Non-Rigid Surface Correspondence

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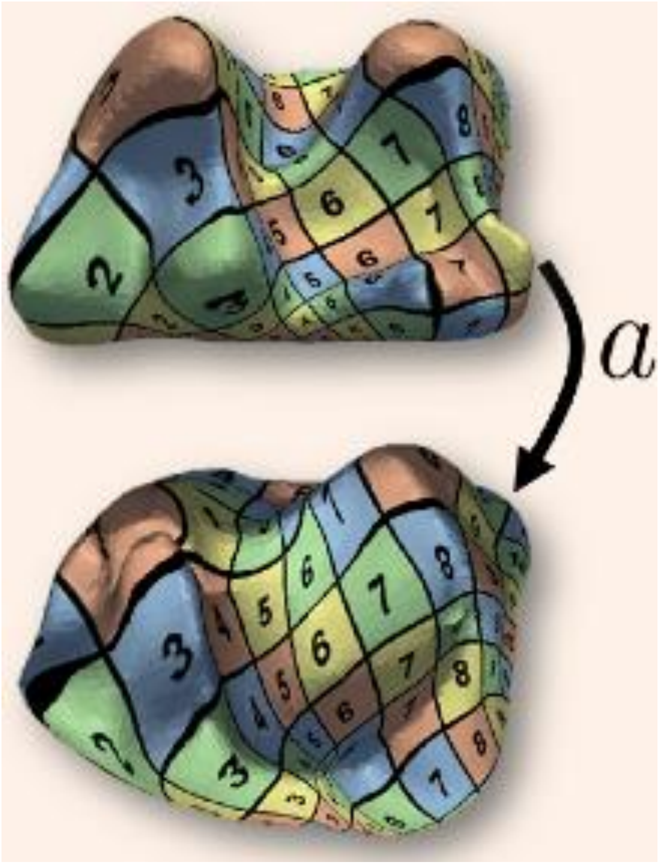
Goal

Find maps between surfaces

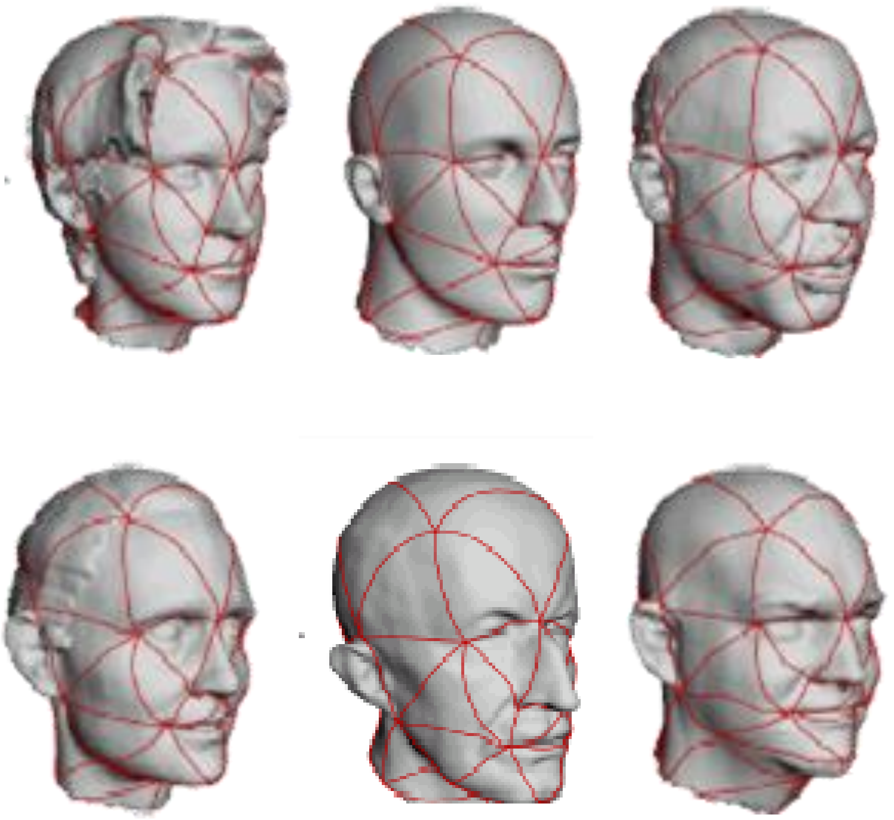
- Non-rigid
- Bijective
- Smooth
- Shape preserving
- Automatic
- Efficient computation
- Provide metric
- Semantic alignment



Motivating Applications

Finding corresponding points on surfaces enables …

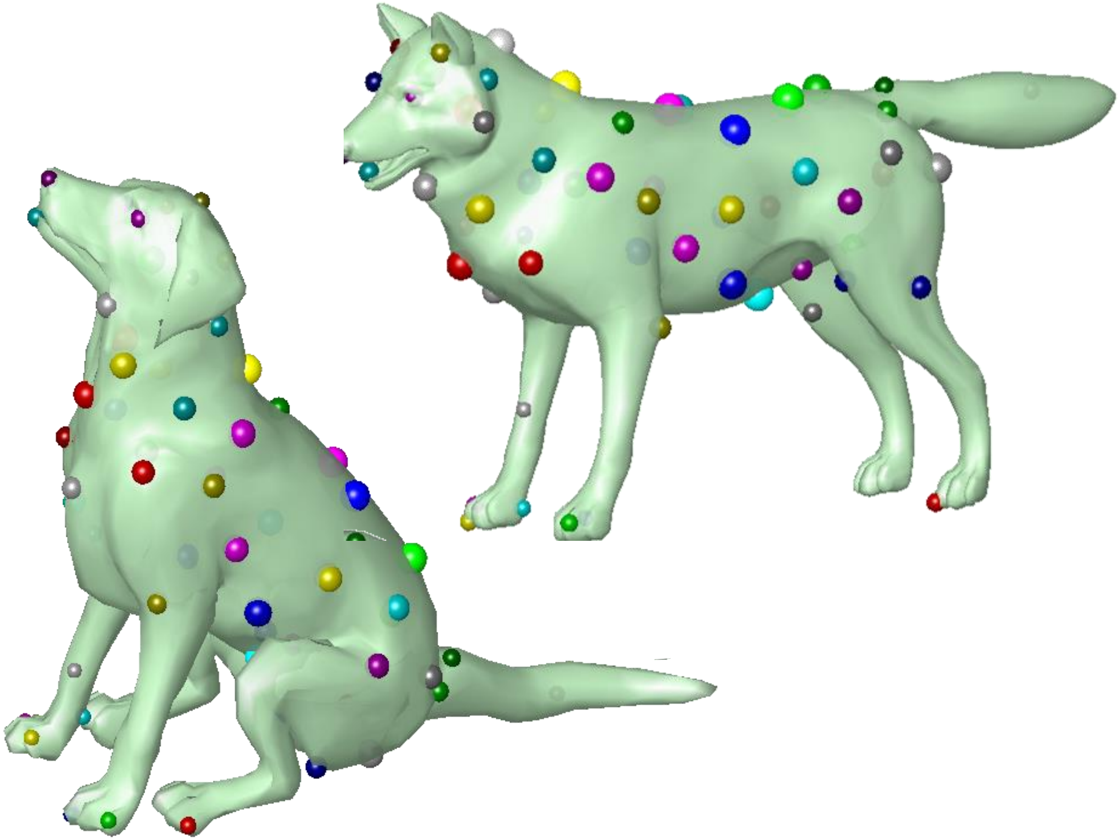
- Surface comparison
- Collection analysis
- Property transfer
- Morphing
- etc.



[Praun et al.]

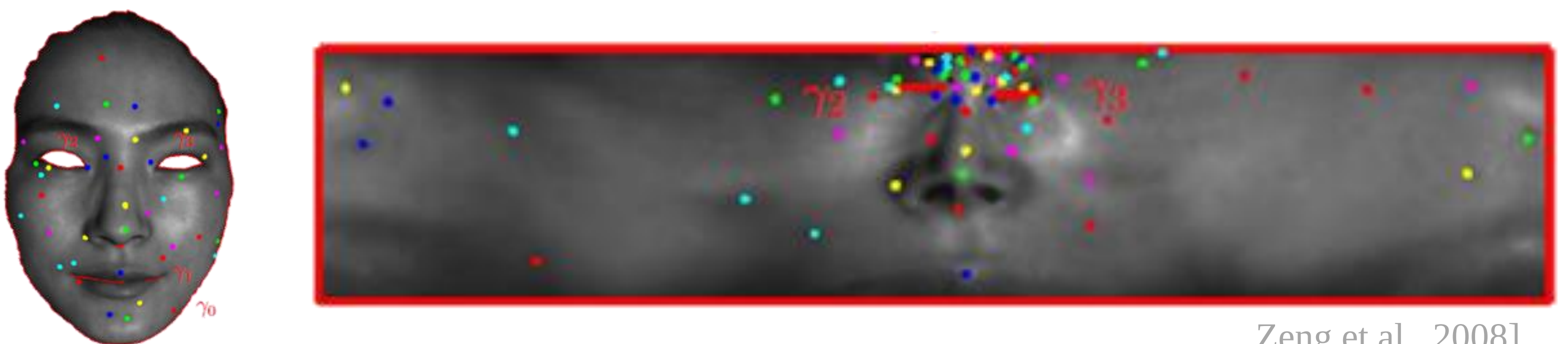
Problem 1

Find a sparse set of feature correspondences

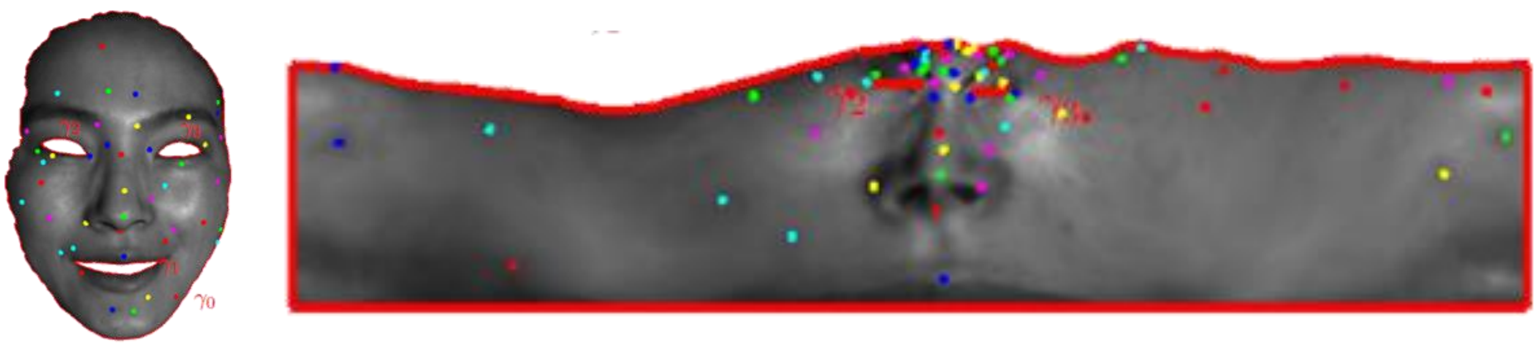


Problem 2

Compute a dense map from
a sparse set of feature correspondences



Zeng et al., 2008]



Least Squares Conformal Map
(preserve angles as best as possible)

Outline

Introduction

Some surface mapping algorithms

- Feature correspondence search
- High-dimensional embedding
- Möbius transformations
- Blended maps

Example Application

Conclusion

Future work

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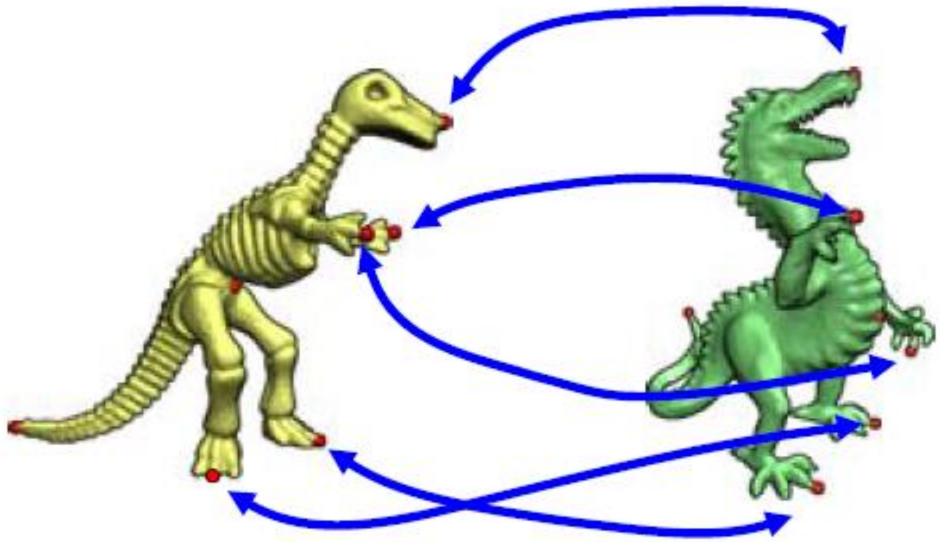
Feature Correspondence Search

For each coarse set of feature correspondences:

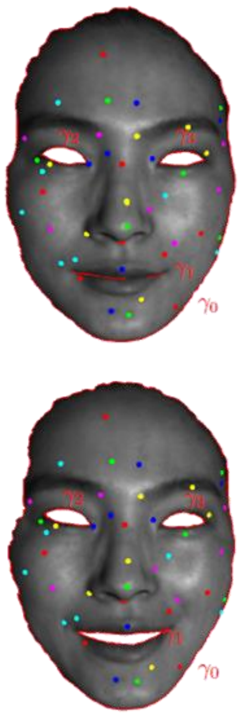
- Measure the deformation required to align them
  - … maybe by solving problem 2
- Remember the one with least deformation



Feature Correspondence Search

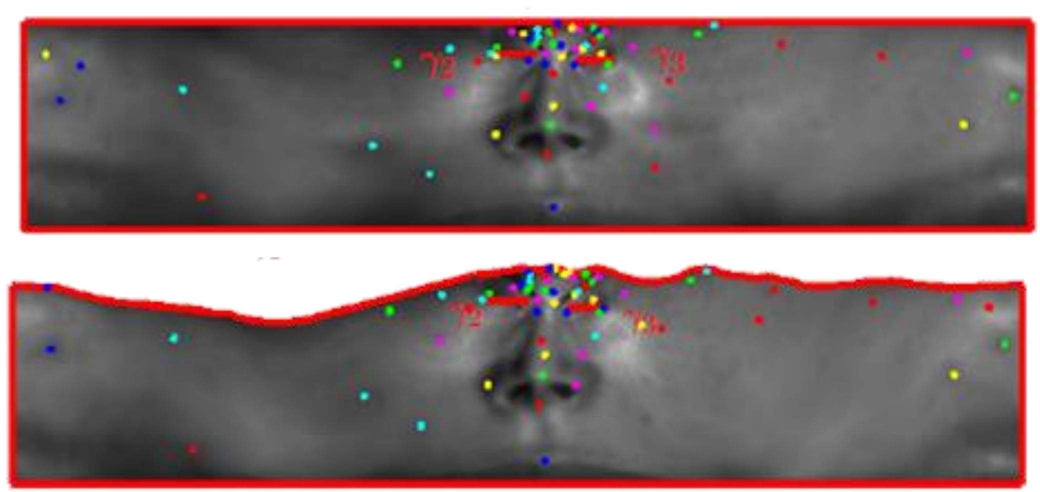
Measures of distortion:

- Differences in geodesic distances
- Differences in conformal factors (angles)
- etc.



