Some
Mesh Surface Properties

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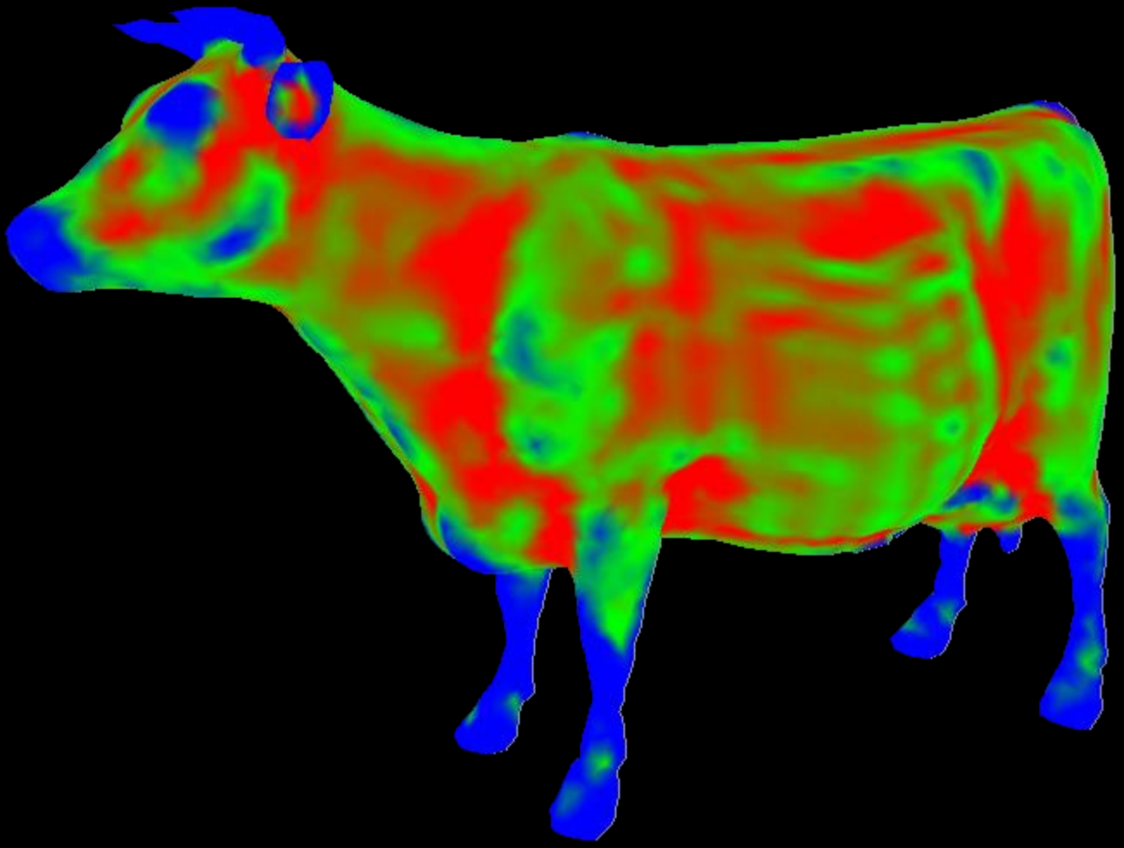
COS 526, Fall 2016

Curvature

Curvature

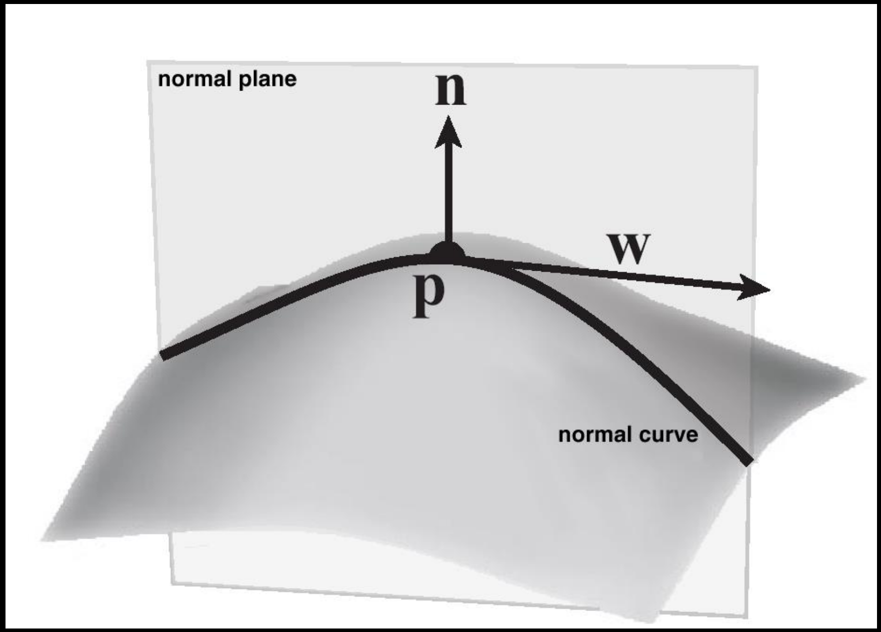


Curvature

Curvature κ of a curve is reciprocal of radius of circle that best approximates it

