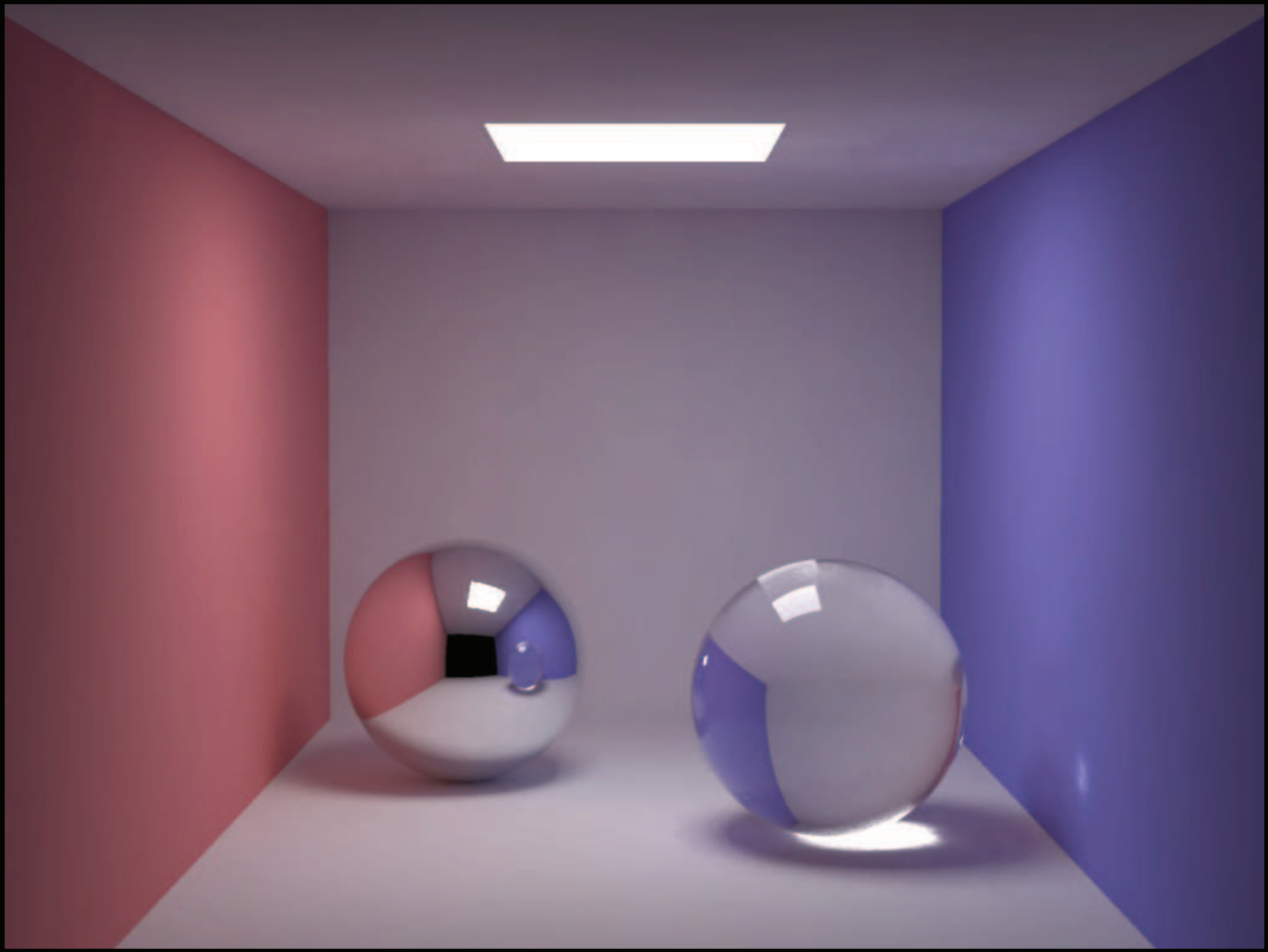
# Photon Mapping



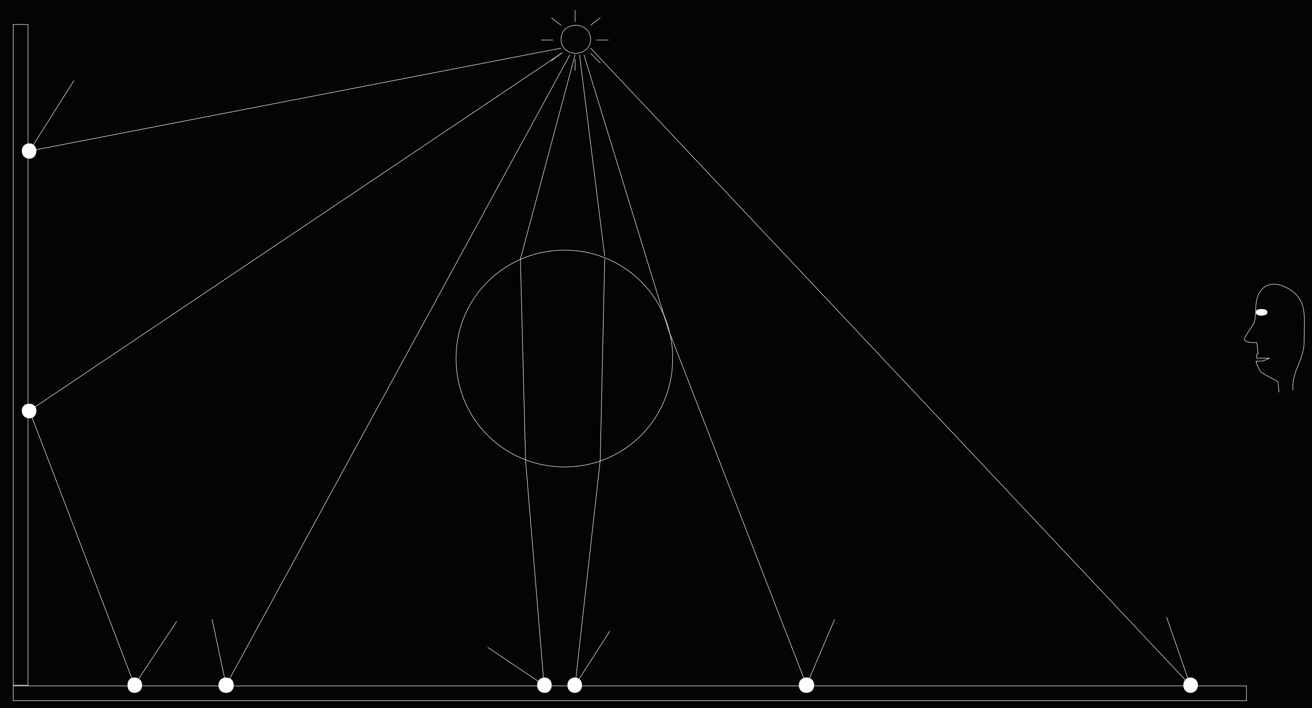
# Photon mapping

A two-pass method

Pass 1: Build the photon map (photon tracing)