Feature points

Branch and bound
Priority-driven search
etc.



Least squares conformal map
aligning corresponding feature points

[Zeng et al., 2008]

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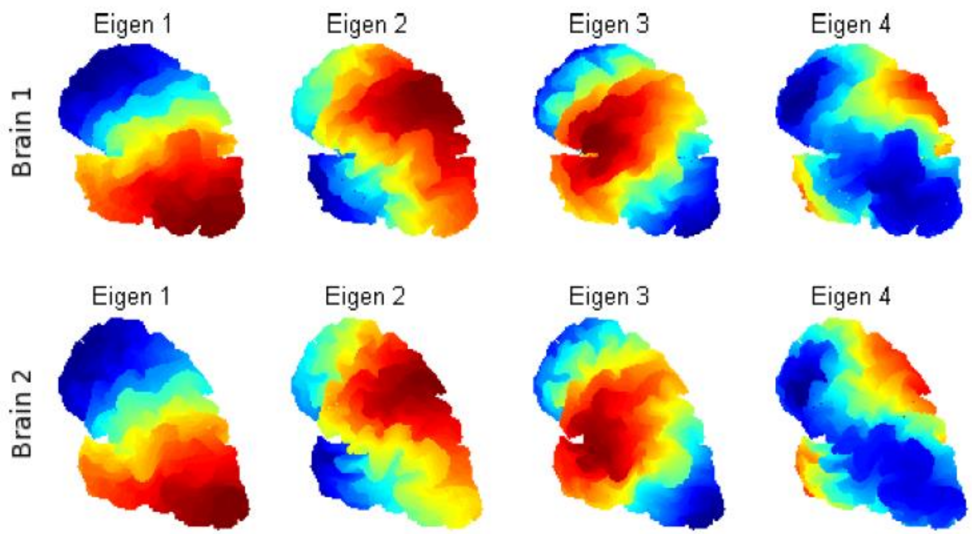
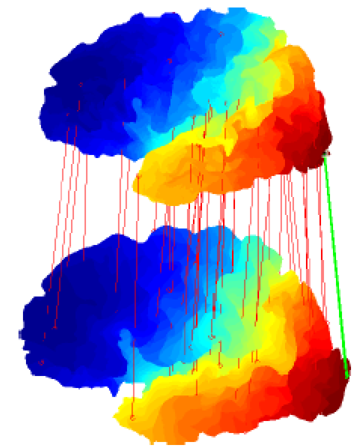
High-Dimensional Embedding

Find nearest neighbors after spectral embedding



Flip, reorder, closest points, etc.



Closest points on embeddings

Eigenfunctions of the Laplacian
[Lombaert et al. 2011]

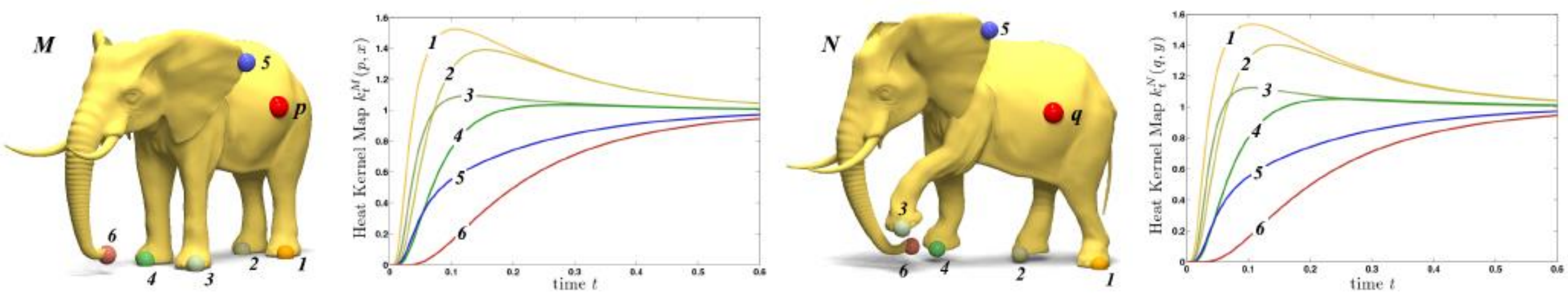
High-Dimensional Embedding

Find nearest neighbors after spectral embedding

Eigenfunctions of the Laplacian
[Lombaert et al. 2011]

High-Dimensional Embedding

Find nearest neighbors after heat kernel embedding implied by a single point correspondence



Heat Kernel Map
[Ovsjanikov et al. 2010]

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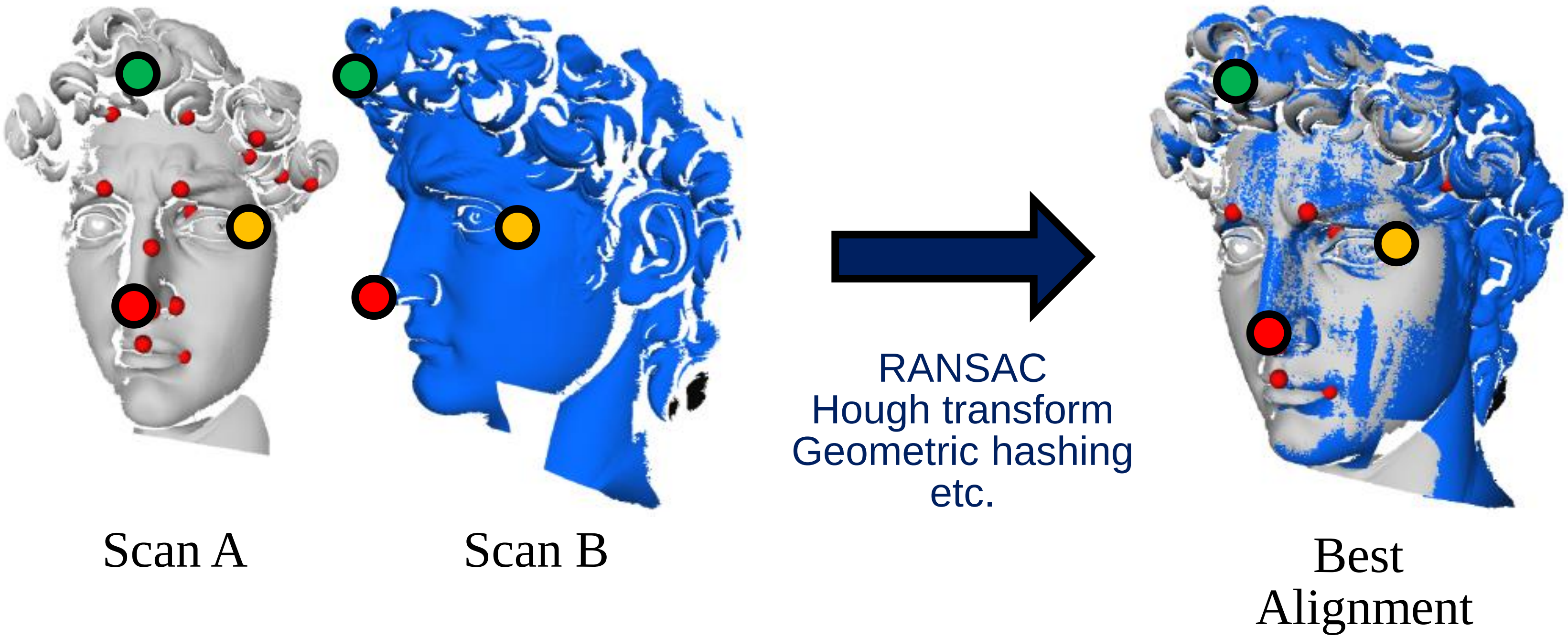
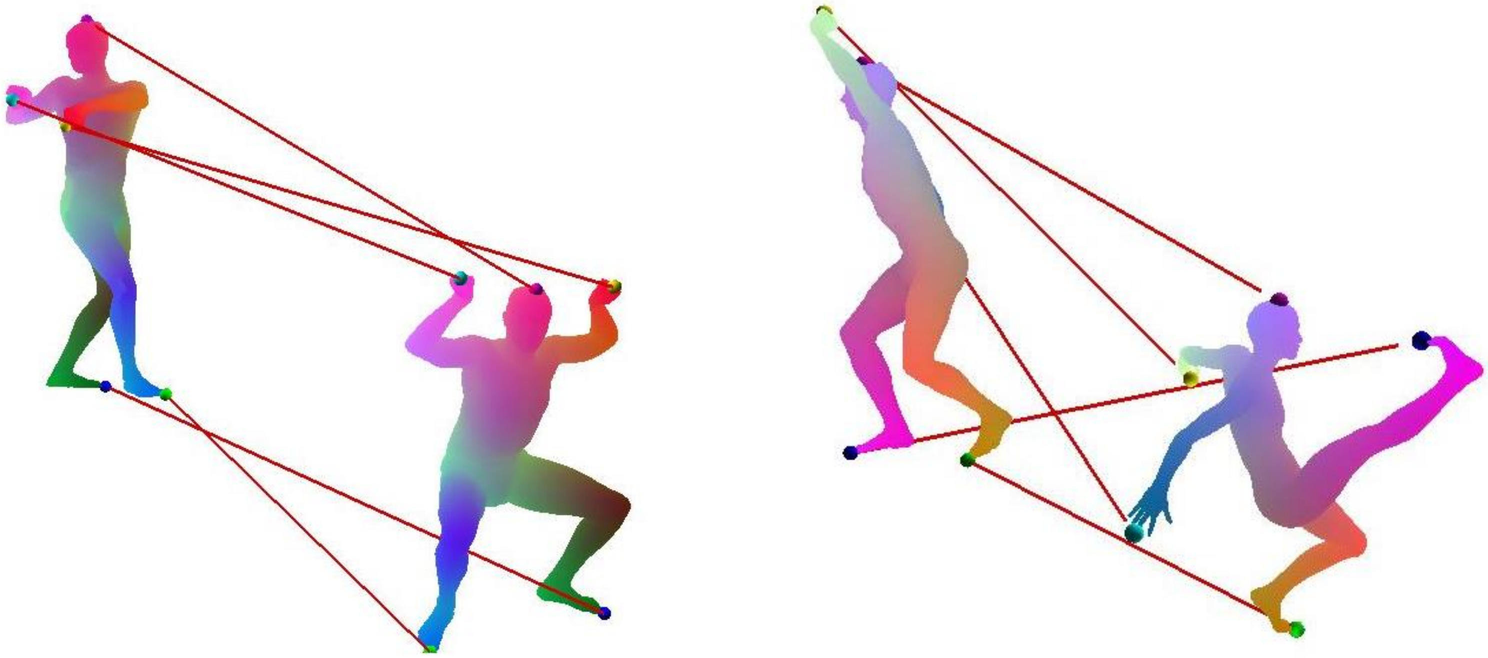
Möbius Transformations

It would be nice to search a low-dimensional space of transformations to align non-rigid surfaces …