Defined at a point $\mathbf{p}$ in a direction $\mathbf{w}$

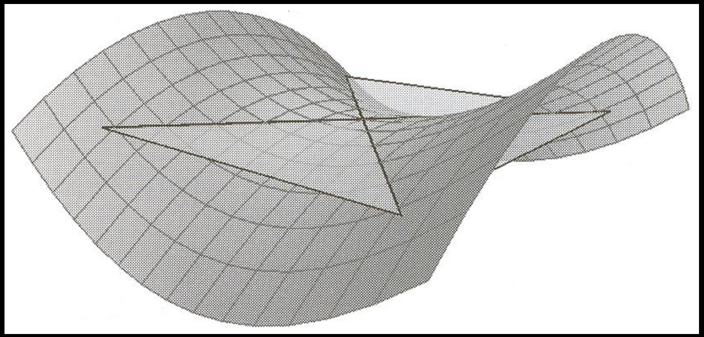
Line has $\kappa = 0$



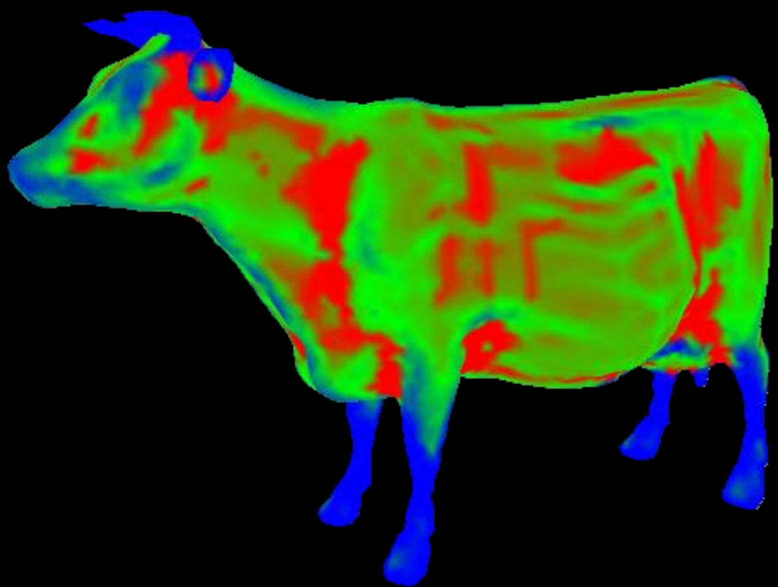
Principal Curvatures

The curvature at a point varies between some minimum and maximum – these are the *principal curvatures* κ_1 and κ_2

They occur in the *principal directions* d_1 and d_2 which are perpendicular to each other

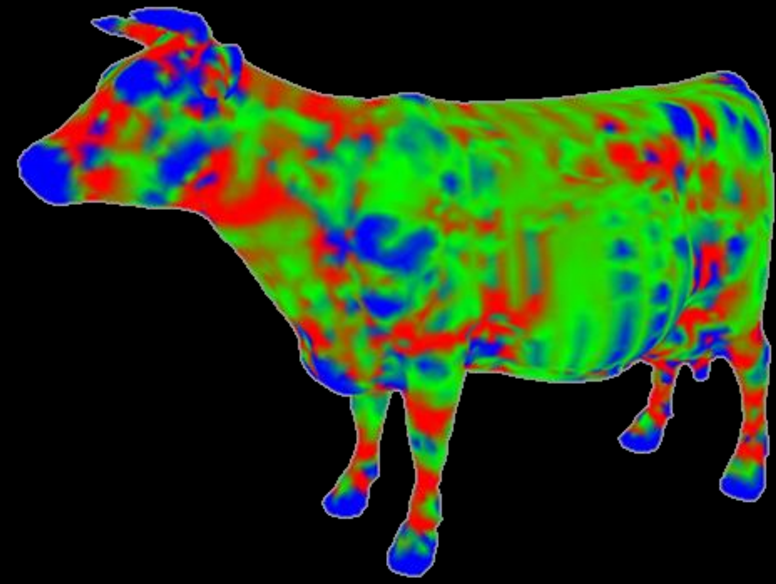


Principal Curvatures



Minimum Curvature

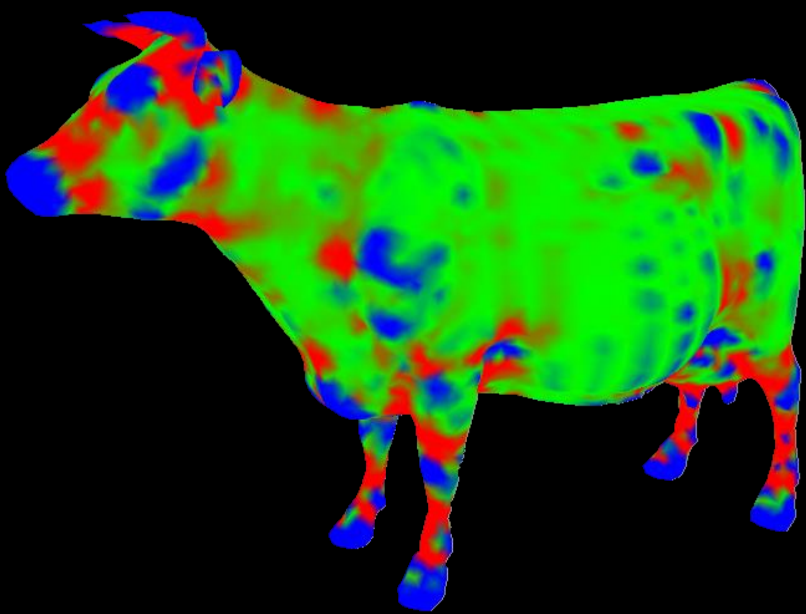
κ_1



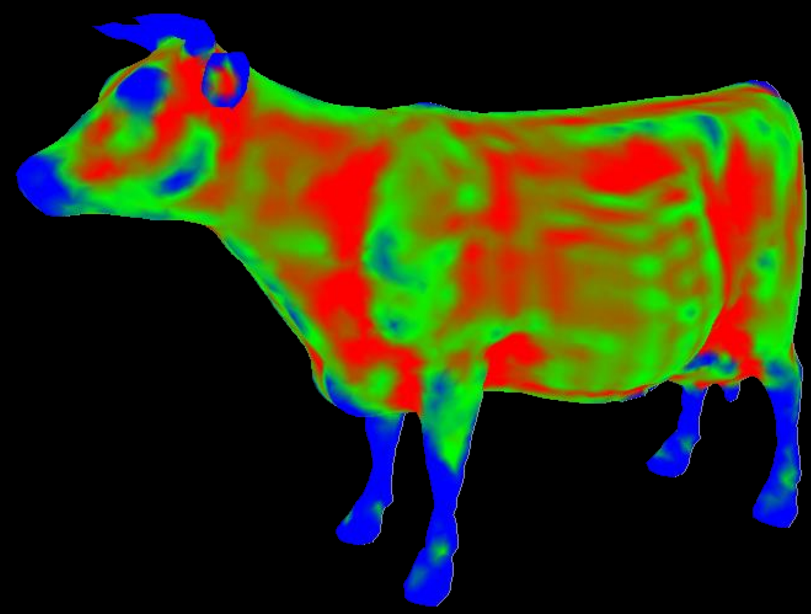
Maximum Curvature

κ_2

Gaussian and Mean Curvature



Gauss Curvature
$K=\kappa_1 \kappa_2$



Mean Curvature
$H=\tfrac{1}{2}(\kappa_1 + \kappa_2)$

What Does Curvature Tell Us?

