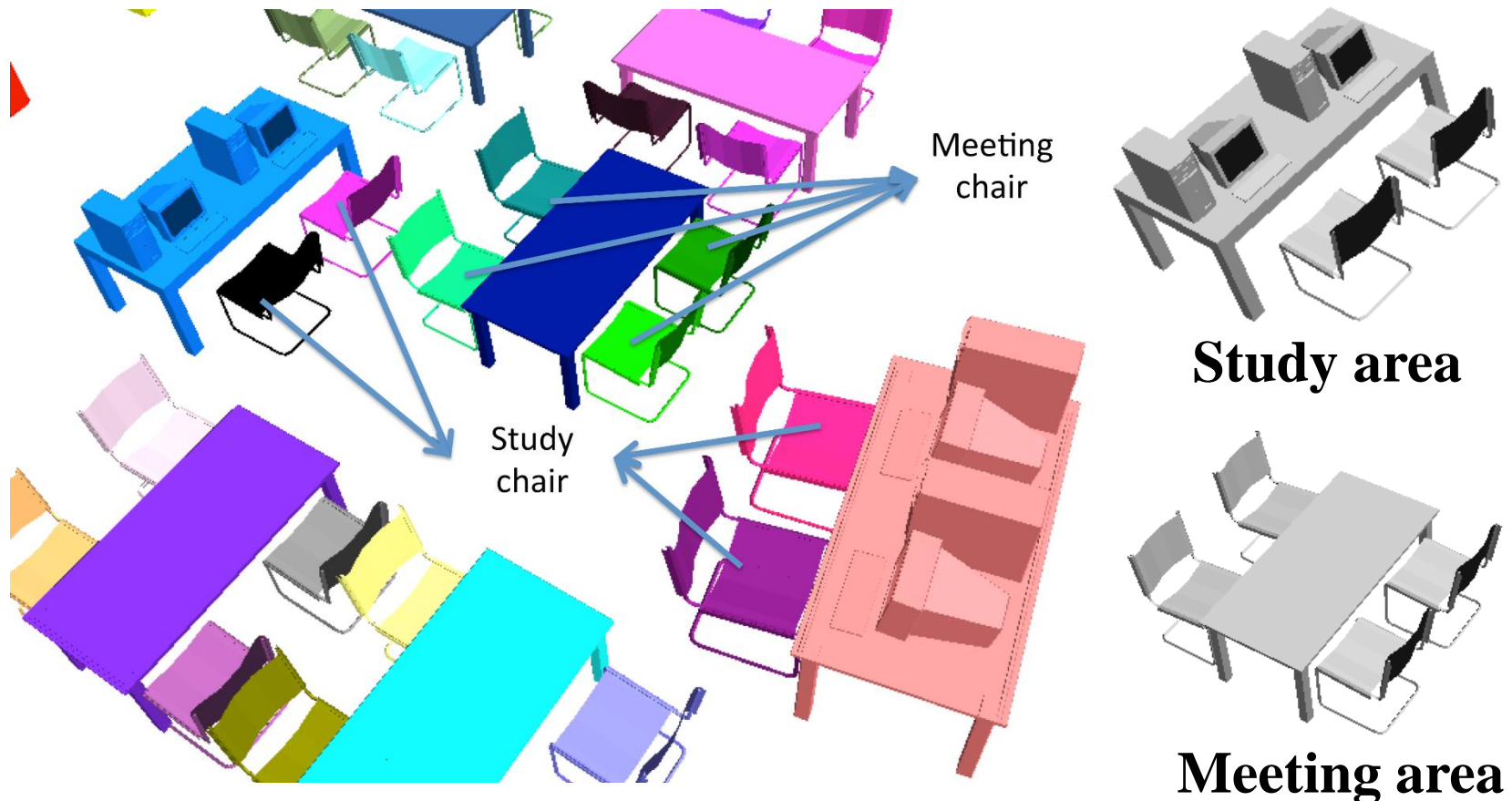
Unlabeled test scene



Labeled test scene graph