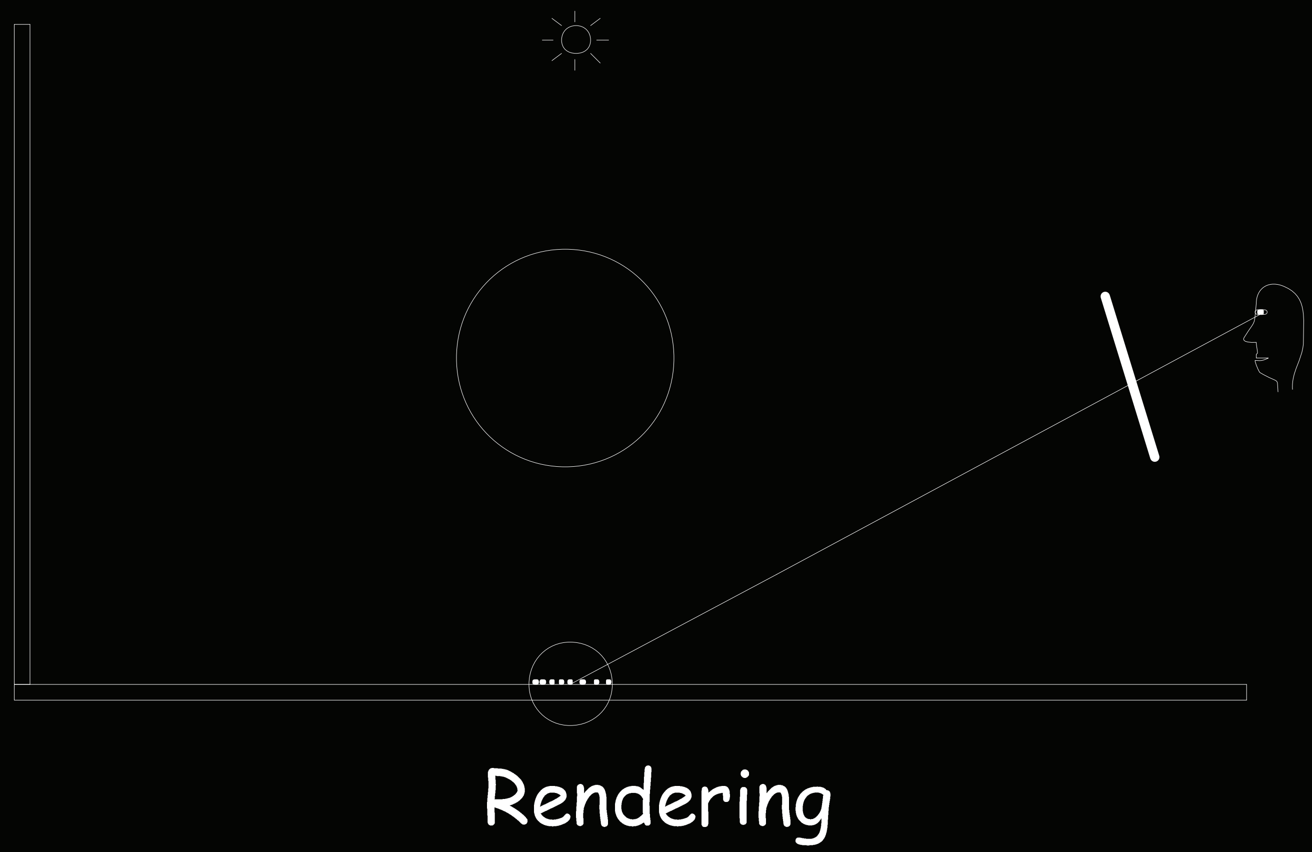
Pass 2: Render the image using the photon map

# Building the Photon Map



Photon Tracing

# Rendering using the Photon Map



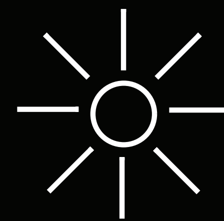
# Photon Tracing

- Photon emission
- Projection maps
- Photon scattering
- Russian Roulette
- The photon map data structure
- Balancing the photon map

# What is a photon?

- Flux (power) - not radiance!
- Collection of physical photons
  - A fraction of the light source power
  - Several wavelengths combined into one entity

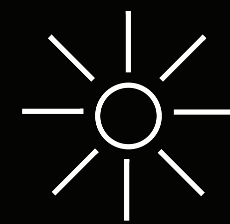
# Photon emission



Given $\Phi$ Watt lightbulb.
Emit $N$ photons.
Each photon has the power $\frac{\Phi}{N}$ Watt.