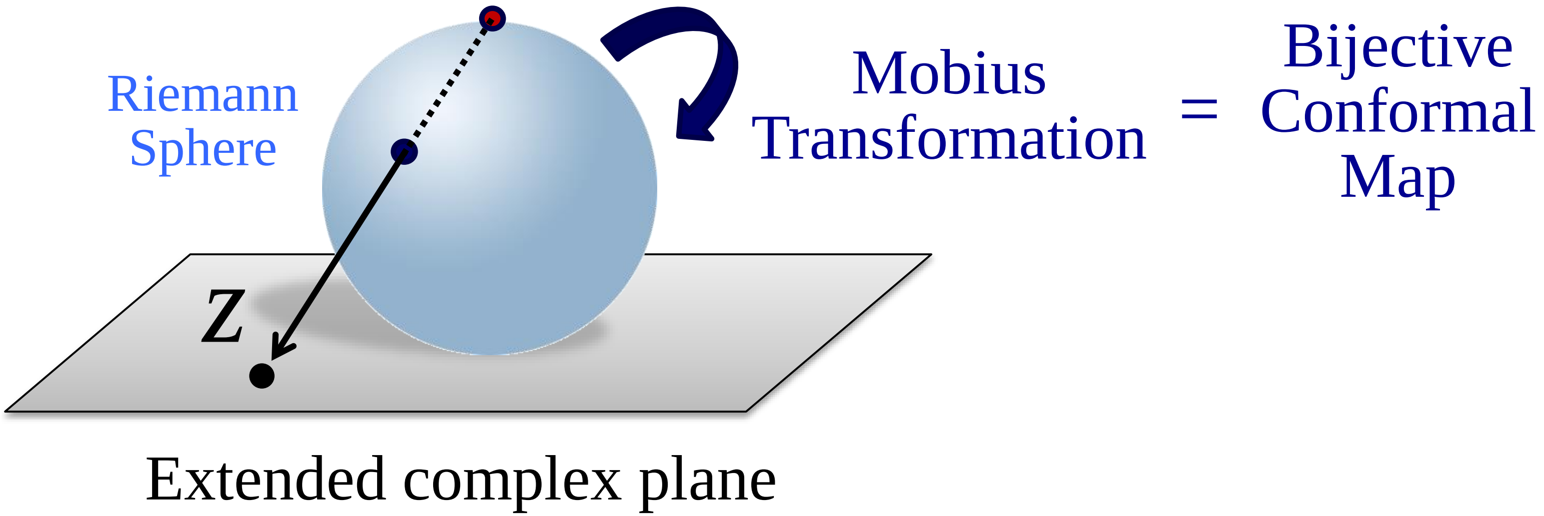
Key Observation

The Möbius group provides a low-dimensional space to search efficiently for the “best” conformal map between genus zero surfaces



Möbius Transformations I

Möbius transformations are a group of functions on the extended complex plane that represent bijective, conformal maps



Möbius Transformations II

Möbius transformations are simple rational functions:

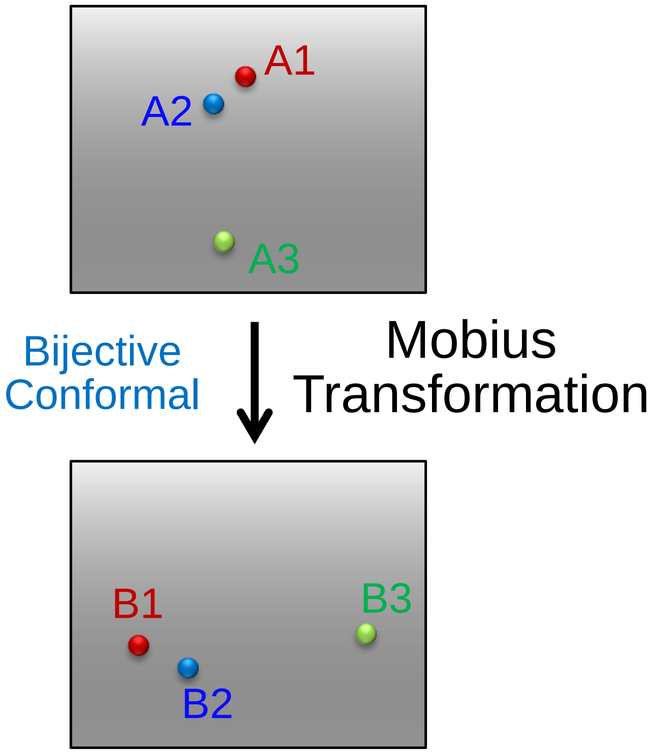
$$f(z) = \frac{az+b}{cz+d}, \quad ad - bc \neq 0, \;\; a,b,c,d \in C$$

They have only six degrees of freedom
(they can be computed analytically
from three point correspondences)

$$f(z_i) = y_i, i = 1,2,3$$

Möbius Transformations III

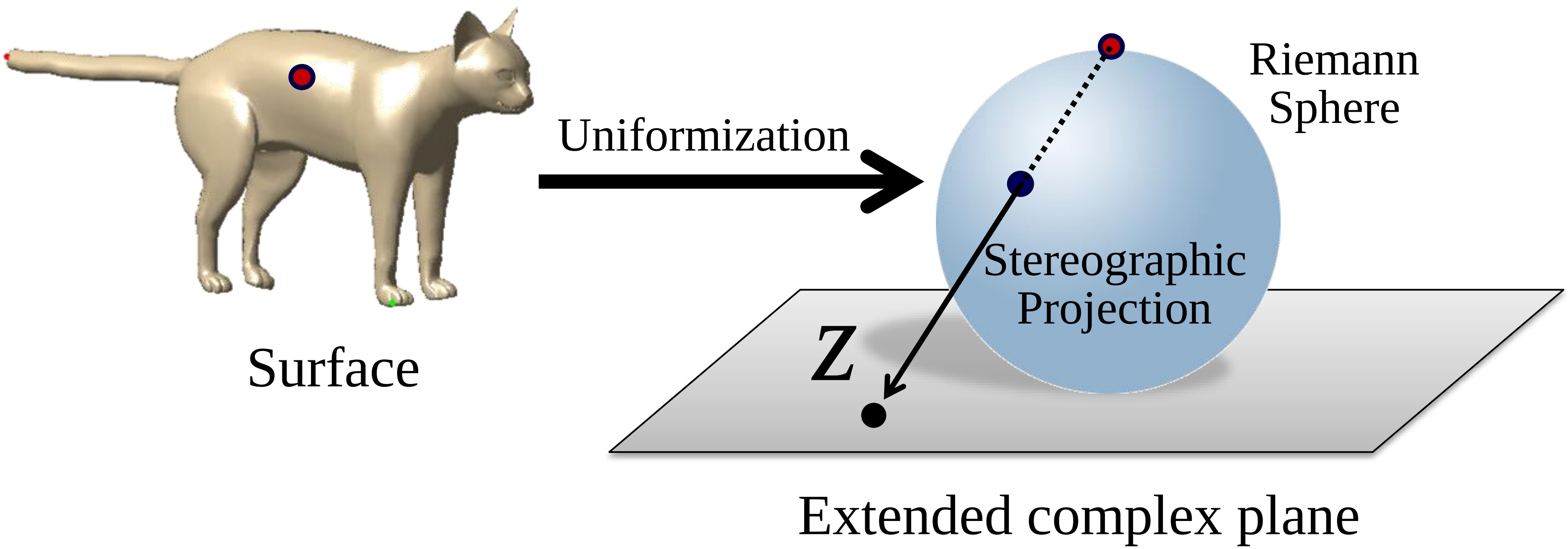
Therefore, any three point correspondences define a bijective, conformal map from the extended complex plane onto itself



Extended complex plane

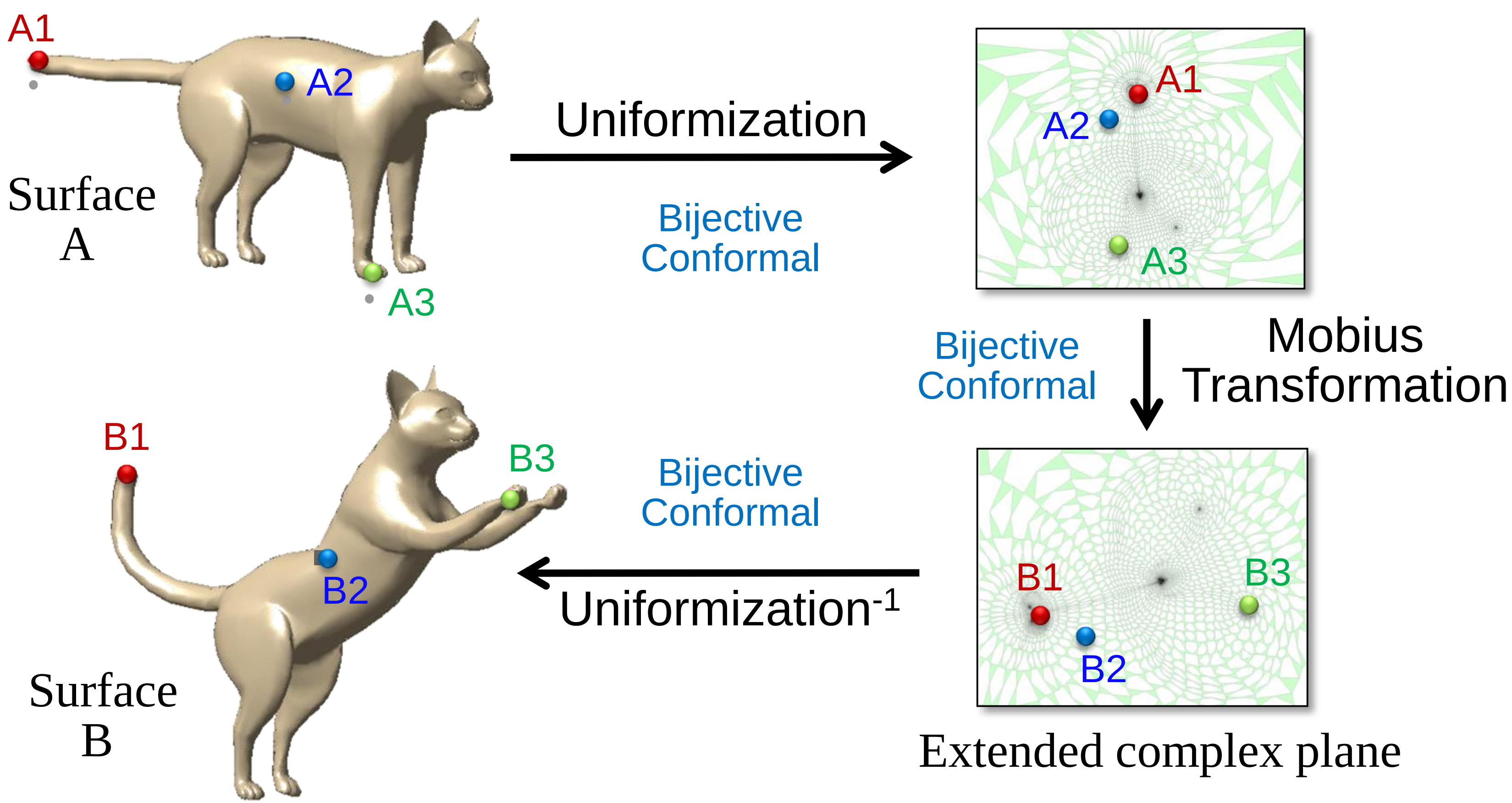
Möbius Transformations IV

Since every genus zero surface can be mapped conformally onto the extended complex plane (Riemann sphere), …



Möbius Transformations V

Any three point correspondences define a bijective, conformal map between genus zero surfaces



Möbius Transformations VI

We can search for the "lowest distortion" bijective, conformal map between genus zero surfaces using algorithms that sample triplets of correspondences(e.g., RANSAC, Hough transform, etc.)

Polynomial-time algorithm
for non-rigid surface mapping

Surface Mapping Algorithm

Example: RANSAC algorithm

- For i = 1 to ~N^3
  - Sample three points (A1,A2,A3) on surface A
  - Sample three points (B1,B2,B3) on surface B
  - Compute conformal map M: (A1,A2,A3)→(B1,B2,B3)
  - Remember M if distortion is smallest

Surface Mapping Algorithm

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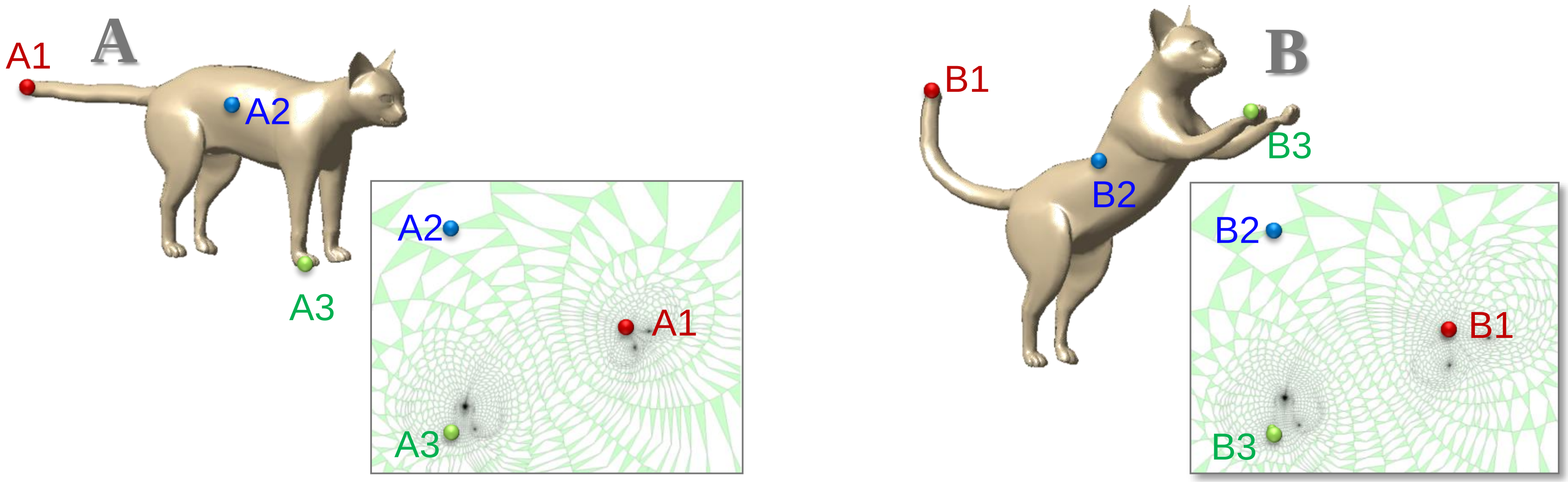