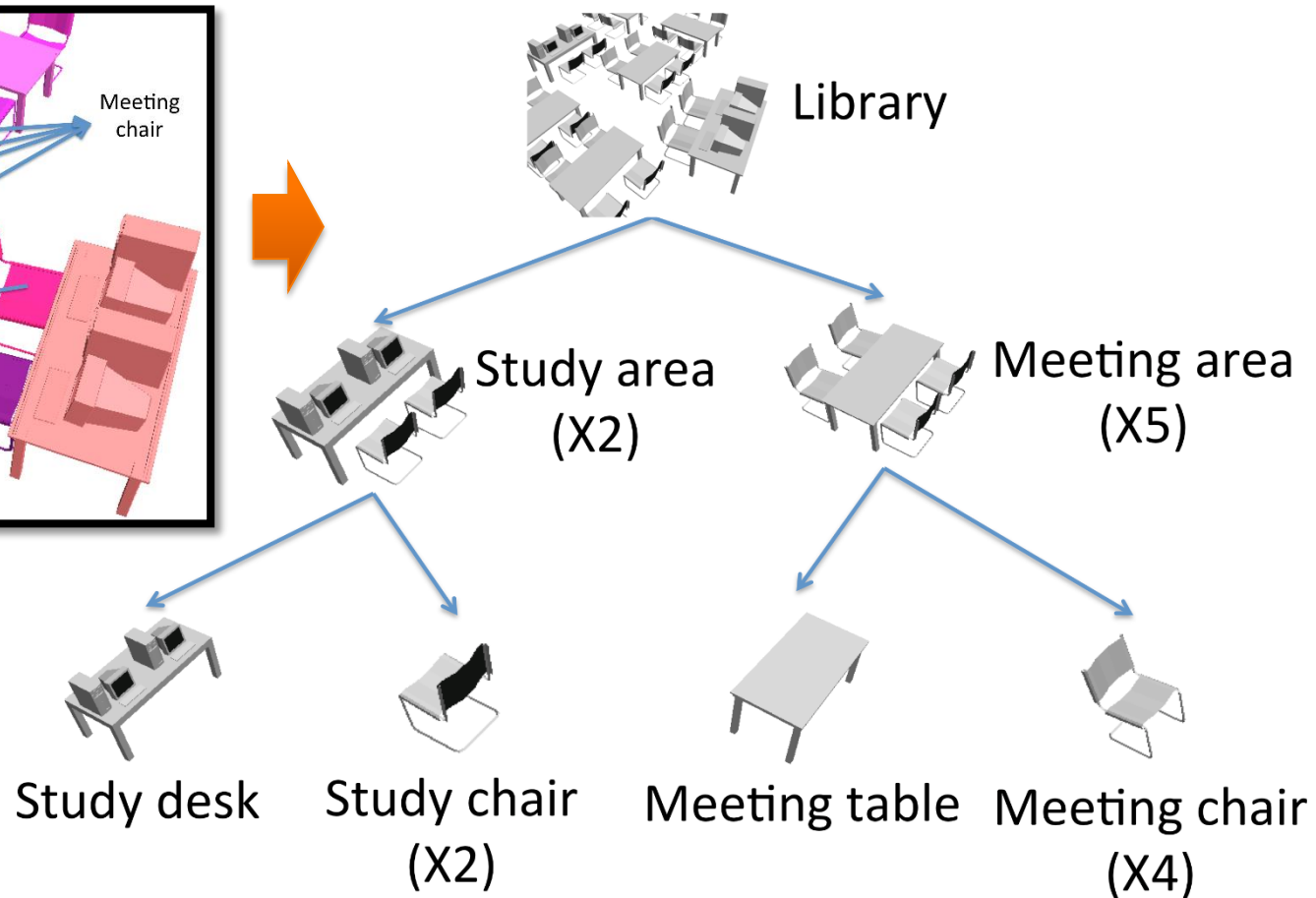
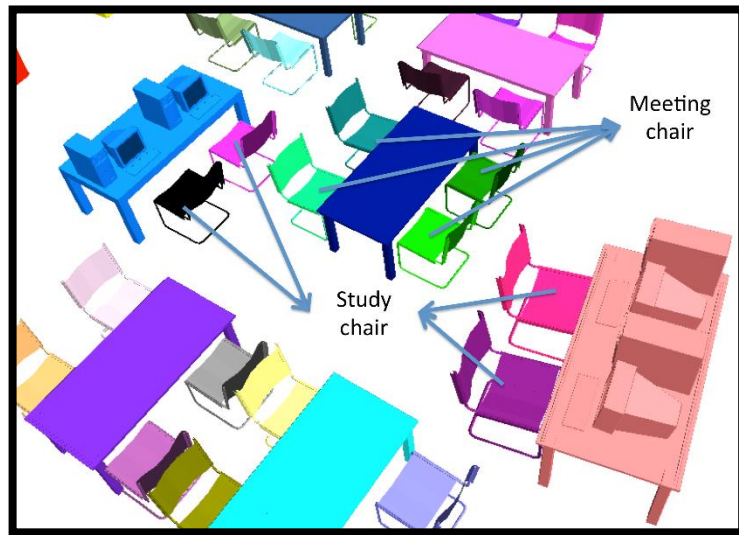
Probabilistic Grammars