- Photon power depends on the number of emitted photons. Not on the number of photons in the photon map.

# Diffuse point light



Generate random direction
Emit photon in that direction

```
// Find random direction
do {
  x = 2.0*random()-1.0;
  y = 2.0*random()-1.0;
  z = 2.0*random()-1.0;
} while ( (x*x + y*y + z*z) > 1.0 );
```

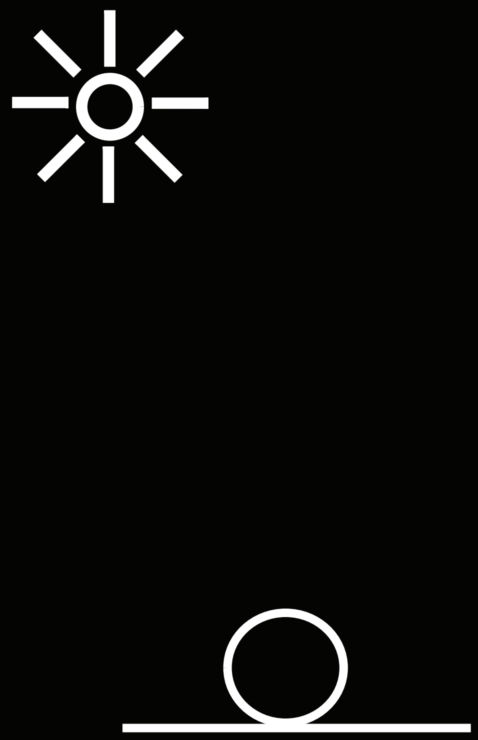
# Example: Diffuse square light

- Generate random position $p$ on square
- Generate diffuse direction $d$
- Emit photon from $p$ in direction $d$

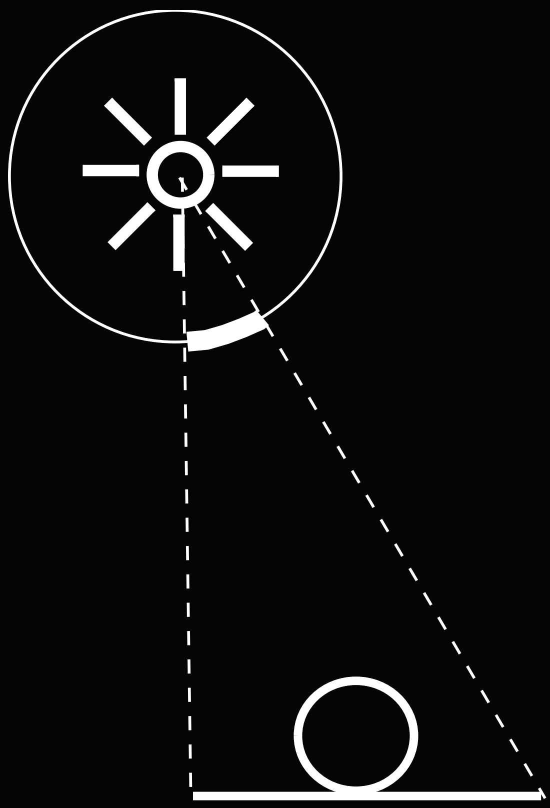
```
// Generate diffuse direction
u = random();
v = 2*π*random();
d = vector( cos(v)√u, sin(v)√u, √(1−u) );
```

$d = \text{vector}(\ cos(v)\sqrt{u},\ sin(v)\sqrt{u},\ \sqrt{1-u}\ );$

# Projection maps



# Projection maps



# Surface interactions

The photon is

- Stored (at diffuse surfaces) and
- Absorbed ($A$) or
- Reflected ($R$) or
- Transmitted ($T$)

$$A + R + T = 1.0$$

# Storing the photon

```
struct photon {
  float x,y,z;        // position
  char p[4];          // power packed as 4 bytes
  char phi,theta;     // incident direction
  short flag;         // flag used for kd-tree
}
```

Memory overhead: 20 bytes/photon.