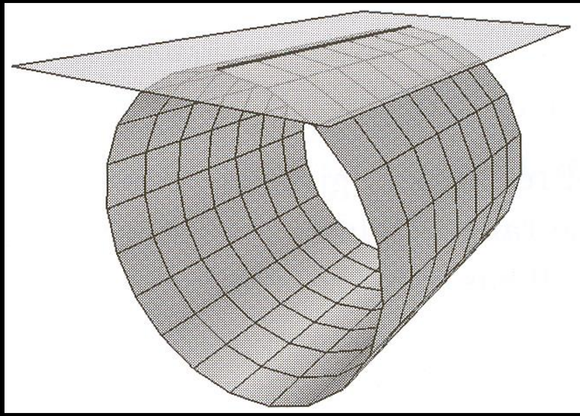
Planar points:

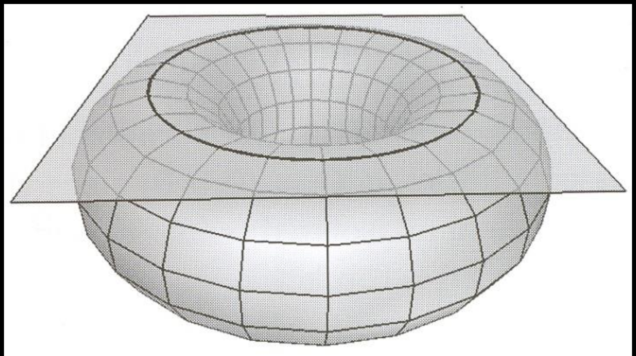
- Zero Gaussian curvature and zero mean curvature
- Tangent plane intersects surface at infinity points

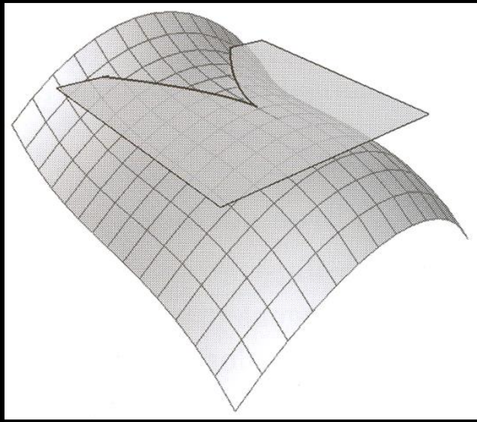
What Does Curvature Tell Us?

Parabolic points:

- Zero Gaussian curvature, non-zero mean curvature
- Tangent plane intersects surface along 1 curves



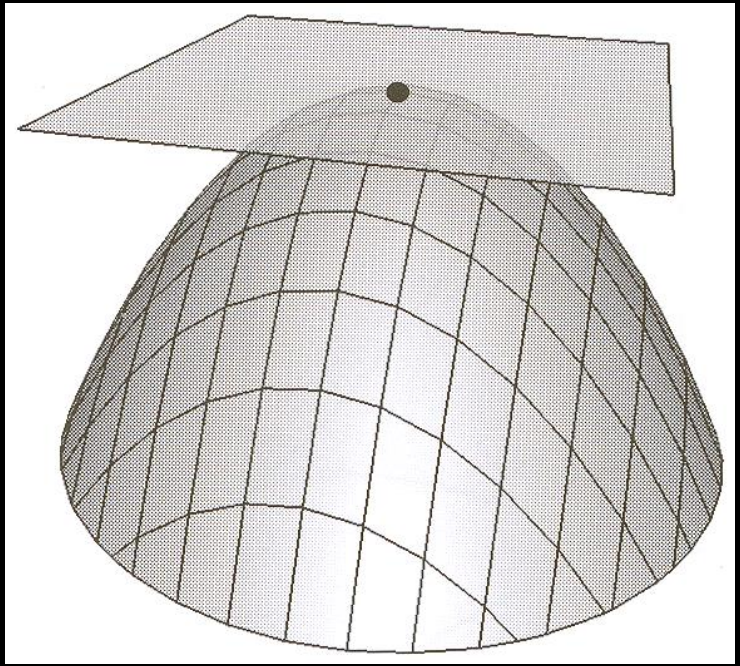




What Does Curvature Tell Us?

Elliptical points:

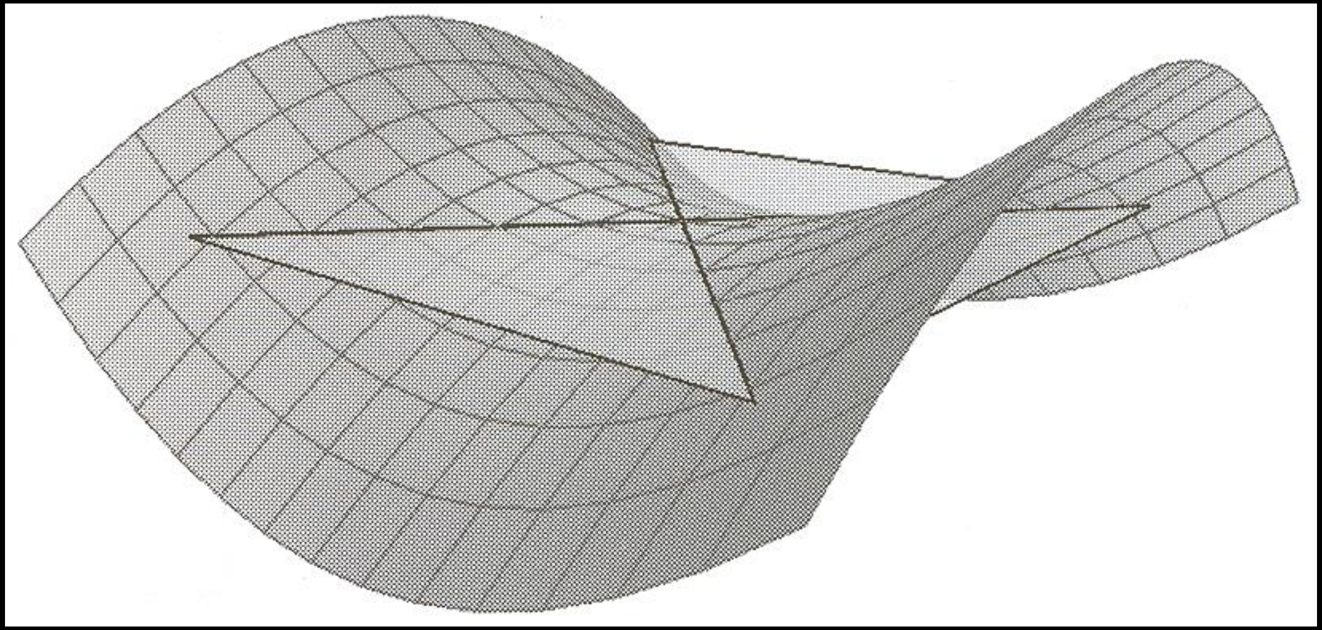
- Positive Gaussian curvature
- Convex/concave depending on sign of mean curvature
- Tangent plane intersects surface at 1 point



What Does Curvature Tell Us?