Measure distortion by relative change of area
(deviation from isometry)

Surface Mapping Algorithm

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Sample three points (A1,A2,A3) on surface A

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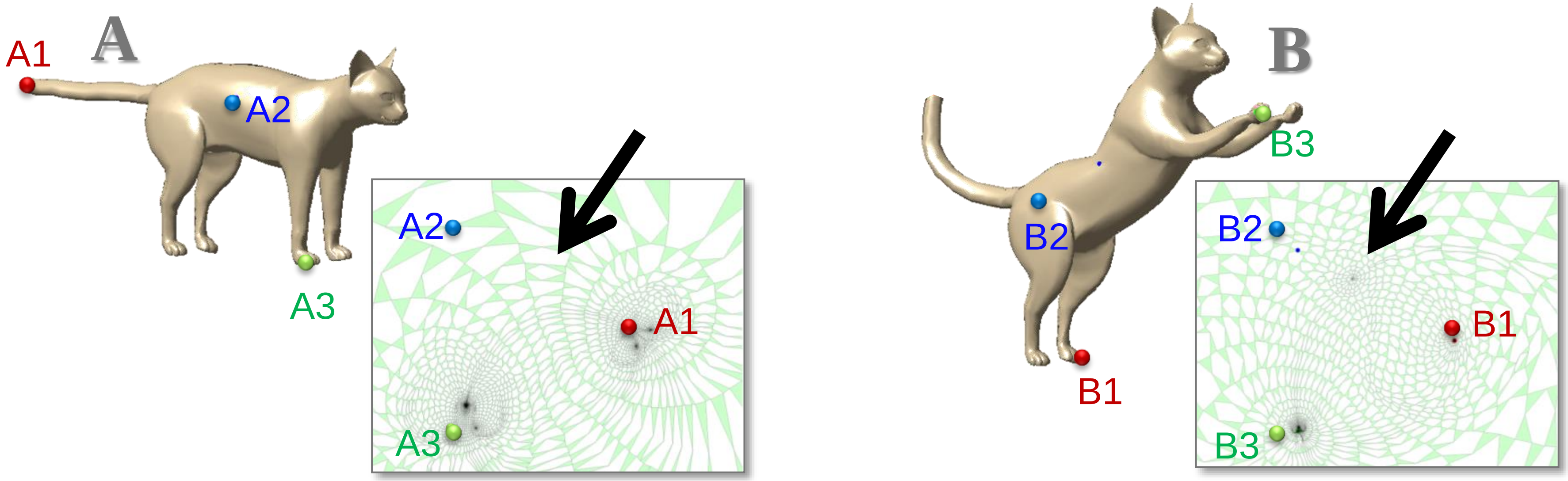
Compute conformal map M: (A1,A2,A3)→(B1,B2,B3)

Remember M if distortion is smallest



Measure distortion by relative change of area
(deviation from isometry)

Surface Mapping Algorithm

RANSAC algorithm properties:

- Non-rigid
- Bijective
- Smooth
- Shape preserving
- Automatic
- Efficient computation
- Provides metric
- Semantic alignment?

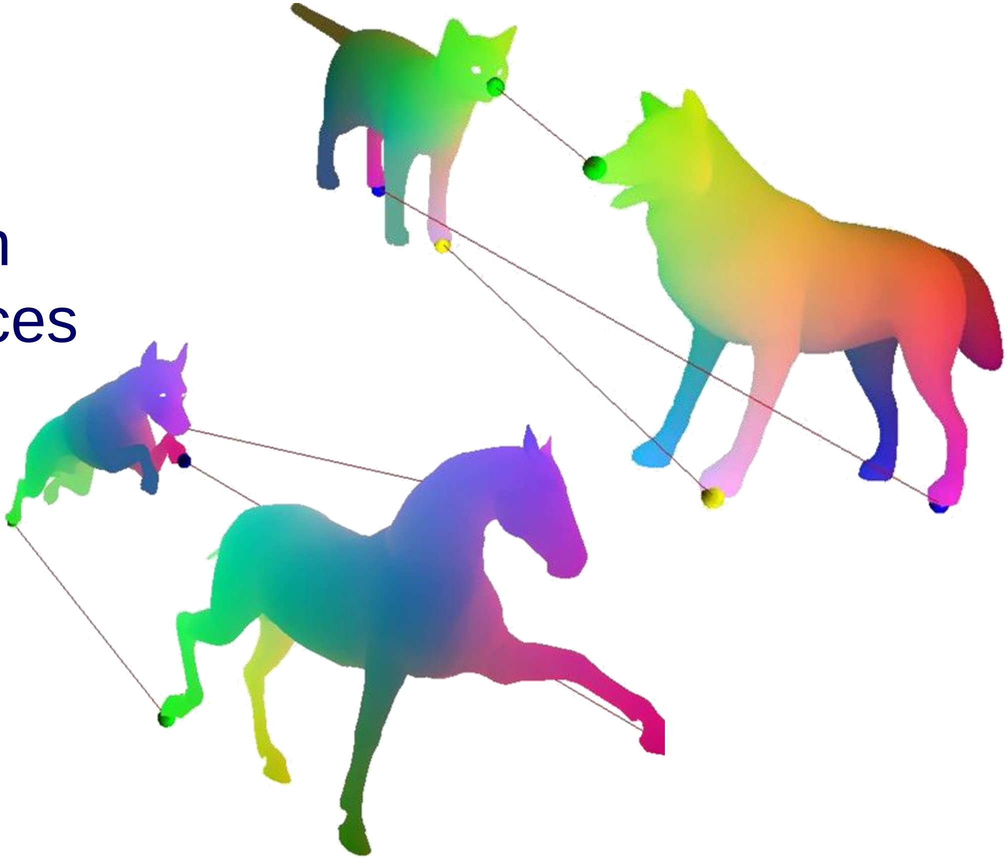
Experimental Results

Data:

- 51 pairs of meshes representing animals from TOSCA and SHREC Watertight data sets

Methodology:

- Predict surface maps
- Compare to ground truth semantic correspondences

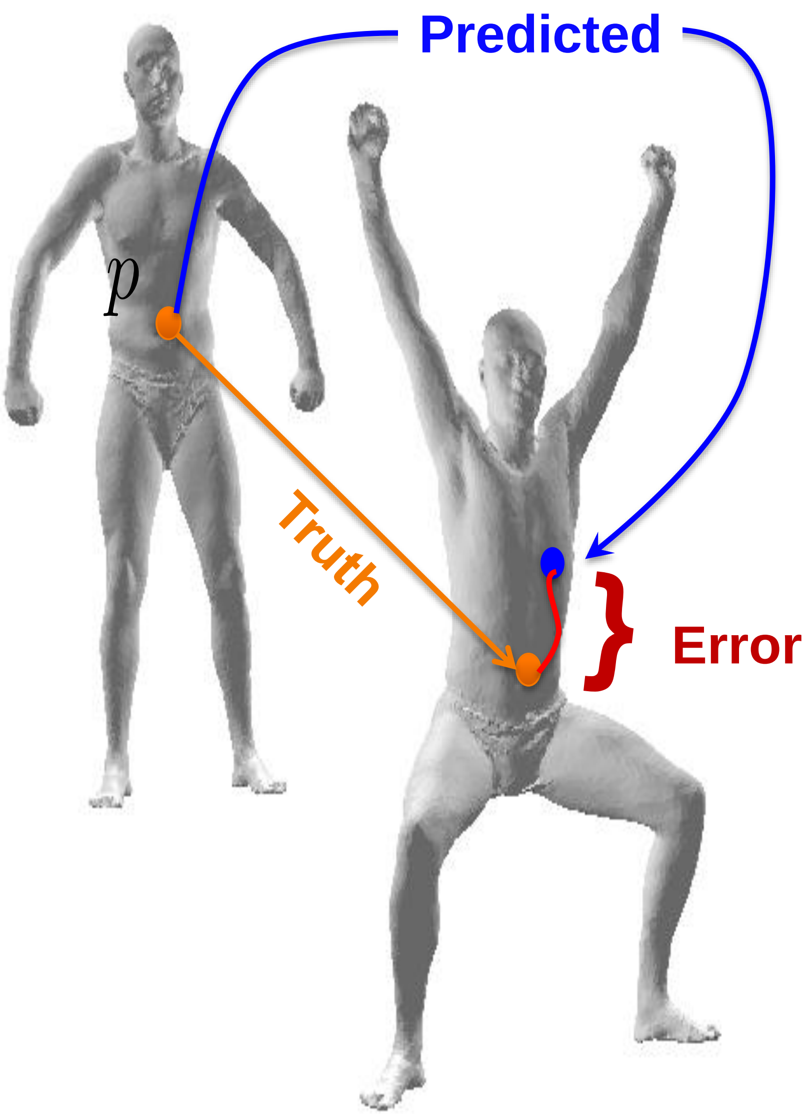


Experimental Results

Evaluation:

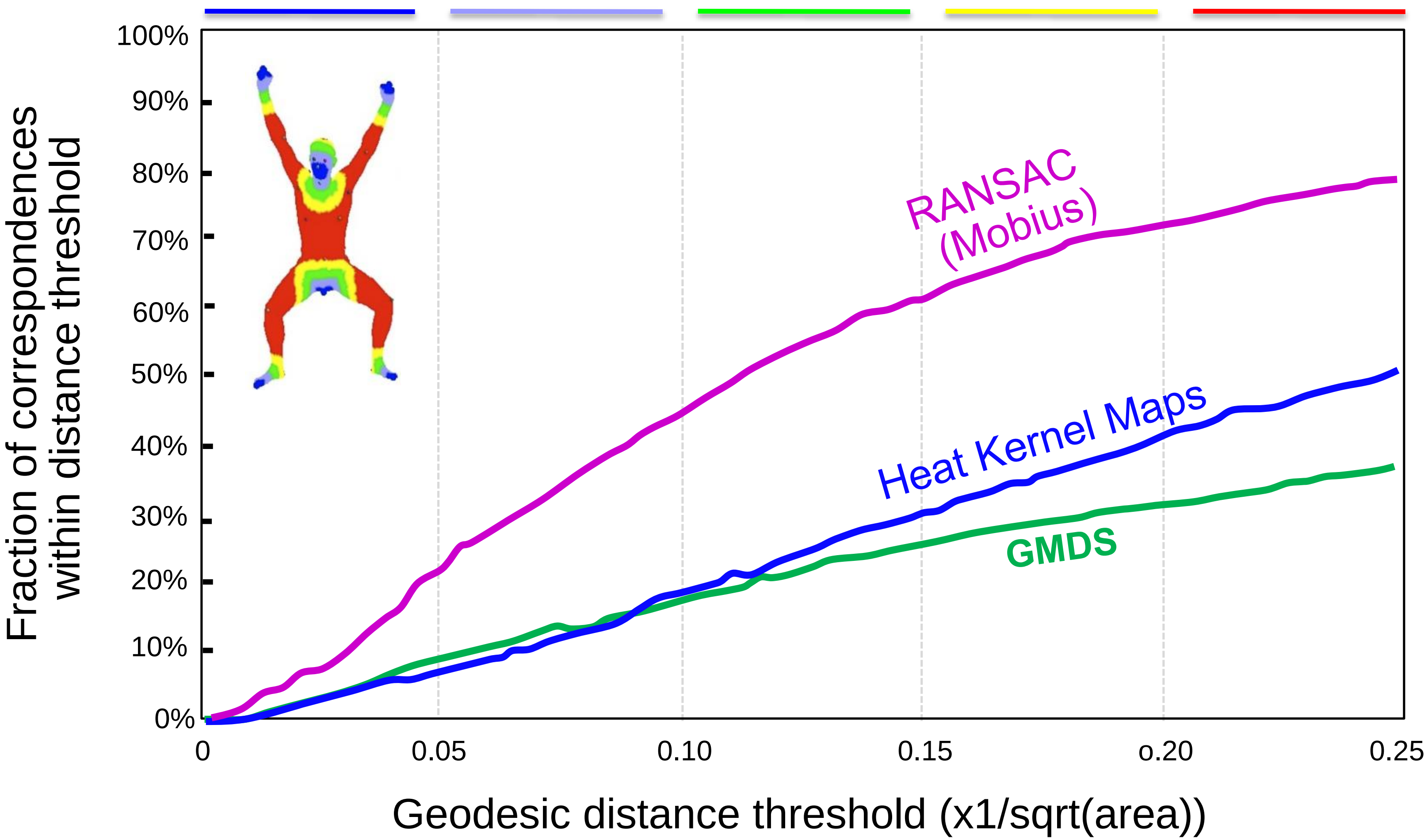
1. For every point with a ground truth correspondence, measure geodesic distance between predicted correspondence and ground truth correspondence

2. Plot fraction of points within geodesic error threshold



Experimental Results

Results:



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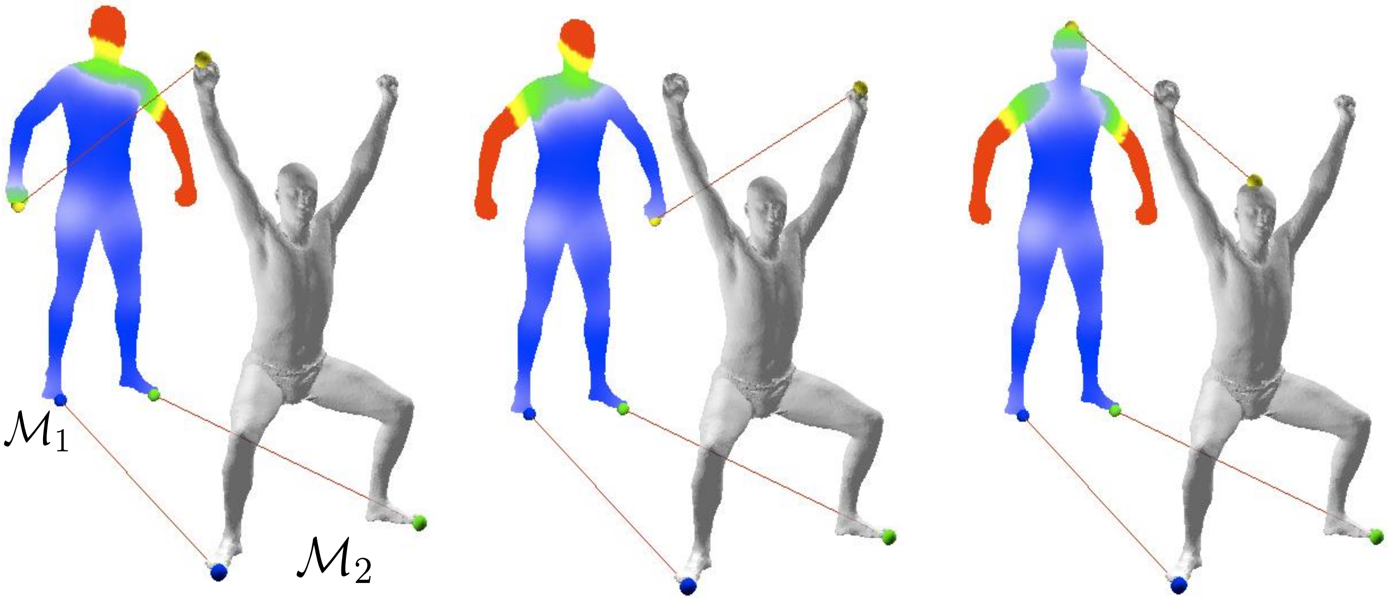
Blended Maps

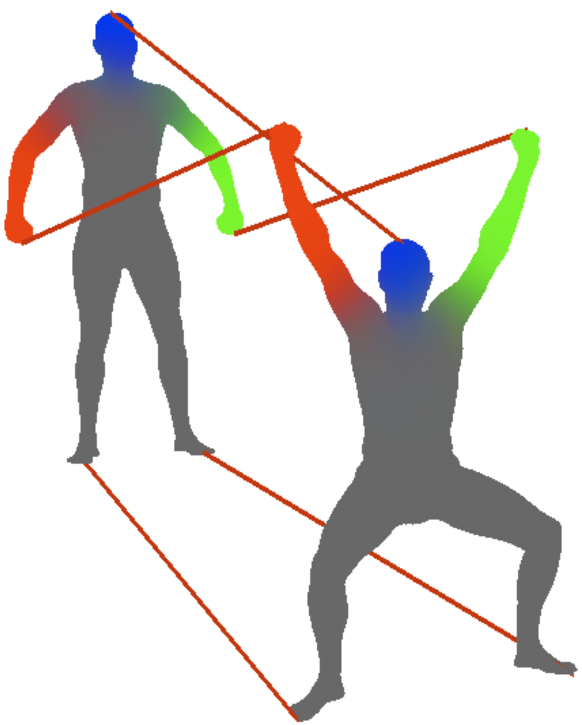
For significantly different surfaces, no single conformal map provides low distortion everywhere



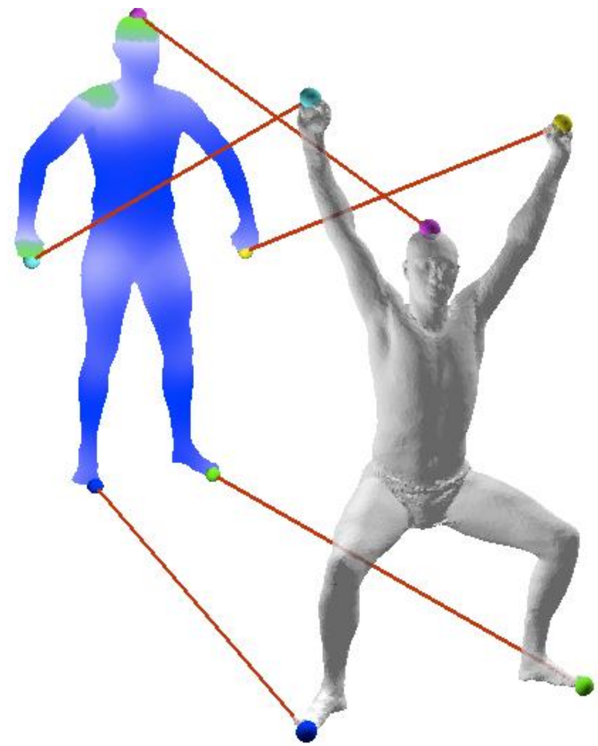
Distortion of m_1 Distortion of m_2 Distortion of m_3