# Photon scattering

The simple way:

Given incoming photon with power $\Phi_p$

Reflect photon with the power $R * \Phi_p$

Transmit photon with the power $T * \Phi_p$

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- Risk: Too many low-powered photons - wasteful!
- When do we stop (systematic bias)?
- Photons with similar power is a good thing.

# Russian Roulette

- Statistical technique
- Known from Monte Carlo particle physics
- Introduced to graphics by Arvo and Kirk in 1990

# Russian Roulette

Probability of termination: $p$

# Russian Roulette

Probability of termination: $p$

$$E\{X\}$$

# Russian Roulette

Probability of termination: $p$

$$E\{X\} = p \cdot 0$$

# Russian Roulette

Probability of termination: $p$

$$E\{X\} = p \cdot 0 + (1 - p)$$

# Russian Roulette

Probability of termination: $p$

$$E\{X\} = p \cdot 0 + (1-p) \cdot \frac{E\{X\}}{1-p}$$

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# Russian Roulette

Probability of termination: $p$

$$E\{X\} = p \cdot 0 + (1-p) \cdot \frac{E\{X\}}{1-p} = E\{X\}$$

Terminate un-important photons and still get the correct result.

# Russian Roulette Example

Surface reflectance: $R$ = 0.5
Incoming photon: $\Phi_p$ = 2 W

```
r = random();
if ( r < 0.5 )
  reflect photon with power 2 W
else
  photon is absorbed
```

# Russian Roulette Intuition

Surface reflectance: $R$ = 0.5
200 incoming photons with power: $\Phi_p$ = 2 Watt

Reflect 100 photons with power 2 Watt instead of 200 photons with power 1 Watt.

# Russian Roulette Example 2

Surface reflectance: $R$ = 0.2
Surface transmittance: $T$ = 0.3
Incoming photon: $\Phi_p$ = 2 W

```
r = random();
if ( r < 0.2 )
  reflect photon with power 2 W
else if ( r < 0.5 )
  transmit photon with power 2 W
else
  photon is absorbed
```

# Russian Roulette

- Very important!
- Use to eliminate un-important photons
- Gives photons with similar power :)

# Sampling a BRDF

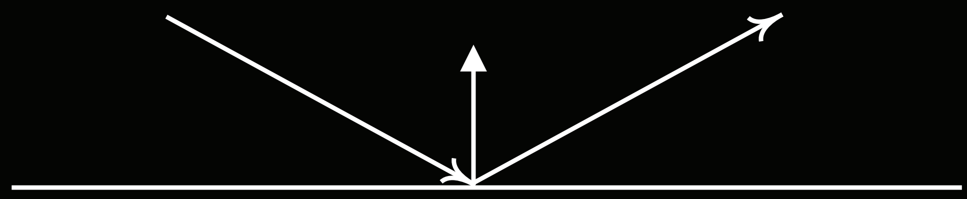
$$f_r(x,\vec{\omega}_i,\vec{\omega}_o) = w_1 f_{r,1}(x,\vec{\omega}_i,\vec{\omega}_o) + w_2 f_{r,2}(x,\vec{\omega}_i,\vec{\omega}_o)$$

# Sampling a BRDF

$$f_r(x, \vec{\omega}_i, \vec{\omega}_o) = w_1 \cdot f_{r,d} + w_2 \cdot f_{r,s}$$

```
r = random()·(w_1 + w_2);
if ( r < w_1 )
  reflect diffuse photon
else
  reflect specular
```

# Specular Reflection



$$\vec{d}_r = \vec{d}_i - 2\vec{n}(\vec{n} \cdot \vec{d}_i)$$

# The photon map datastructure

Requirements:

- Compact (we want many photons)
- Fast insertion of photons
- Fast nearest neighbor search

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A left-balanced kd-tree

# The kd-tree

- Introduced by Bentley in 1975
- A multidimensional "binary" tree

# A left-balanced kd-tree

Post-process:

- No pointers (save 40% memory)
- Faster search

Recursively split photons along the median of the "largest" dimension.