Hyperbolic points:

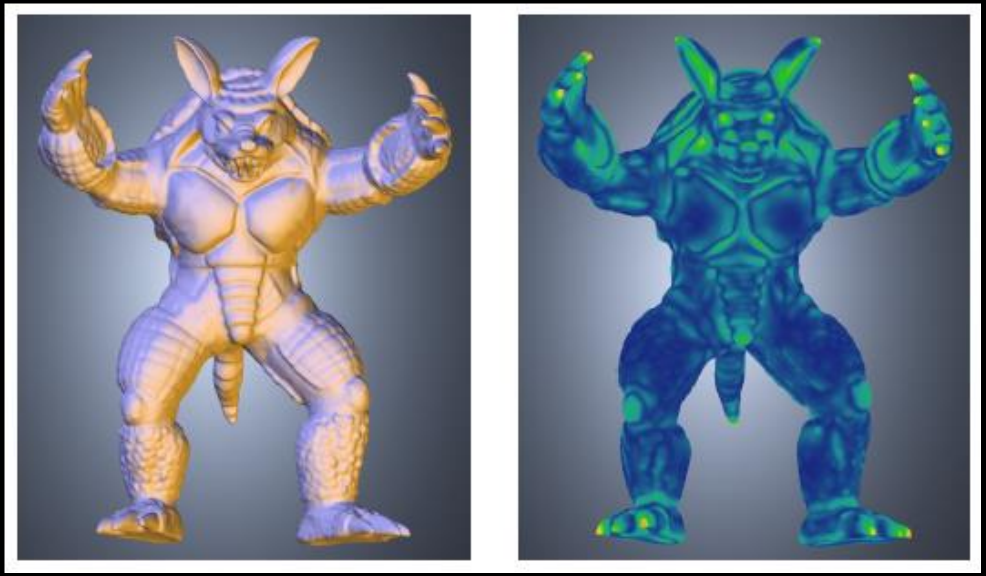
- Negative Gaussian curvature
- Tangent plane intersects surface along 2 curves



What Does Curvature Tell Us?

Mesh Saliency:

- Motivated by models of perceptual salience
- Difference between mean curvature blurred with σ and blurred with 2σ



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Principal Component Analysis (PCA)

Principal Component Analysis (PCA)

Tensor voting

- Extract points $\{q_i\}$ in neighborhood
- Compute covariance matrix M
- Analyze eigenvalues and eigenvectors of M (via SVD)
- Eigenvectors are Principal Axes

$$\mathbf{M} = \frac{1}{n}\sum_{i=1}^{n}\begin{bmatrix} q_i^x q_i^x & q_i^x q_i^y & q_i^x q_i^z \\ q_i^y q_i^x & q_i^y q_i^y & q_i^y q_i^z \\ q_i^z q_i^x & q_i^z q_i^y & q_i^z q_i^z \end{bmatrix}$$

Covariance Matrix

$$\mathbf{M} = \mathbf{USU}^t$$

$$\mathbf{S} = \begin{bmatrix} \lambda_a & 0 & 0 \\ 0 & \lambda_b & 0 \\ 0 & 0 & \lambda_c \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix}$$

Eigenvalues & Eigenvectors

Principal Component Analysis (PCA)

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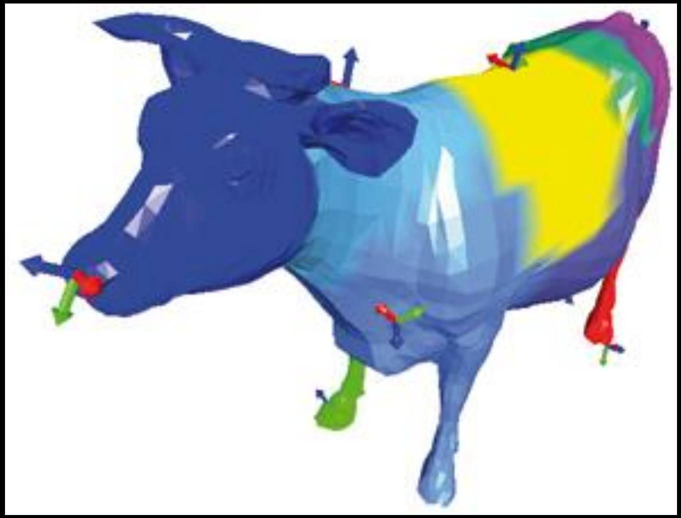
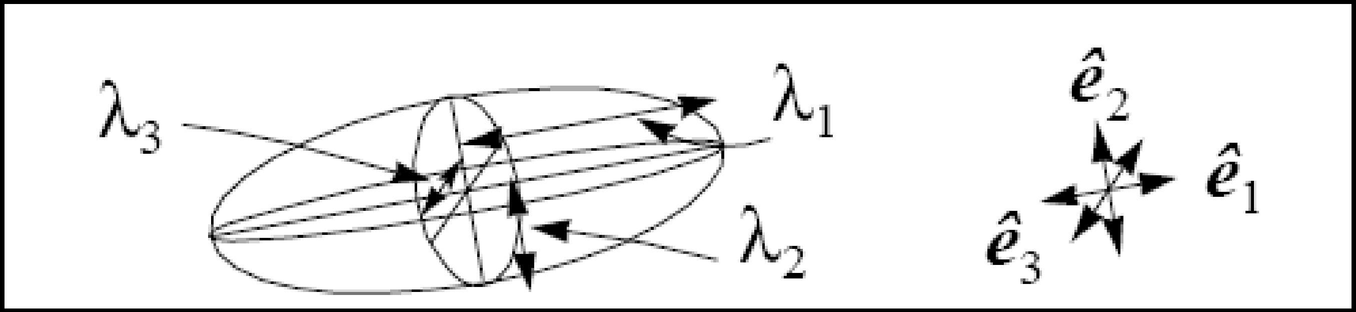
Eigenvalues & Eigenvectors