Semantic and functional relationships are often more prominent within hierarchical contexts



Hierarchical Grammar

We learn a probabilistic grammar from examples, and then use it to parse new test scenes



Hierarchical Grammar

Labels: object group, object category, object part
sleep area, bed, curtain piece

Rules: derivation from a label to a list of labels

bed → bed frame mattress

Hierarchical Grammar

Probabilities:

Derivation: $P_{nt}(rule / lhs)$

bed \rightarrow frame mattress
 $P = 0.8$

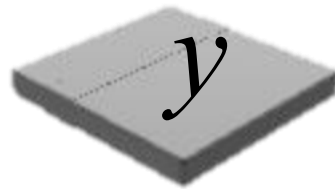
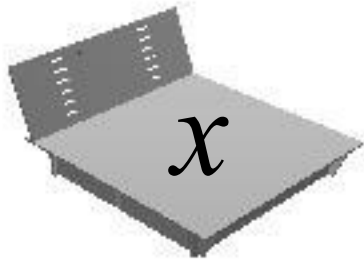
Cardinality distribution: $P_{Card}(\#, rhs / lhs)$

sleep area \rightarrow bed nightstand rug ...

$P_{card}(* sleeparea)$	0	1	2	3	4+
bed
nightstand	0.3	0.3	0.4	0	0
rug

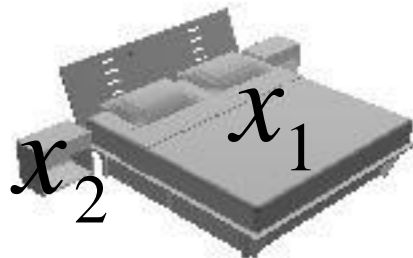
Hierarchical Grammar

Shape descriptor probability: $P_g(x / label)$



$$P_g(x | bedframe) > P_g(y | bedframe)$$

Spatial relationships: $P_g(v / lhs, rhs1, rhs2)$

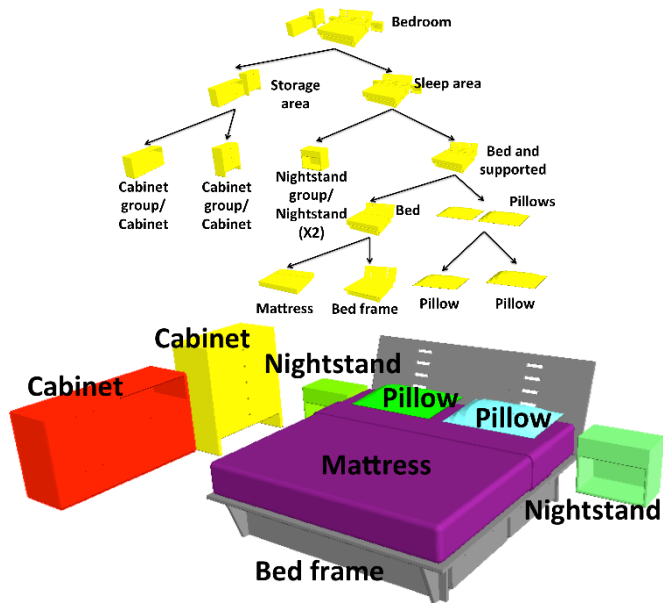


$$P_s(x_1, x_2 | sleeparea, bed, nightstand) >$$

$$P_s(x_1, x_3 | sleeparea, bed, nightstand)$$

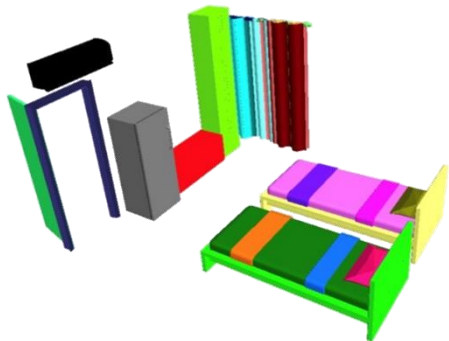
x_3

Grammar Learning and Parsing



Training set of labeled scene graphs

+

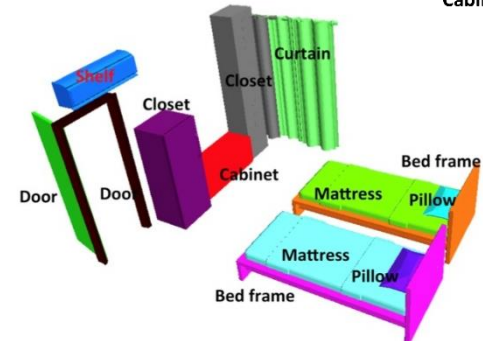
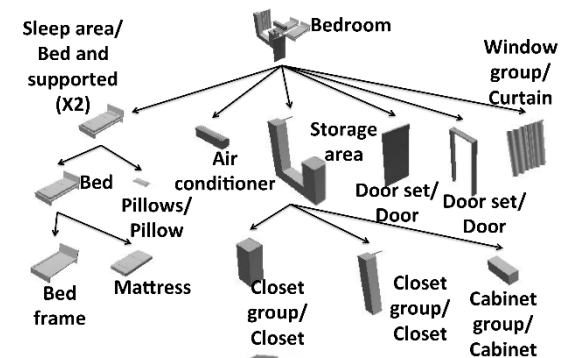