# Photon tracing

## Overview:

```
While (we want more photons) {
  Emit a photon
  while (photon hits a surface) {
   Store photon
   Use Russian Roulette to scatter photon
  }
}
Build balanced kd-tree
```

# Rendering

We want a Radiance value, $L$, per pixel.

The photon map stores flux/power.

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Radiance is the differential flux per differential solid angle per differential cross-sectional area:

$$L(x, \vec{\omega}) = \frac{d\Phi^2(x, \vec{\omega})}{d\omega \, \cos\theta \, dA}$$

# Rendering

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Radiance is the differential flux per differential solid angle per differential cross-sectional area:

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How do we get a radiance estimate from the photon map?

# Radiance estimate

$$L(x,\vec{\omega}) \;=\; \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega})L'(x,\vec{\omega}')\cos\theta'\,d\omega'$$

# Radiance estimate

$$
\begin{aligned}
L(x,\vec{\omega}) &= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) L'(x,\vec{\omega}') \cos\theta' \, d\omega' \\
&= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi'^2(x,\vec{\omega}')}{d\omega' \, \cos\theta' \, dA} \, \cos\theta' d\omega'
\end{aligned}
$$

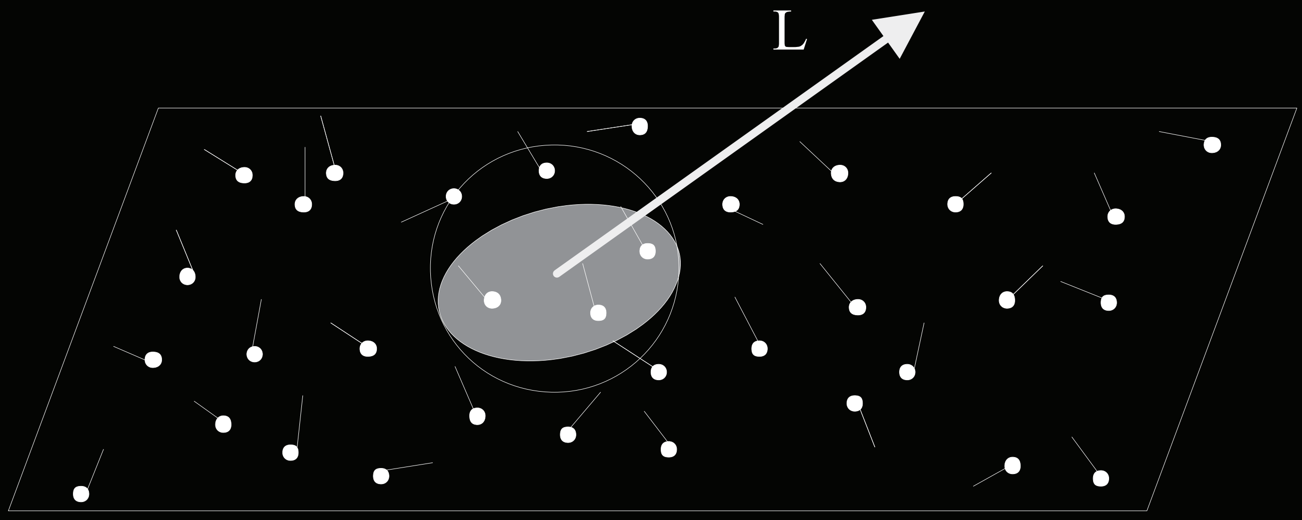
# Radiance estimate

$$
\begin{aligned}
L(x,\vec{\omega}) &= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) L'(x,\vec{\omega}') \cos\theta' \, d\omega' \\
&= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi'^2(x,\vec{\omega}')}{d\omega' \, \cos\theta' \, dA} \, \cos\theta' d\omega' \\
&= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi'^2(x,\vec{\omega}')}{dA}
\end{aligned}
$$