Blended Maps

Idea: blend conformal maps with smooth weights

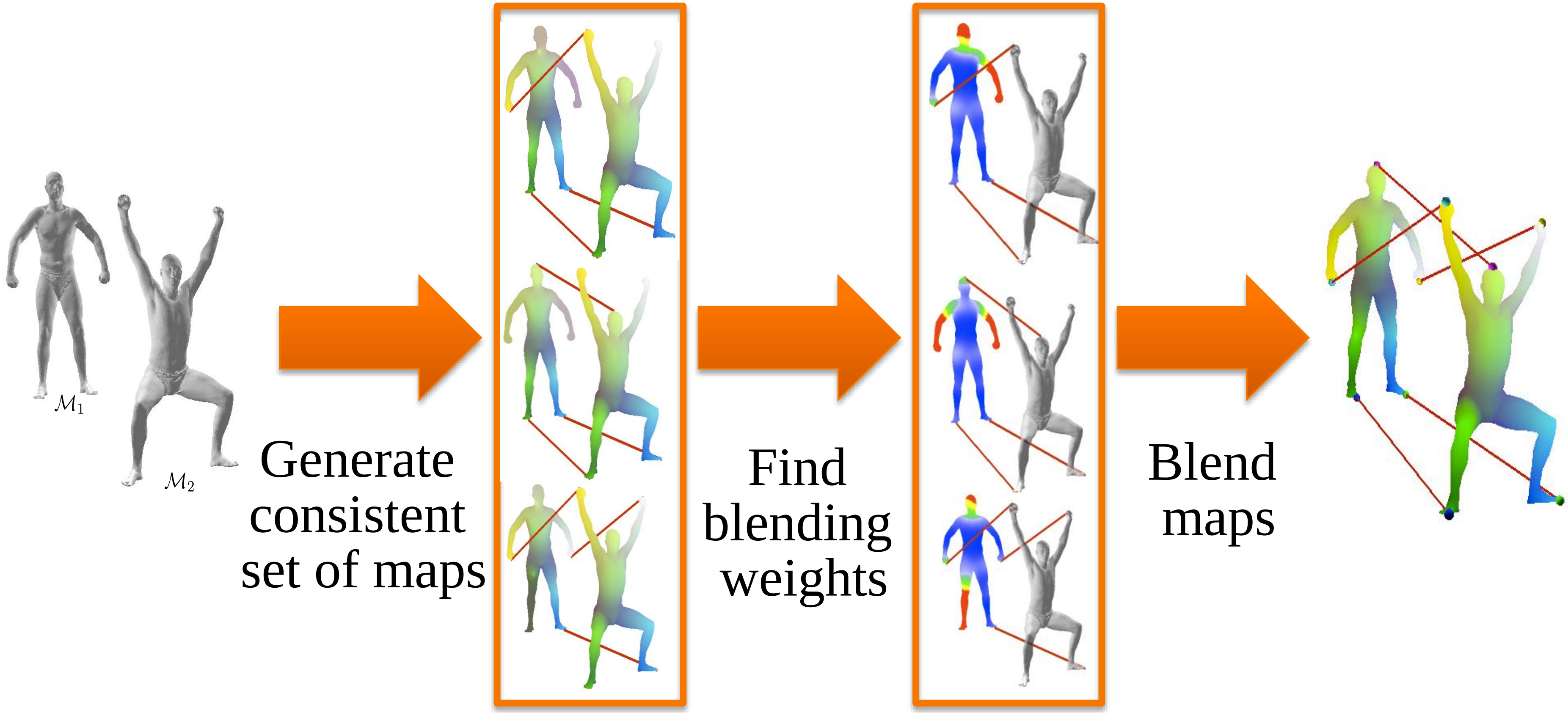


Blending Weights
for m_1, m_2, and m_3



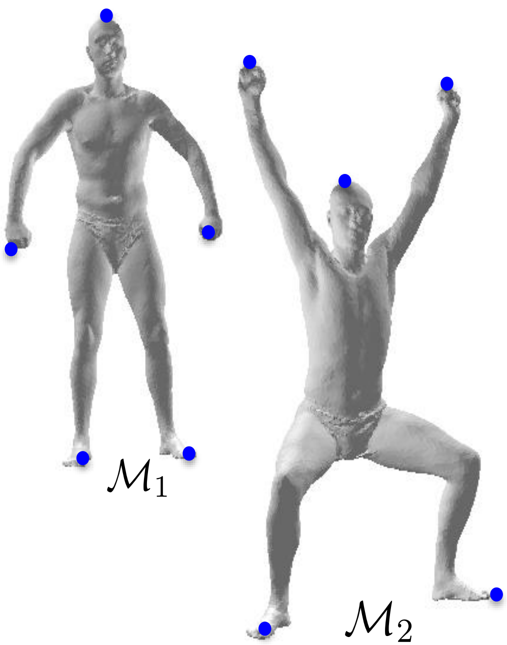
Distortion of the
Blended Map

Computing Blended Maps

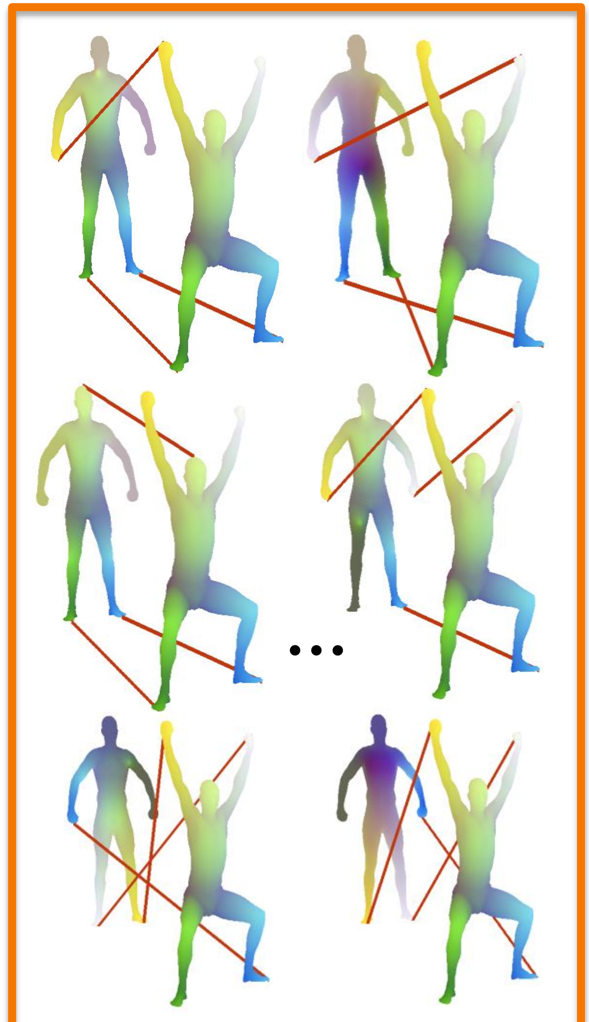


Computing Blended Maps

1. Generate candidate maps by enumerating triplets of feature correspondences

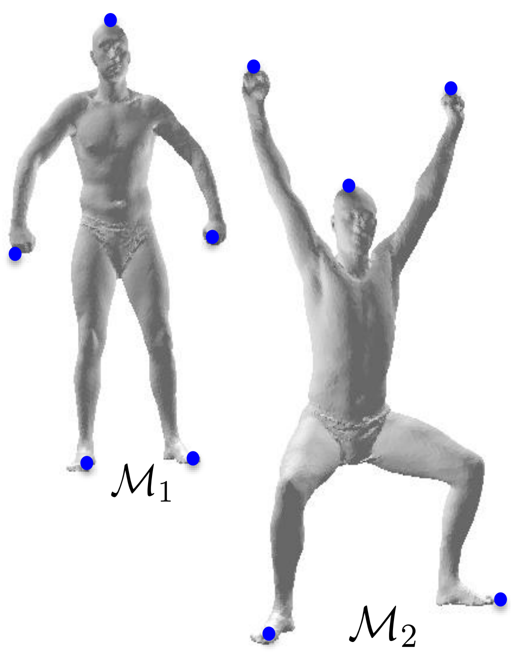


Set of candidate maps

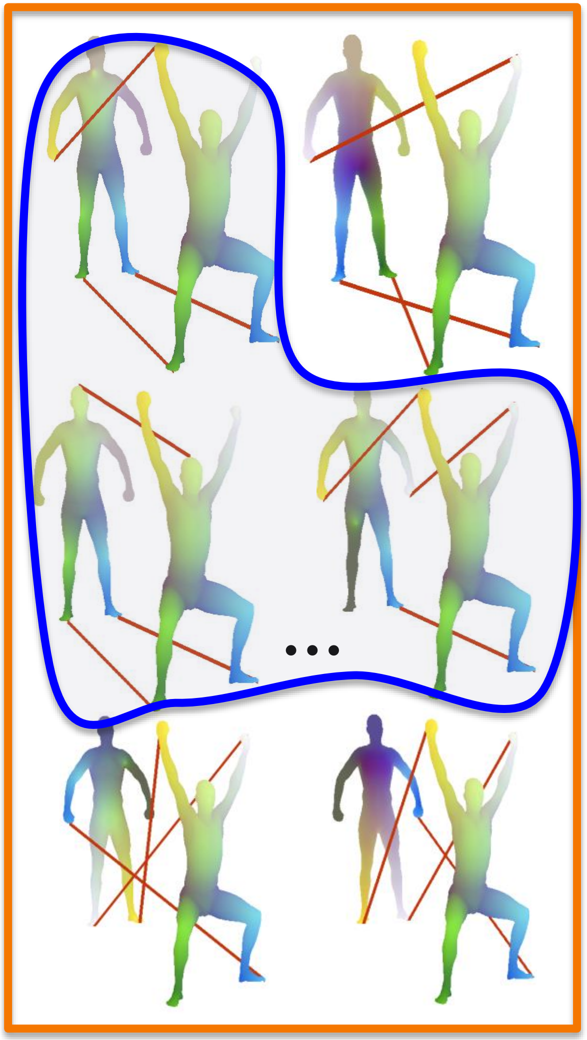


Computing Blended Maps

2. Select consistent set of
 low-distortion candidate maps



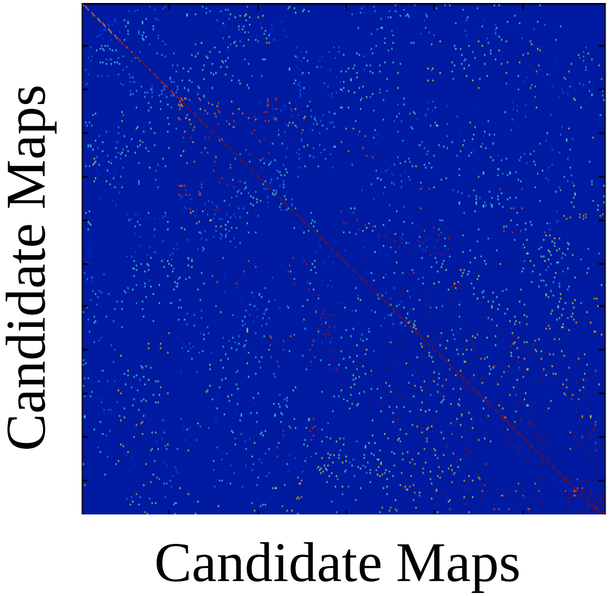
Set of
candidate
maps

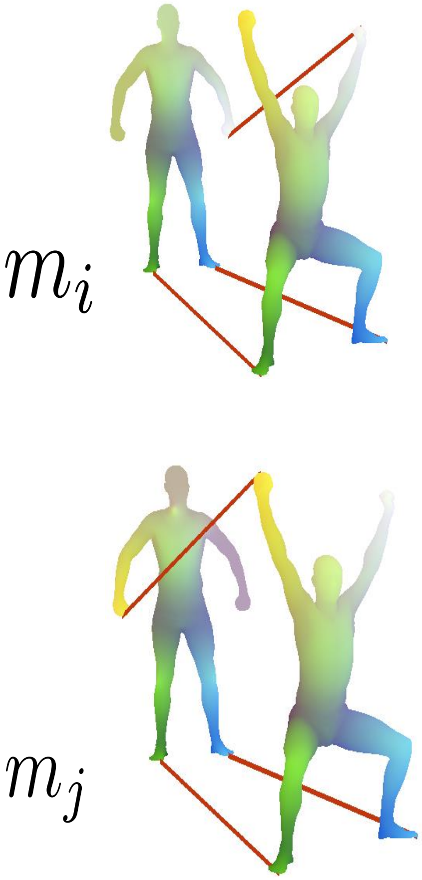


Computing Blended Maps

2a. Define a matrix **B** where every entry (i,j) indicates the distortion of m_i and m_j and their pairwise similarity $S_{i,j}$

$$\mathbf{B}_{i,j} = \int_{M_1} c_i(p) c_j(p) S_{i,j}(p) dA(p)$$



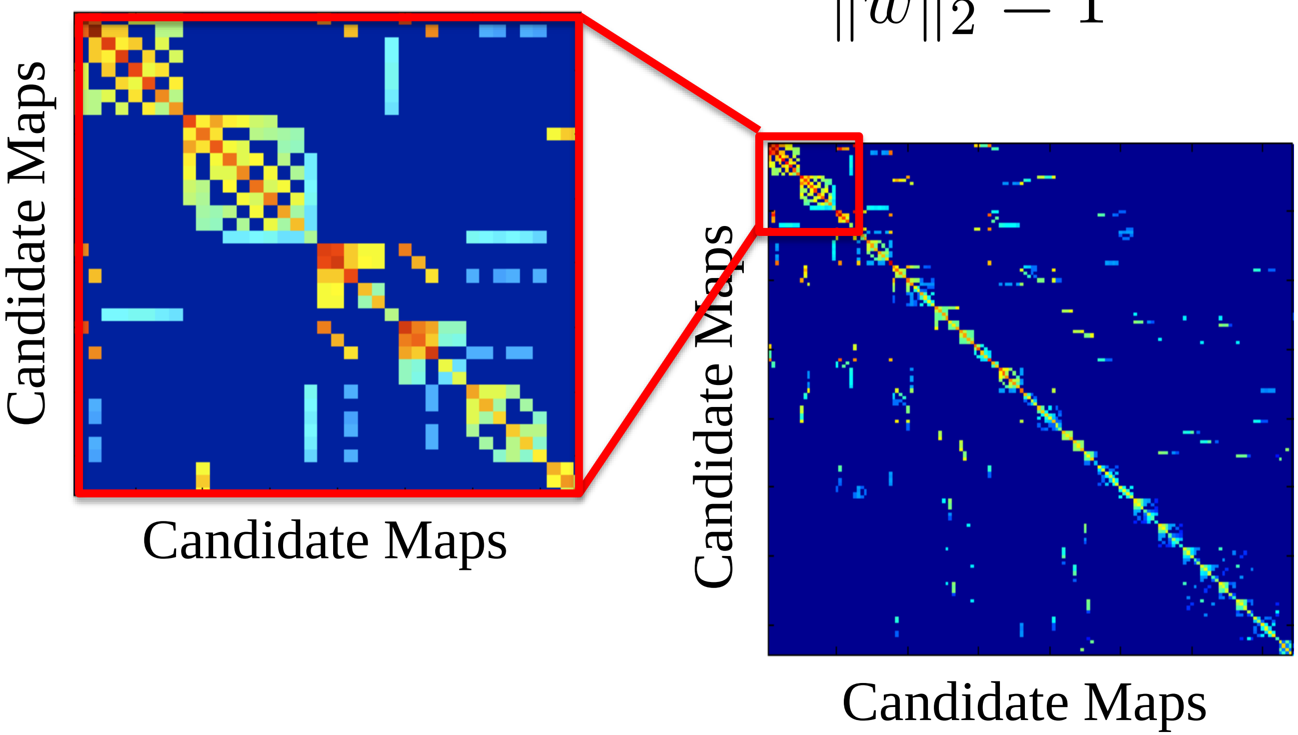


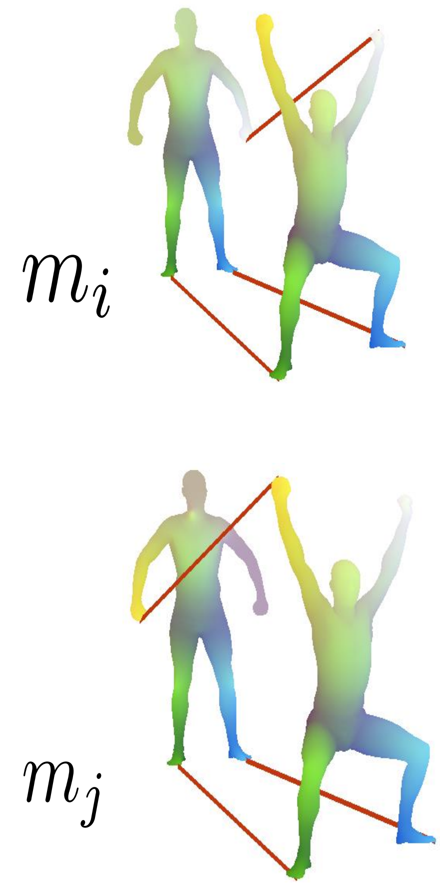
Computing Blended Maps

2b. Find block of consistent, low-distortion maps using top eigenvector(s) of $\mathbf{B}$