Unlabeled test scene

Learn
→

Probabilistic Hierarchical Grammar

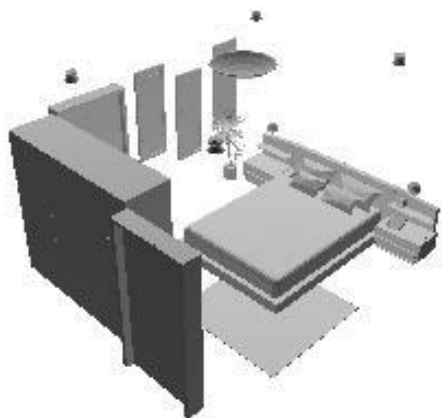
↓ Parse



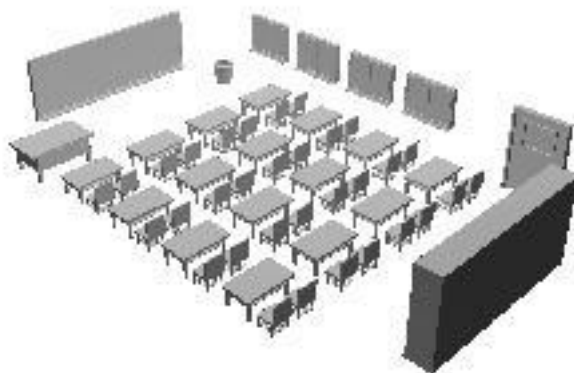
Labeled test scene graph

Hierarchical Grammar Results

Learned hierarchical probabilistic grammars from scenes in Trimble 3D Warehouse



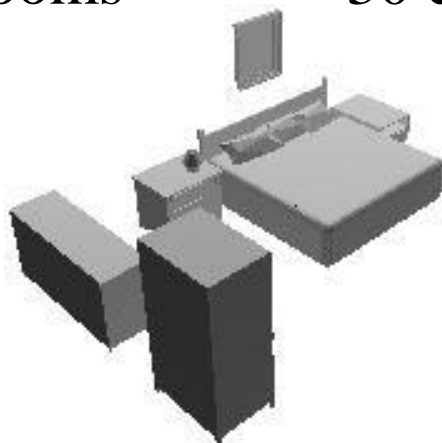