# Radiance estimate

$$
\begin{aligned}
L(x,\vec{\omega}) &= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) L'(x,\vec{\omega}') \cos\theta'\, d\omega' \\
&= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi'^2(x,\vec{\omega}')}{d\omega' \, \cos\theta' \, dA} \, \cos\theta' d\omega' \\
&= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi'^2(x,\vec{\omega}')}{dA} \\
&\approx \sum_{p=1}^{n} f_r(x,\vec{\omega}'_p,\vec{\omega}) \frac{\Delta\Phi_p(x,\vec{\omega}'_p)}{\Delta A}
\end{aligned}
$$

# Radiance estimate



# The radiance estimate

Locate the $n$ nearest photons using kd-tree

$$\boxed{L(x,\vec{\omega}) \approx \sum_{p=1}^{n} f_r(x,\vec{\omega}'_p,\vec{\omega}) \frac{\Delta\Phi_p(x,\vec{\omega}'_p)}{\pi r^2}}$$

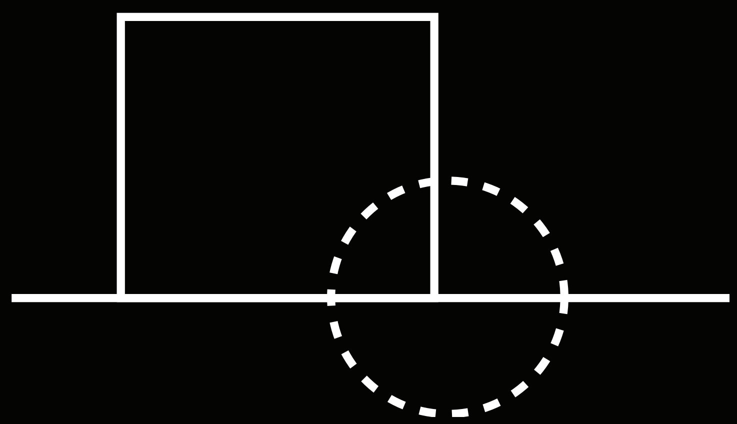
# The radiance estimate

Locate the $n$ nearest photons using kd-tree

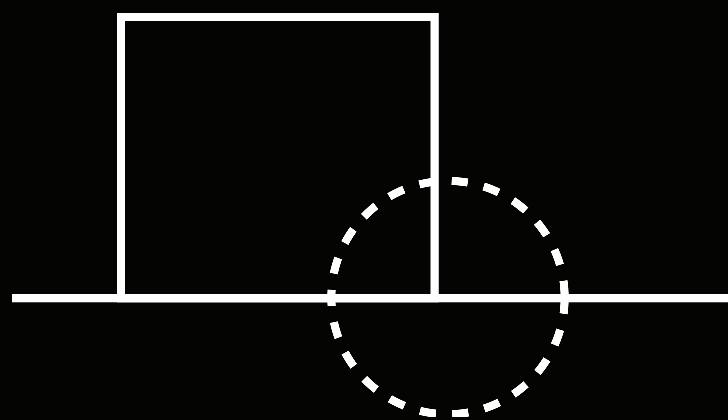
$$\boxed{L(x,\vec{\omega}) \approx \sum_{p=1}^{n} f_r(x,\vec{\omega}'_p,\vec{\omega})\frac{\Delta\Phi_p(x,\vec{\omega}'_p)}{\pi r^2}}$$