Principal Component Analysis (PCA)

Eigenvectors are
"Principal Axes of Inertia"

Eigenvalues are variances of the point distribution in those directions

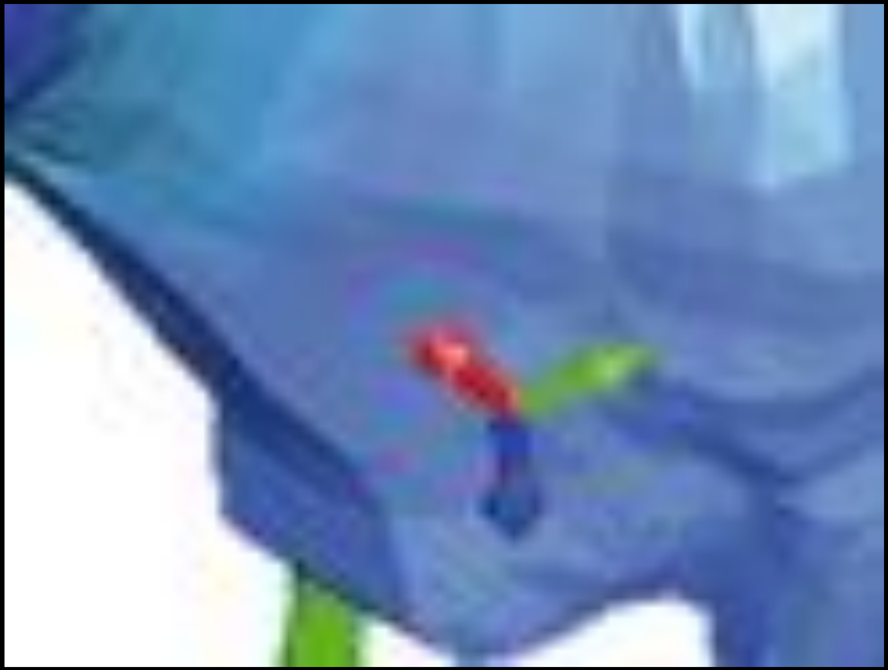




What Does PCA Tell Us?

Provides estimate of normal direction

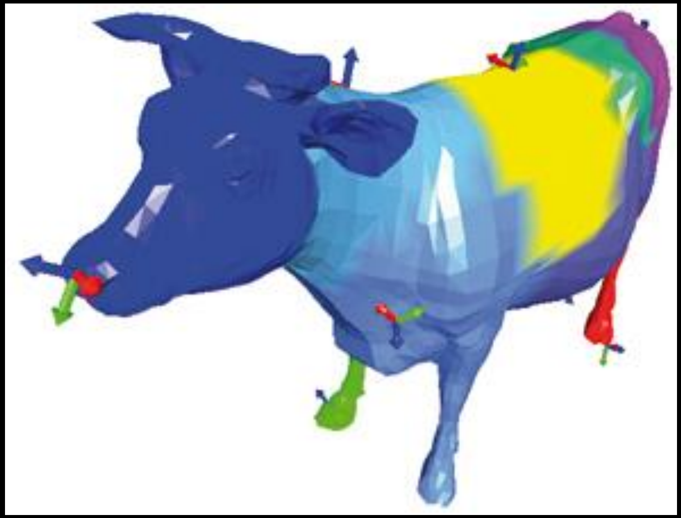
- Eigenvector (principal axis) associated with smallest eigenvalue

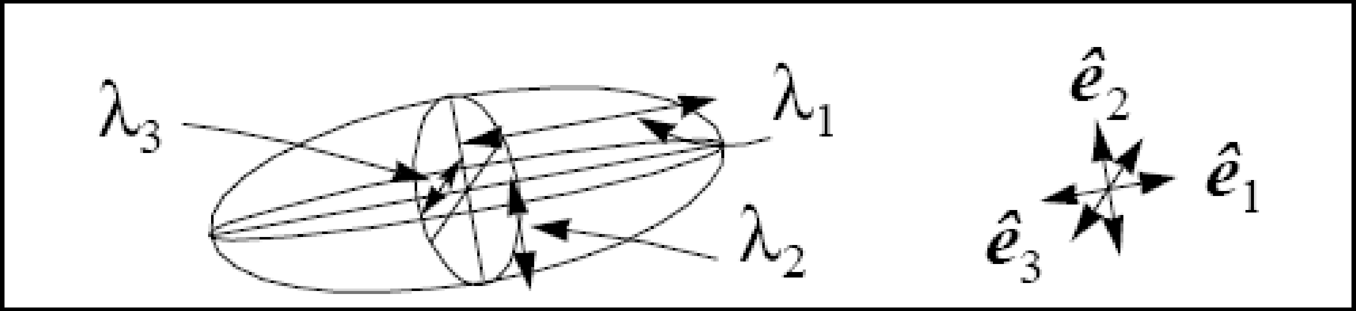


What Does PCA Tell Us?