77 bedrooms



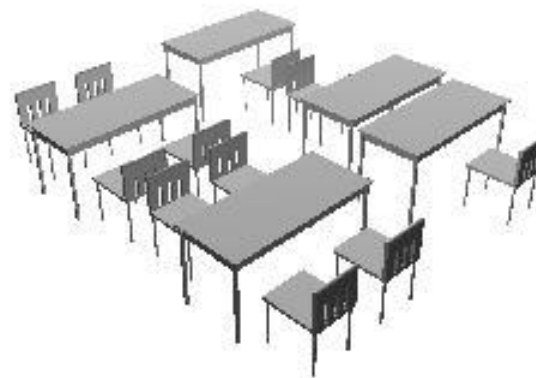
30 classrooms



8 libraries



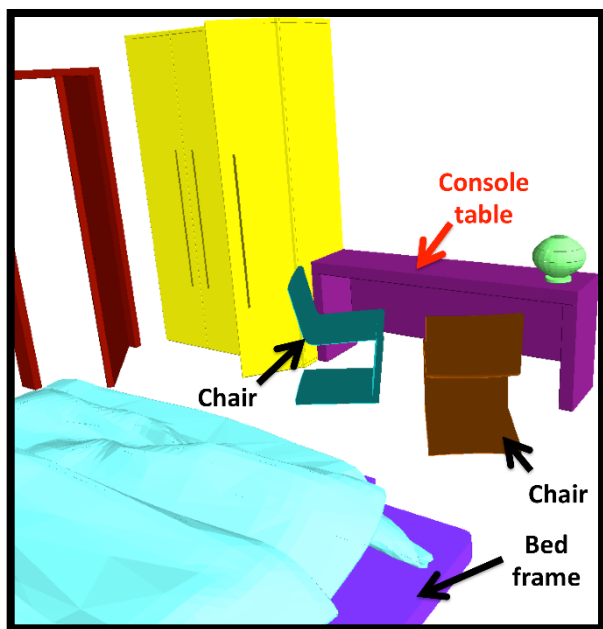
17 small bedrooms



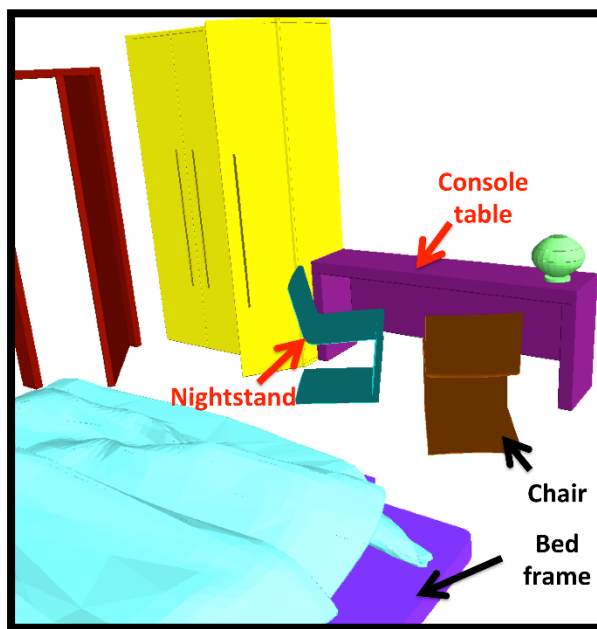
8 small libraries

Hierarchical Grammar Results

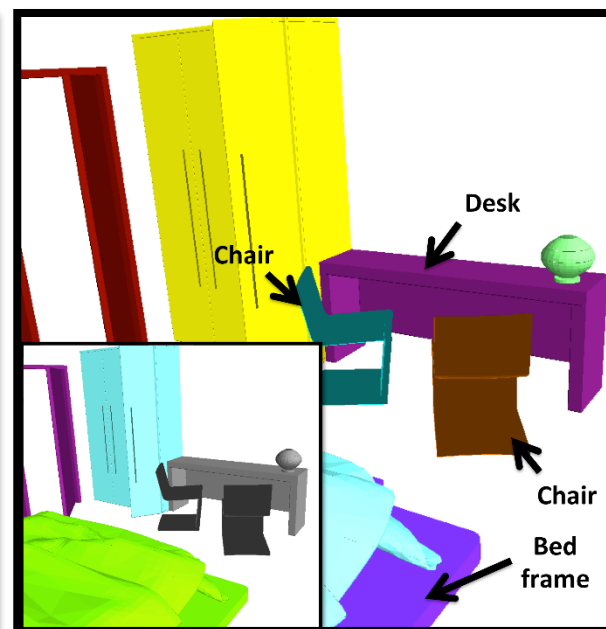
Parsed left-out scenes with learned grammar