$$E_{\mathcal{M}}(\vec{w}) = \vec{w}^T \mathbf{B} \vec{w}$$

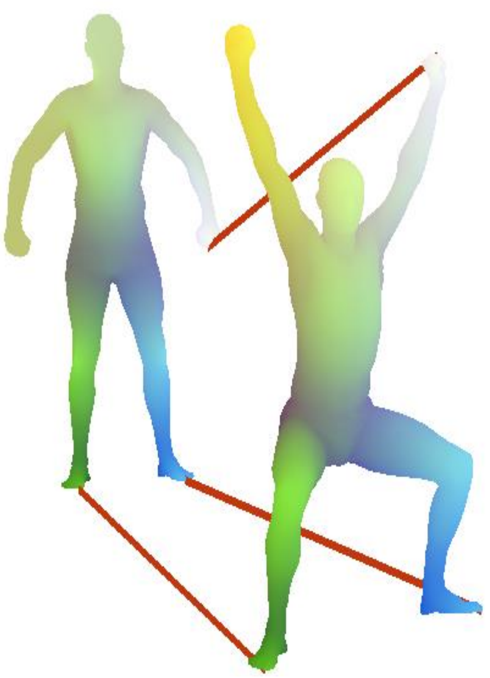
$$\|\vec{w}\|_2 = 1$$



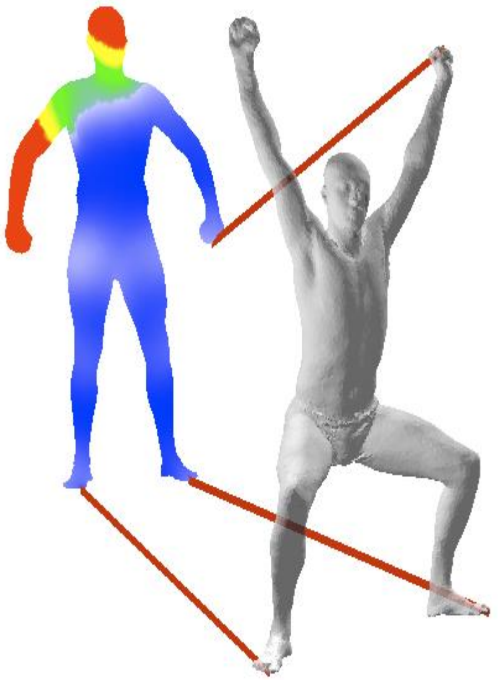


Computing Blended Maps

3. Compute blending weight $c_i(p)$ for every map m_i at every point p based on distortion of m_i at p



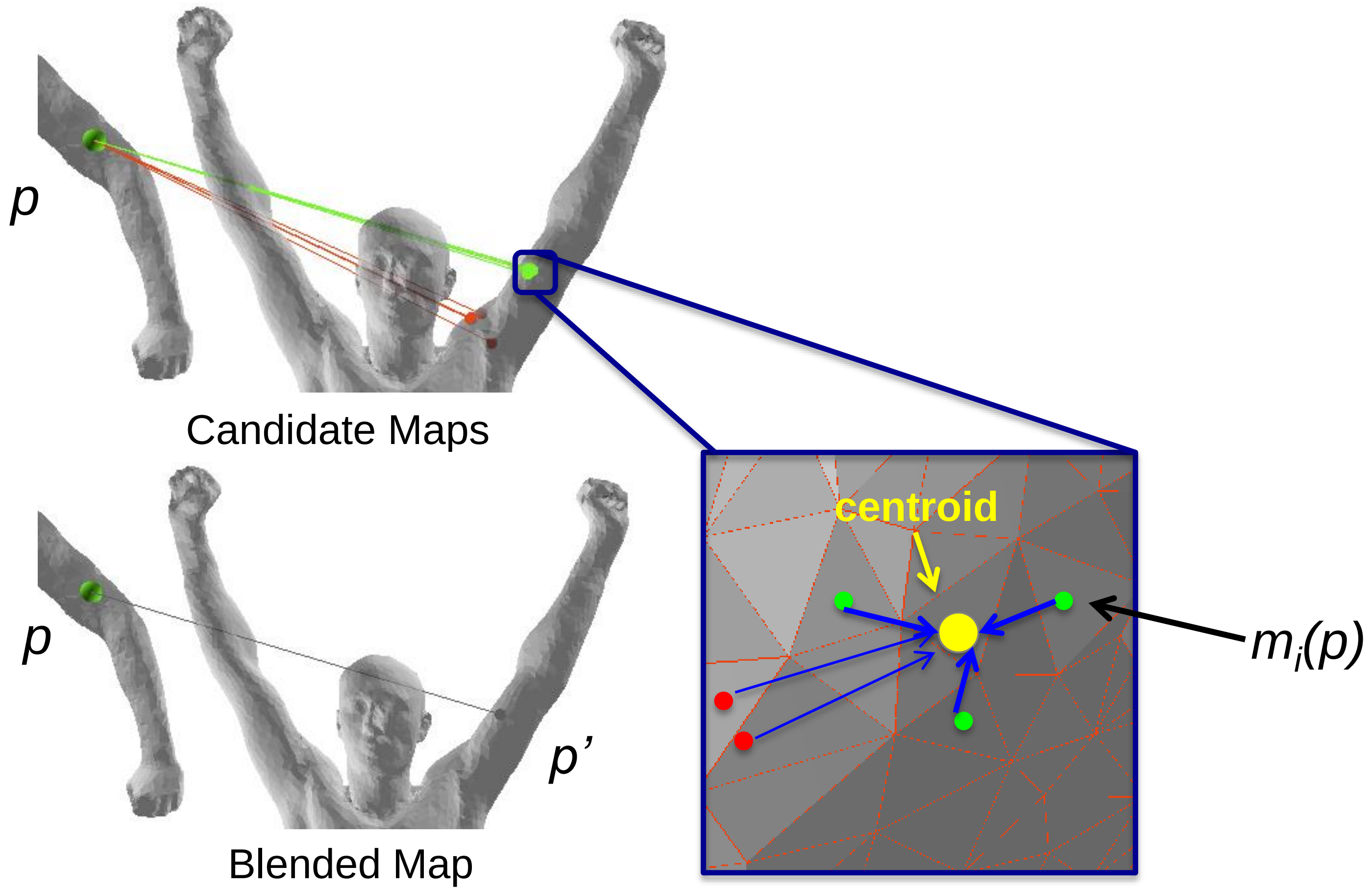
Candidate Map



Blending Weight
$c_i(p)$

Computing Blended Maps

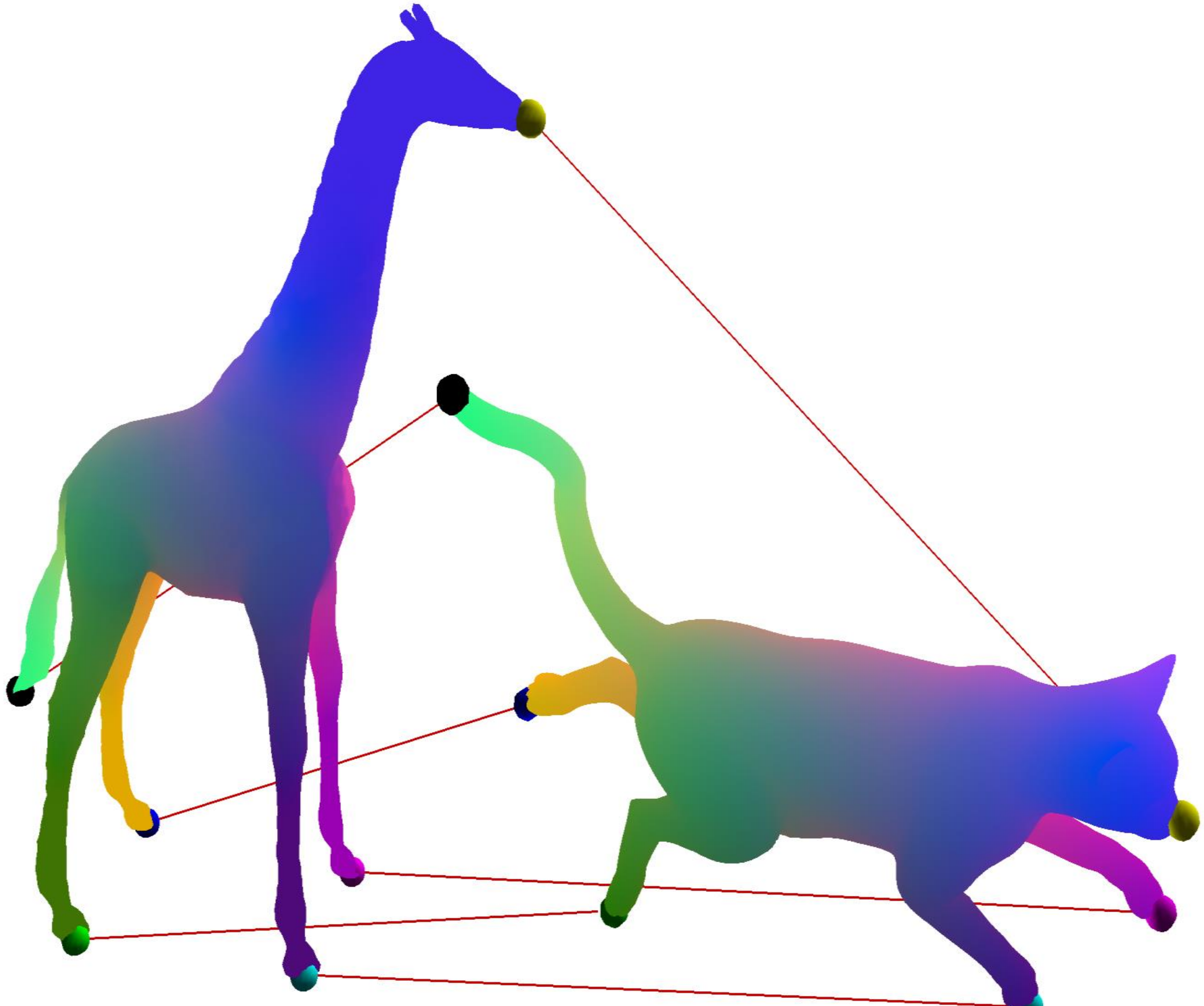
4. Define image *p'* of every point *p* as the weighted geodesic centroid of $m_i(p)$



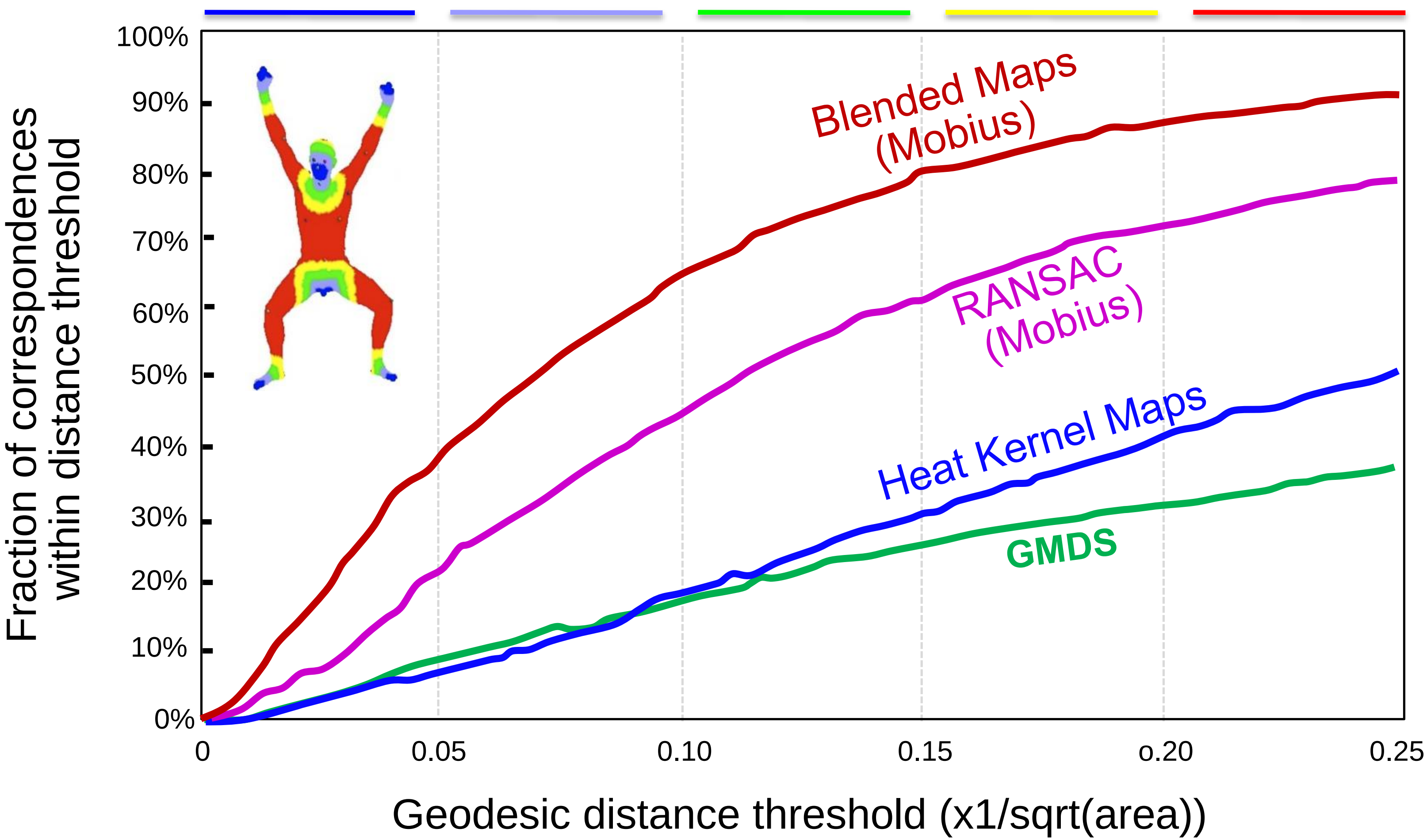
Candidate Maps

Blended Map

Computing Blended Maps



Experimental Results



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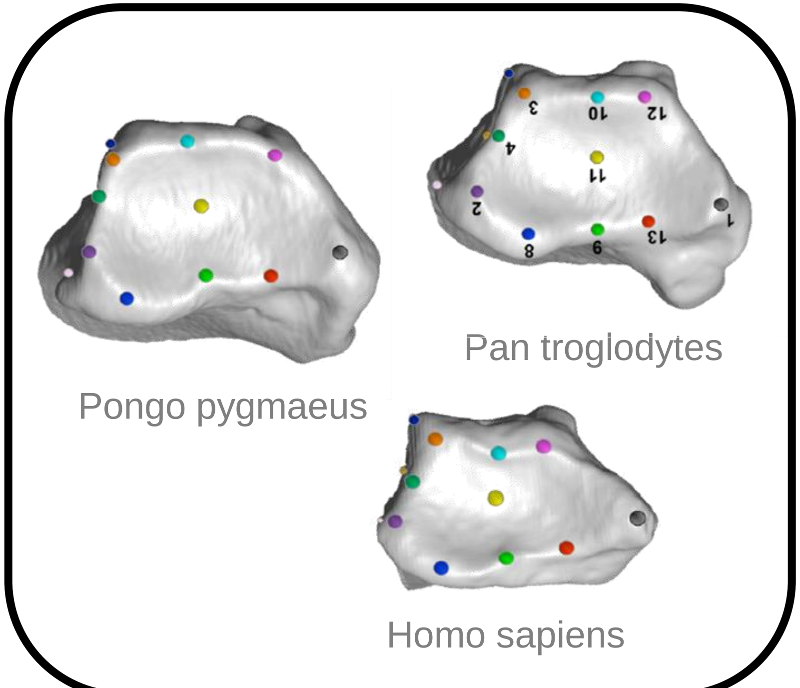
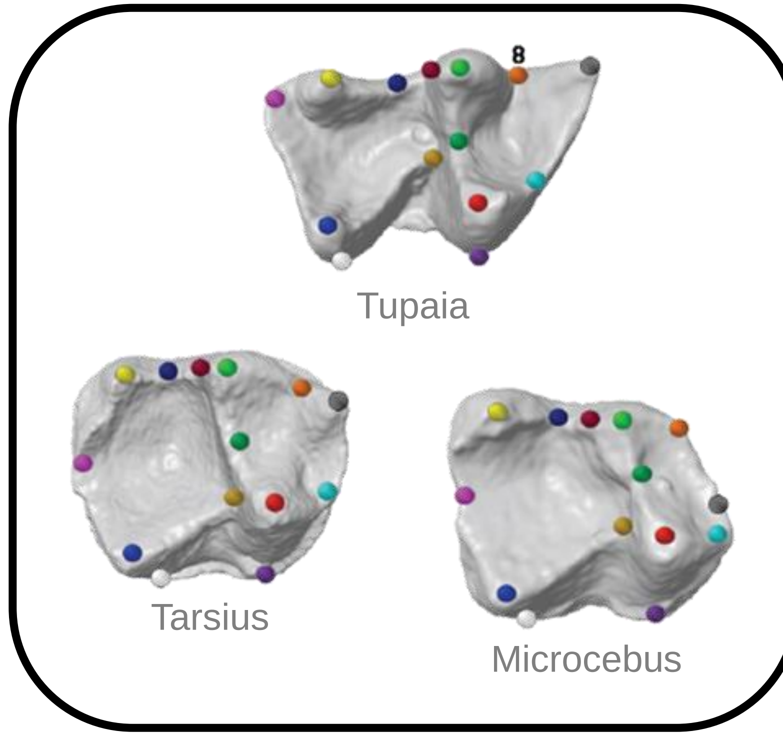
Application

Automatically quantify the geometric similarity of anatomical surfaces



Distal Radius



Mandibular Molar

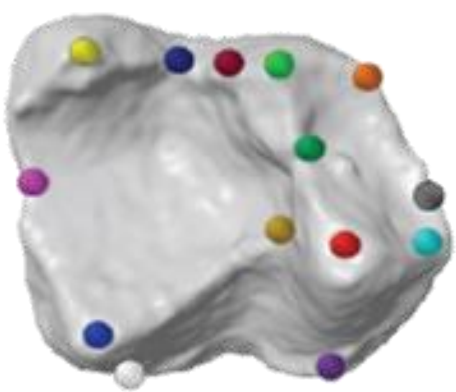
[Boyer, Lipman, St. Clair, Puente, Patel, Funkhouser, Jernvall, and Daubechies, 2011]