Helps us construct a local coordinate frame for every point

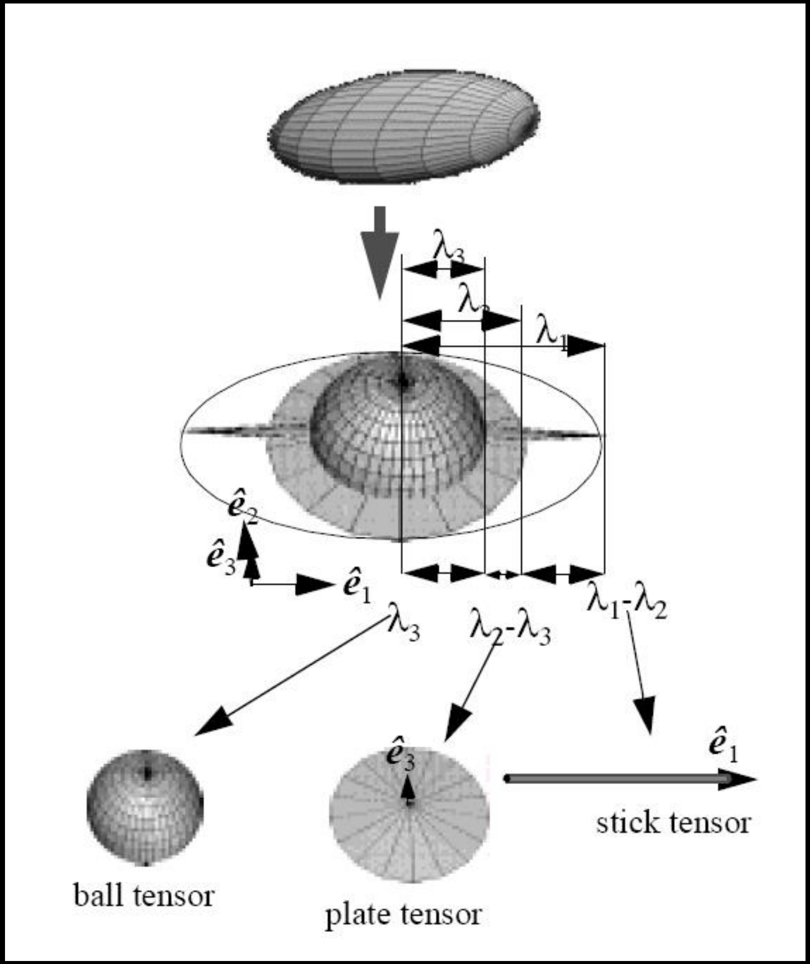
- Map $\hat{e}_1$ to X axis
- Map $\hat{e}_2$ to Y axis
- Map $\hat{e}_3$ to Z axis





What Does PCA Tell Us?

Helps differentiate
  nearly plane-like,
  from stick-like,
  from sphere-like,
  etc.



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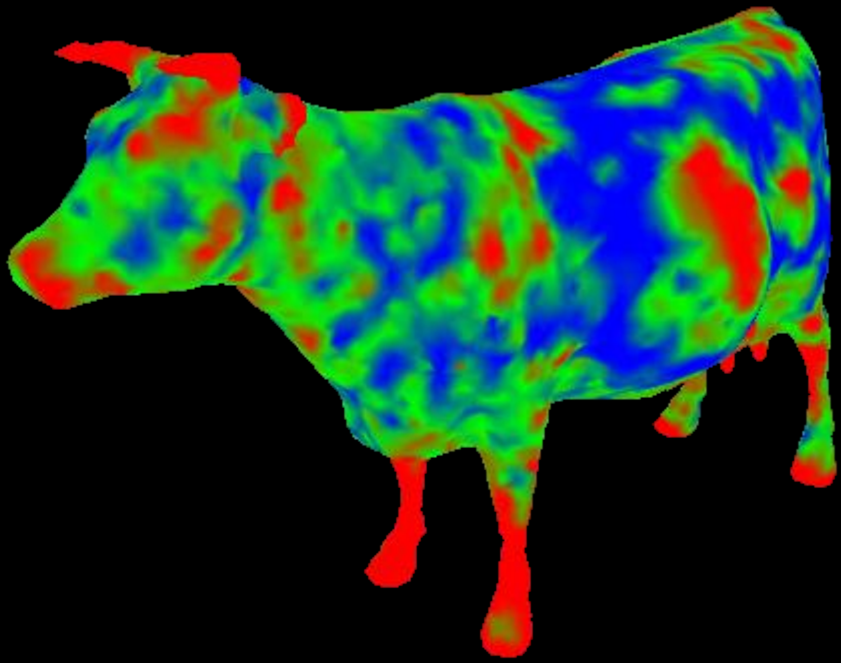


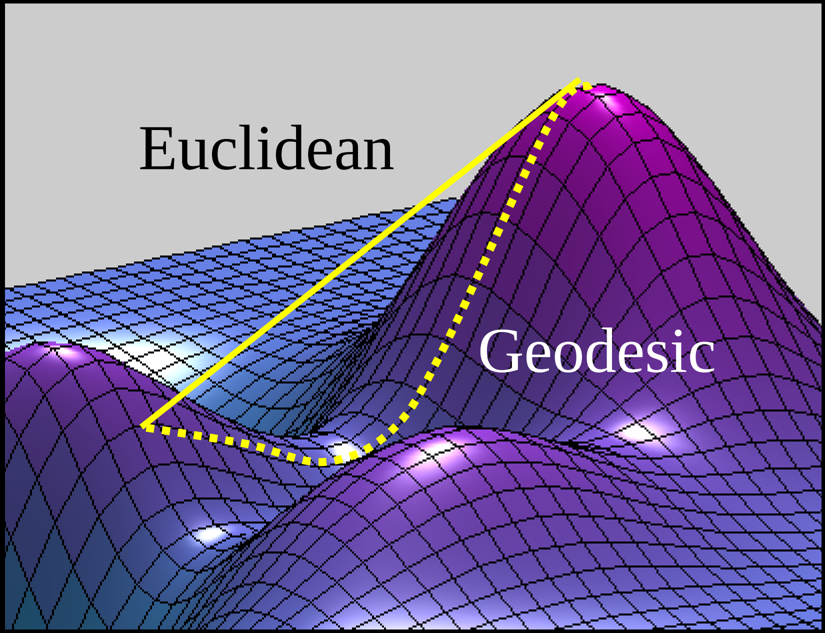
$\lambda_2 / (\lambda_1+\lambda_2+\lambda_3)$

Statistics of Distances

Statistics of Distances

Distances can be along surface (geodesic) or as a crow flies (Euclidean)