Shape Only



Flat Grammar

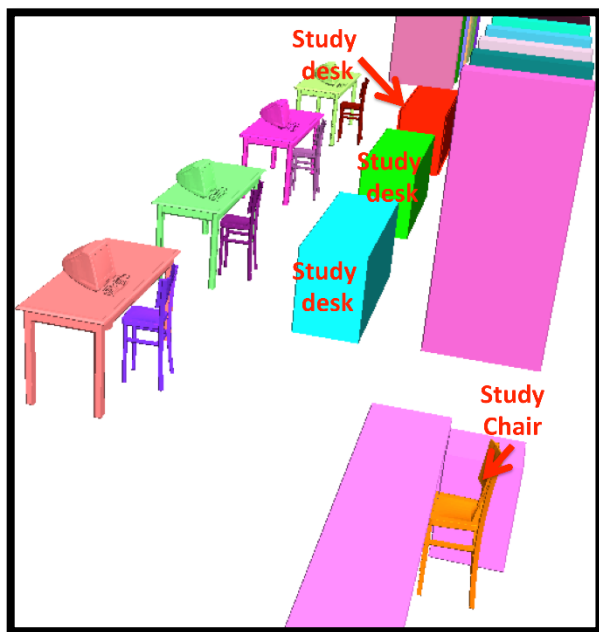


Our Hierarchical Grammar

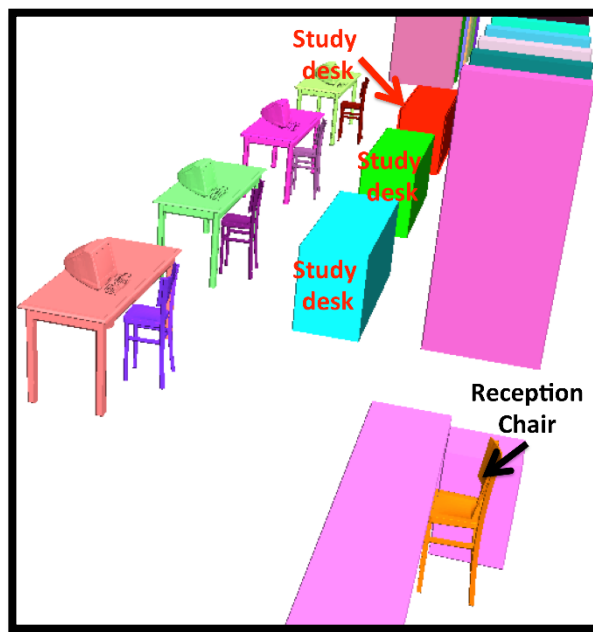
Comparison of our parsing results to other methods

Hierarchical Grammar Results

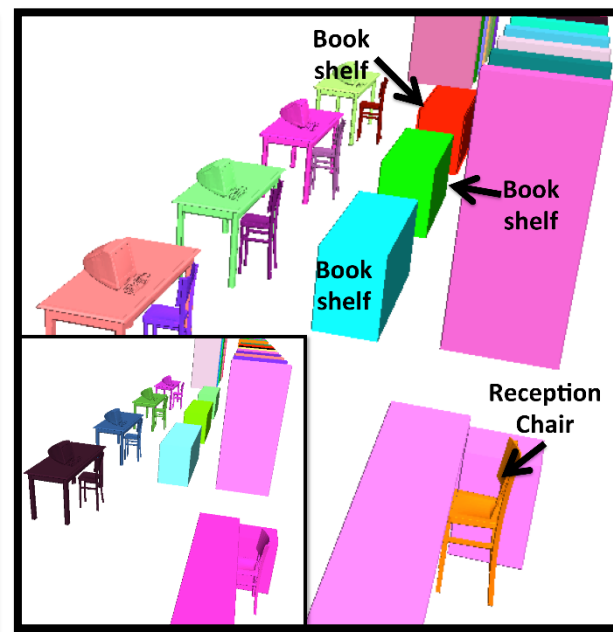
Parsed left-out scenes with learned grammar