Application

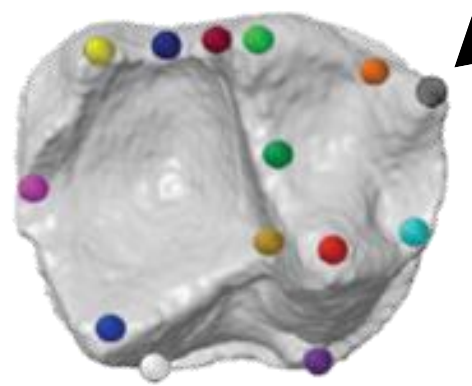
Traditional Procrustes distance:

$$d(X,Y) = min_R \left[\left(\sum_{i=1}^{N} \|R(X_i) - Y_i\|^2 \right)^{1/2} \right]$$

Human Specified Landmarks



$X = \{ X_i \}$

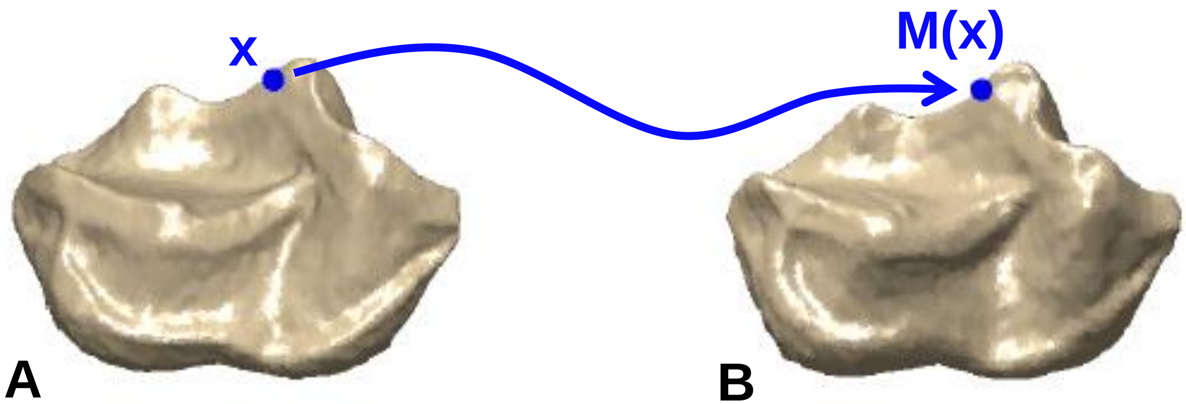


$Y = \{ Y_i \}$

Application

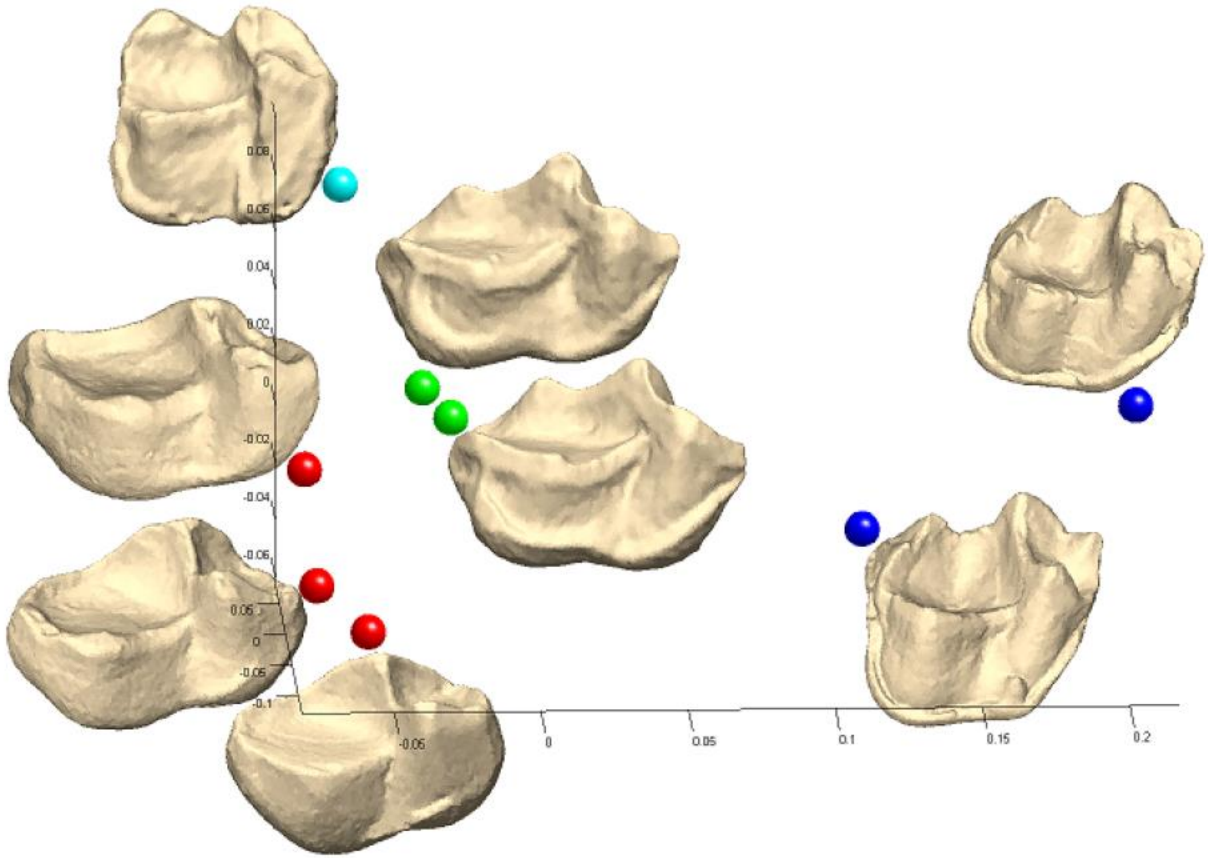
New continuous Procrustes distance:

$$d(A,B) = min_{R,M}\left[\left(\int_A \|R(x) - M(x)\|^2\, dx\right)^{1/2}\right]$$



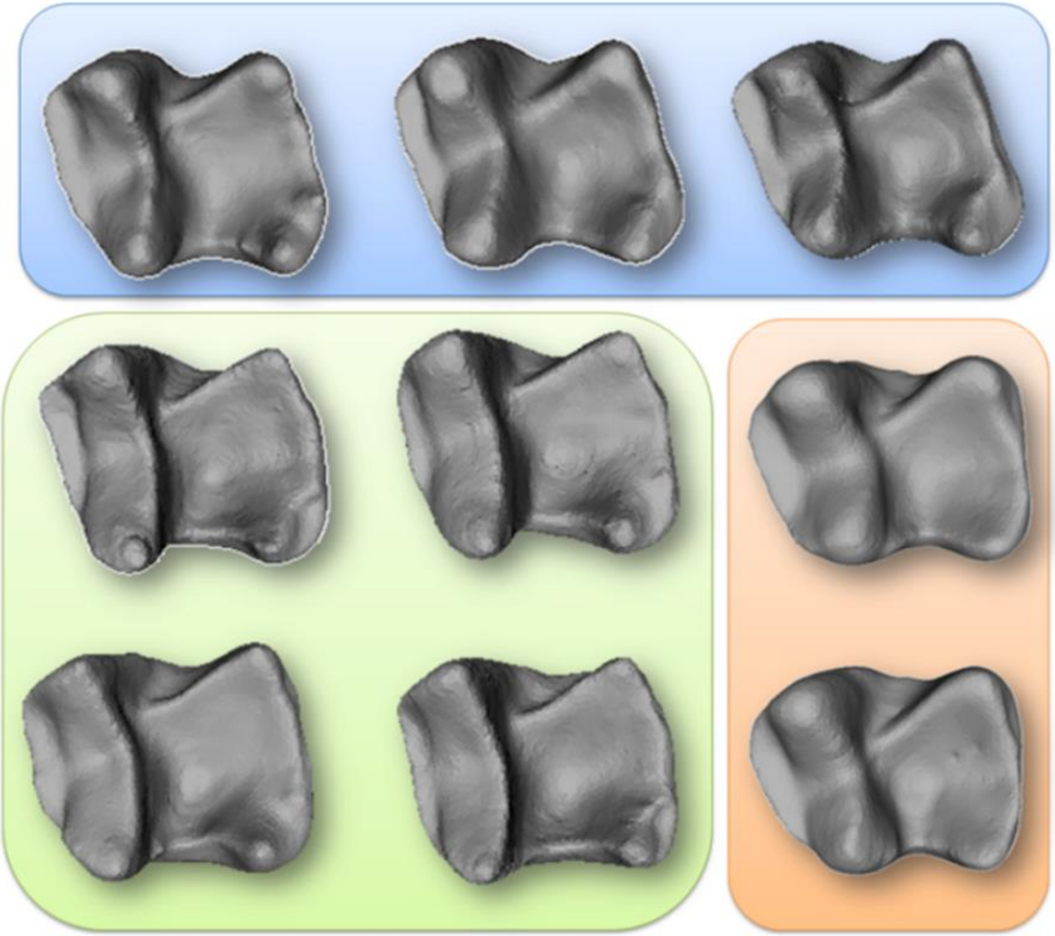
Application

Embedding based on new distance



Application

Clustering based on new distance



Species Groups of Galaga Genus

Application

Classification based on nearest-neighbors

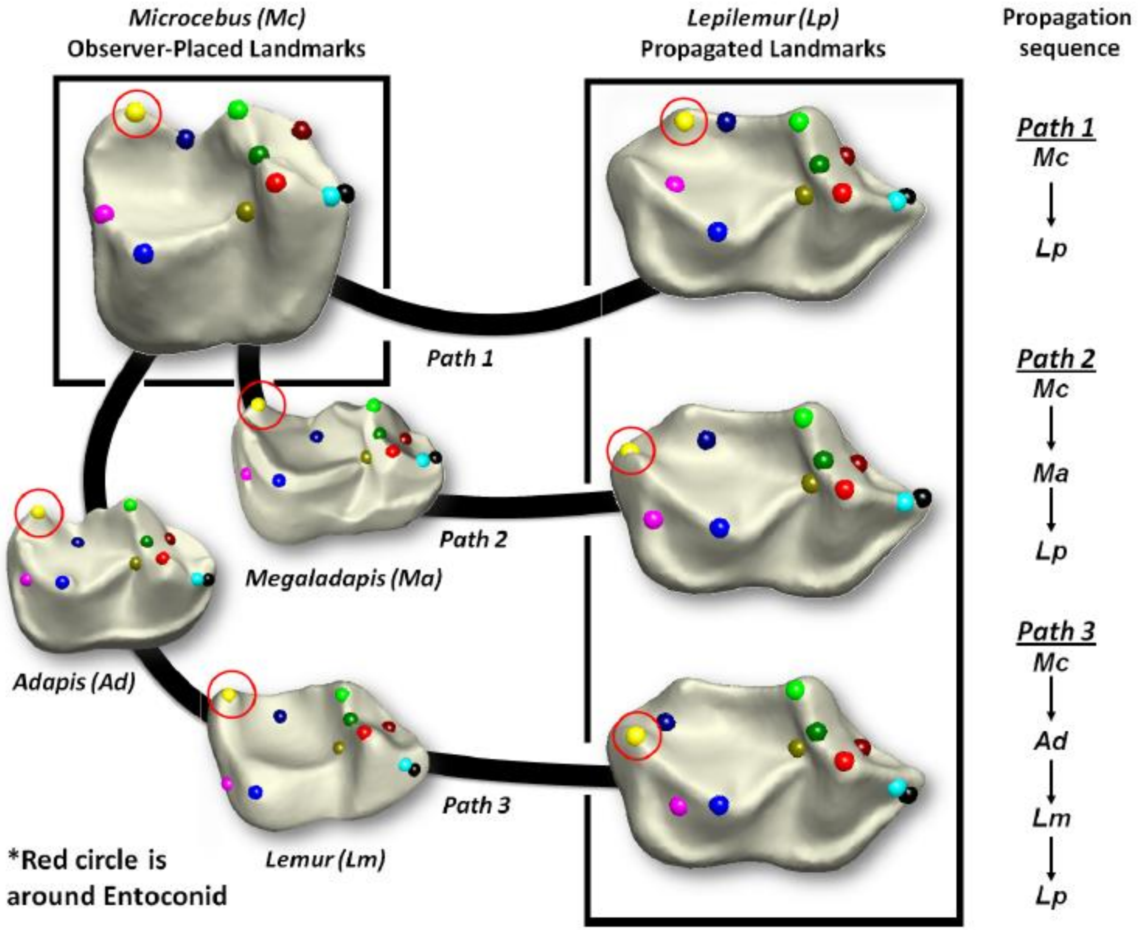
Mandibular Molar	# Groups	# Objects	New Distance	Human Landmarks
Genus	24	99	90.9%	91.9%
Family	17	106	92.5%	94.3%
Order	5	116	94.8%	95.7%

First Metatarsal	# Groups	# Objects	New Distance	Human1 Landmarks	Human2 Landmarks
Genus	13	59	79.9%	76.3%	88.1%
Family	9	61	91.8%	83.6%	93.4%
Superfamily	2	61	100%	100%	100%

Distal Radius	# Groups	# Objects	New Distance	Human Landmarks
Genus	4	45	84.4%	77.7%

Application

Propagating correspondences



Acknowledgments

Test data

- Giorgi et al. (SHREC Watertight), Anguelov et al. (SCAPE), Bronstein et al. (TOSCA)

Test code:

- Ovsjanikov et al. (HKM), Bronstein et al. (GMDS)

Application

- Boyer, St. Clair, Patel, Jernvall, Puente, Daubechies

Funding:

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Thank You!