Geodesic distance to point

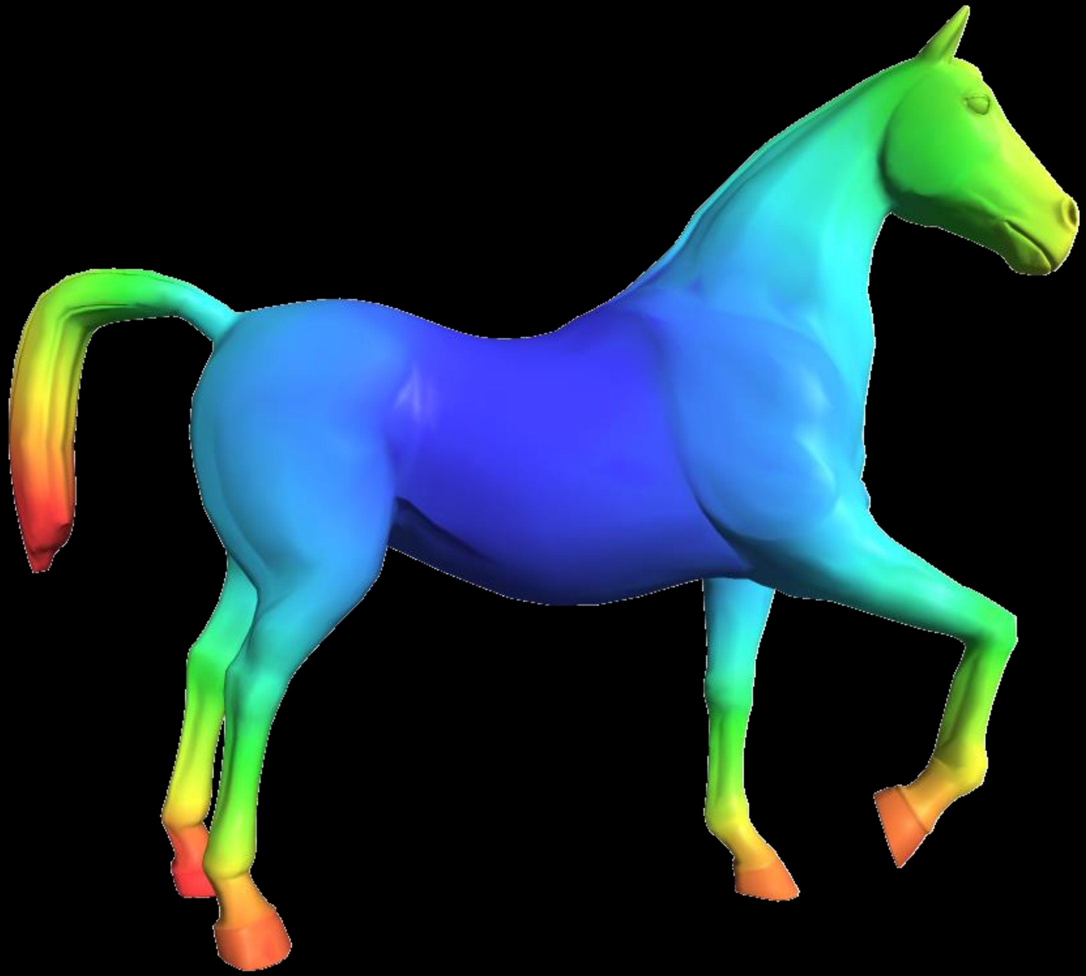


Geodesic vs. Euclidean distance

Shamir

Statistics of Distances

Average geodesic distance to other points on surface



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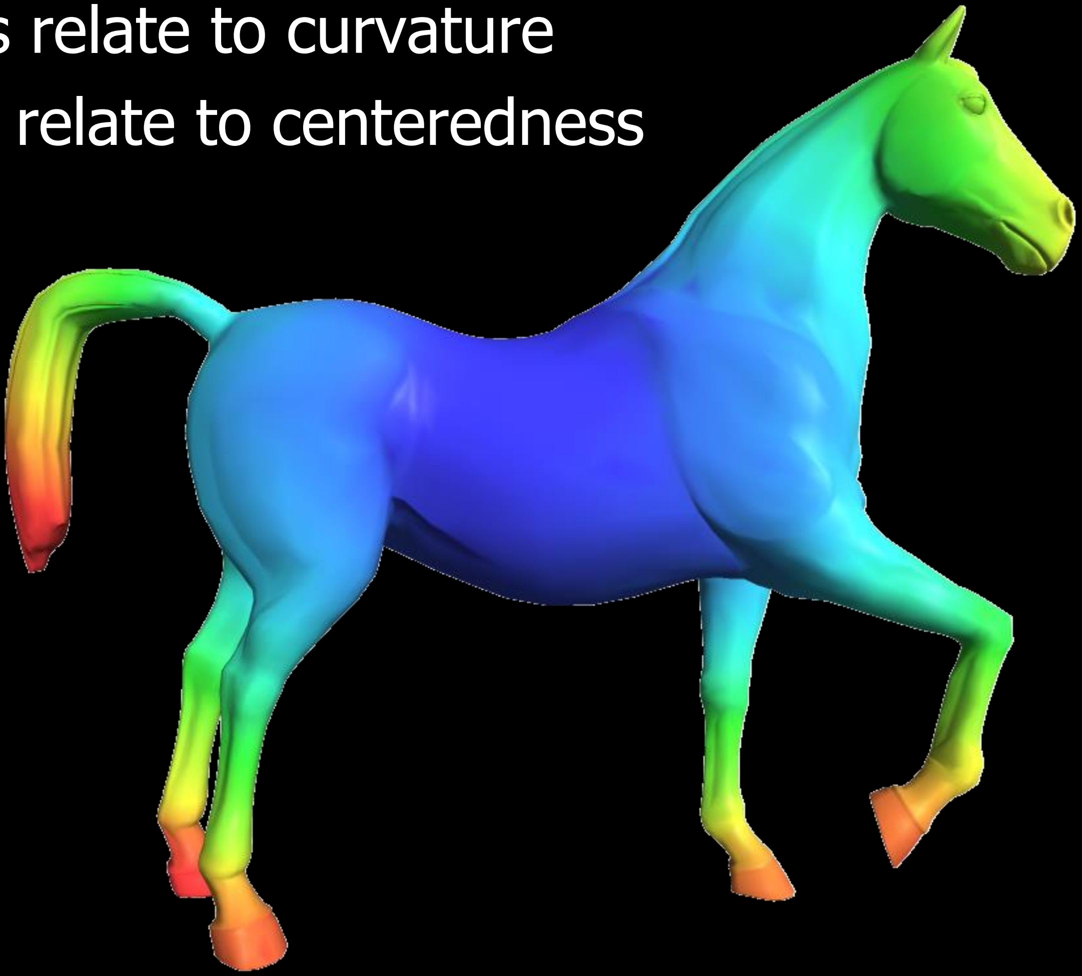
What Do Statistics of Distance Tell Us?