Shape Only



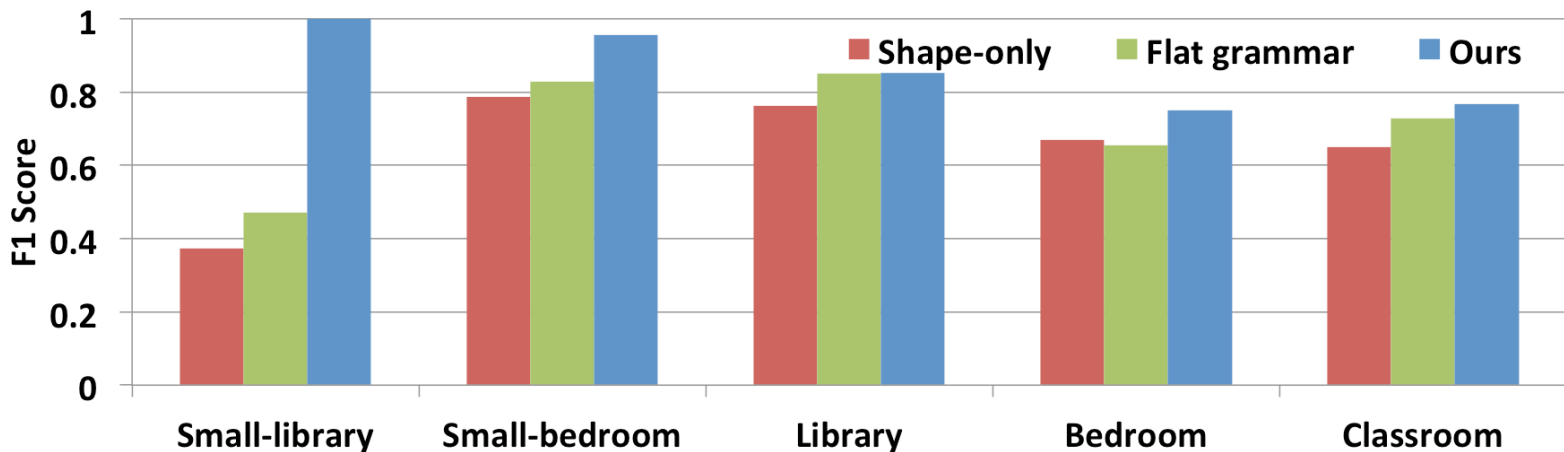
Flat Grammar



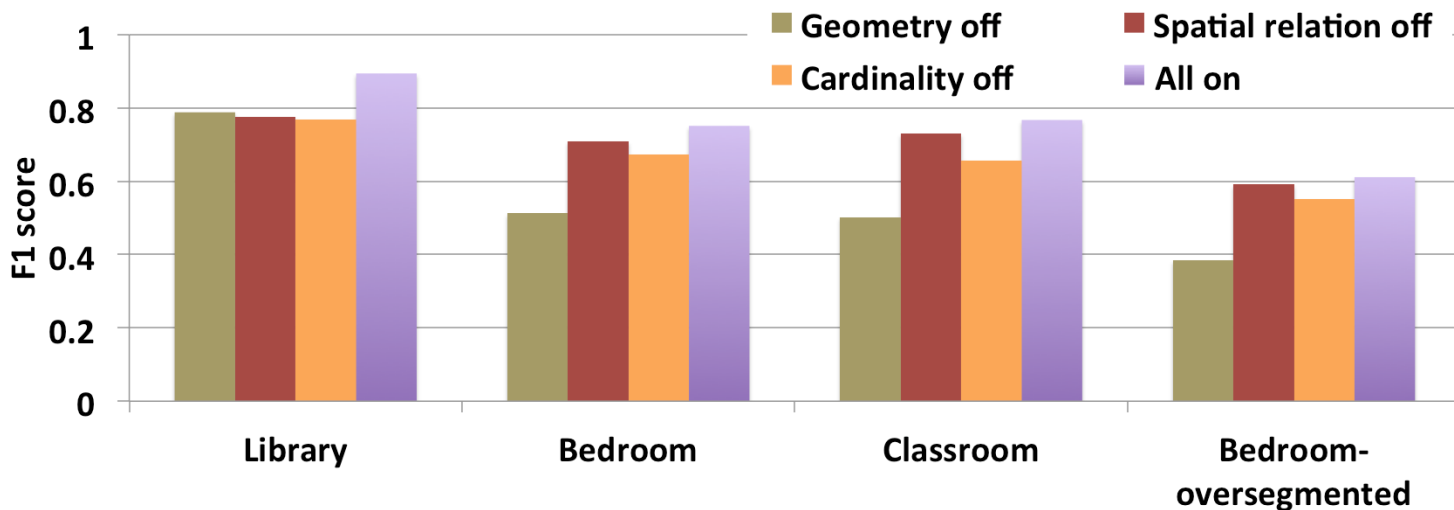
Our Hierarchical Grammar

Comparison of our parsing results to other methods

Hierarchical Grammar Results



Comparison of object classification



Impact of Individual Energy Terms

Procedural Modeling

Sweeps

Fractals

Grammars

Probabilistic models

Probabilistic grammars

Probabilistic programs ???

Procedural Modeling Summary

Motivation:

- Describe 3D models algorithmically

Methods:

- Sweeps, fractals, grammars
- Probabilistic models, grammars, and programs (?)

Advantages:

- Automatic generation
- Concise representation
- Parameterized classes of models
- **Learned from examples**