Histograms of geodesic distances

- Small distances relate to curvature
- Long distances relate to centeredness



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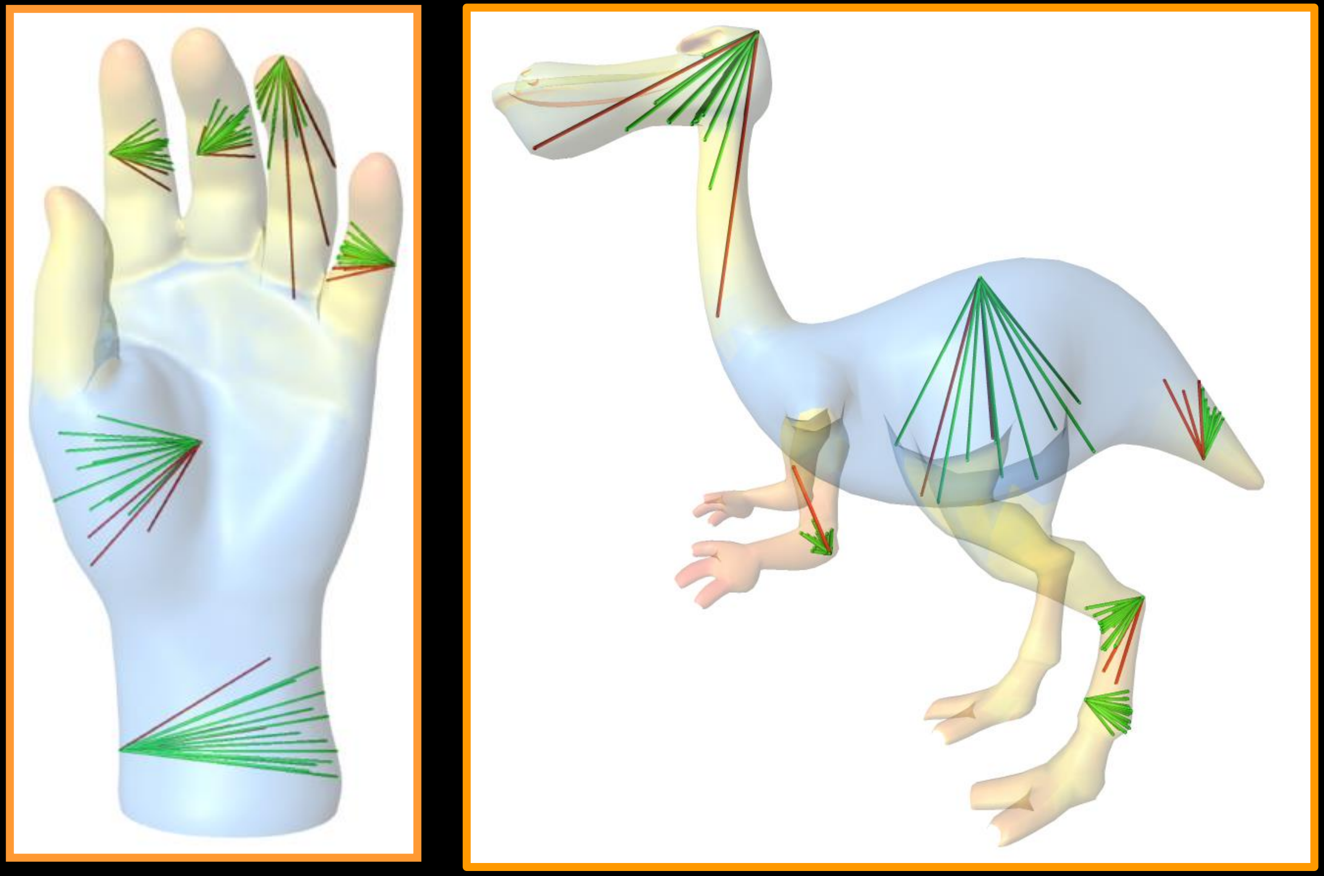
Shape Diameter Function

Shape Diameter Function

Median distance along sampling of rays through interior



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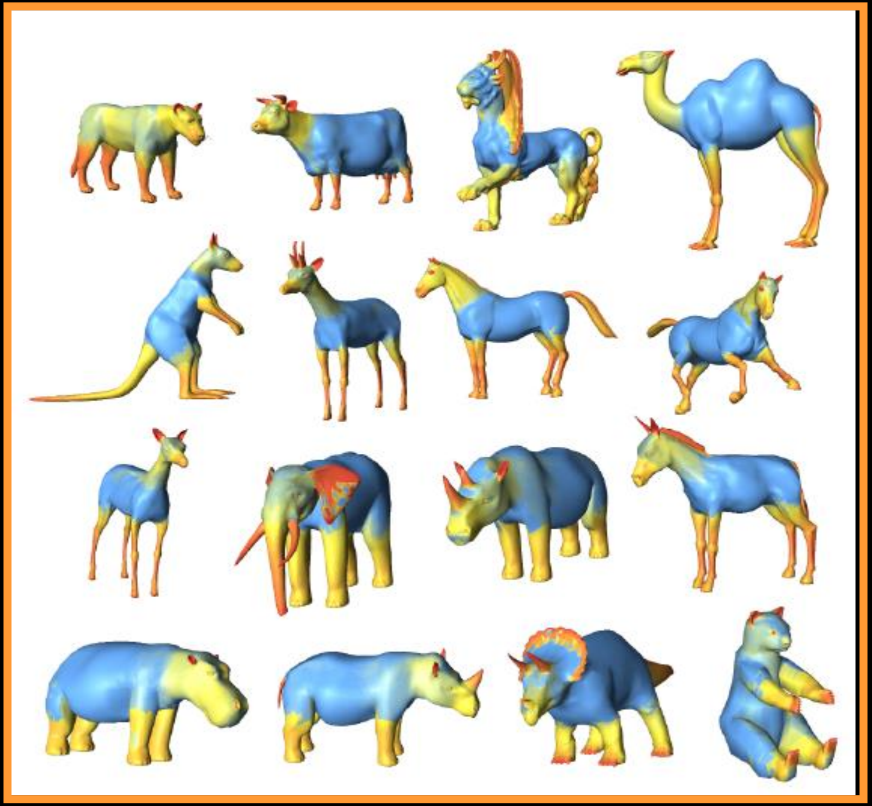
Shape Diameter Function

Distinguish between thin and thick parts in a model

Sharp changes often correlate with part boundaries



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Mesh Surface Properties in Assignment 2