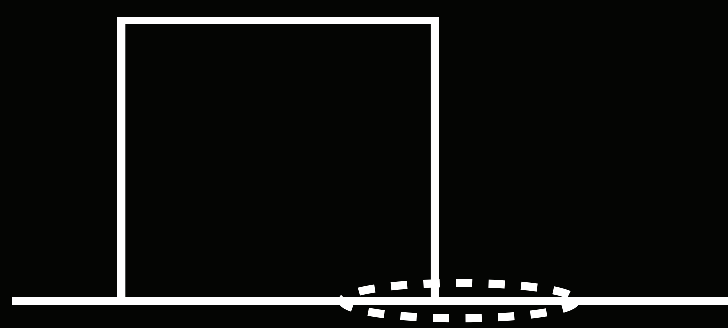
Where the surface is locally flat this is a consistent estimator:
Converges to the correct result :)

# Disc filtering



# Disc filtering





# Filtering

Weight nearby photons higher.

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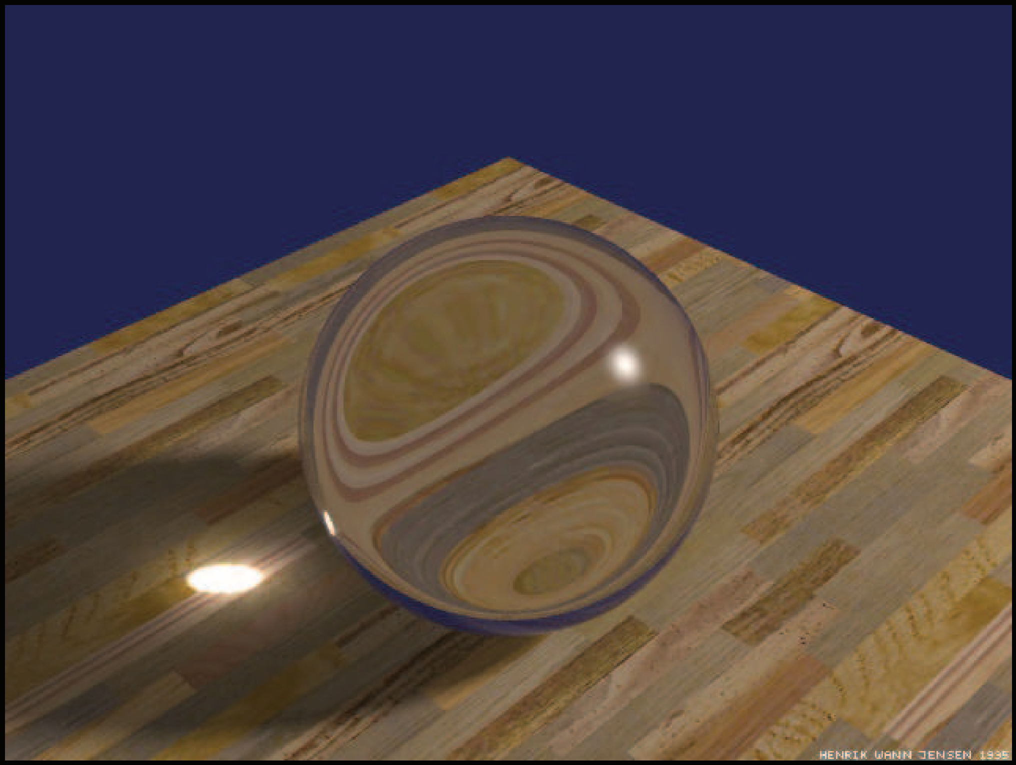
Cone filtering:

$$w_{pc} = 1 - \frac{d_p}{k\,r},$$

$$L_r(x,\vec{\omega}) \approx \sum_{p=1}^{N} f_r(x,\vec{\omega}_p,\vec{\omega}) \frac{\Delta\Phi_p(x,\vec{\omega}_p)}{\pi r^2} \frac{w_{pc}}{(1-\frac{2}{3k})}$$

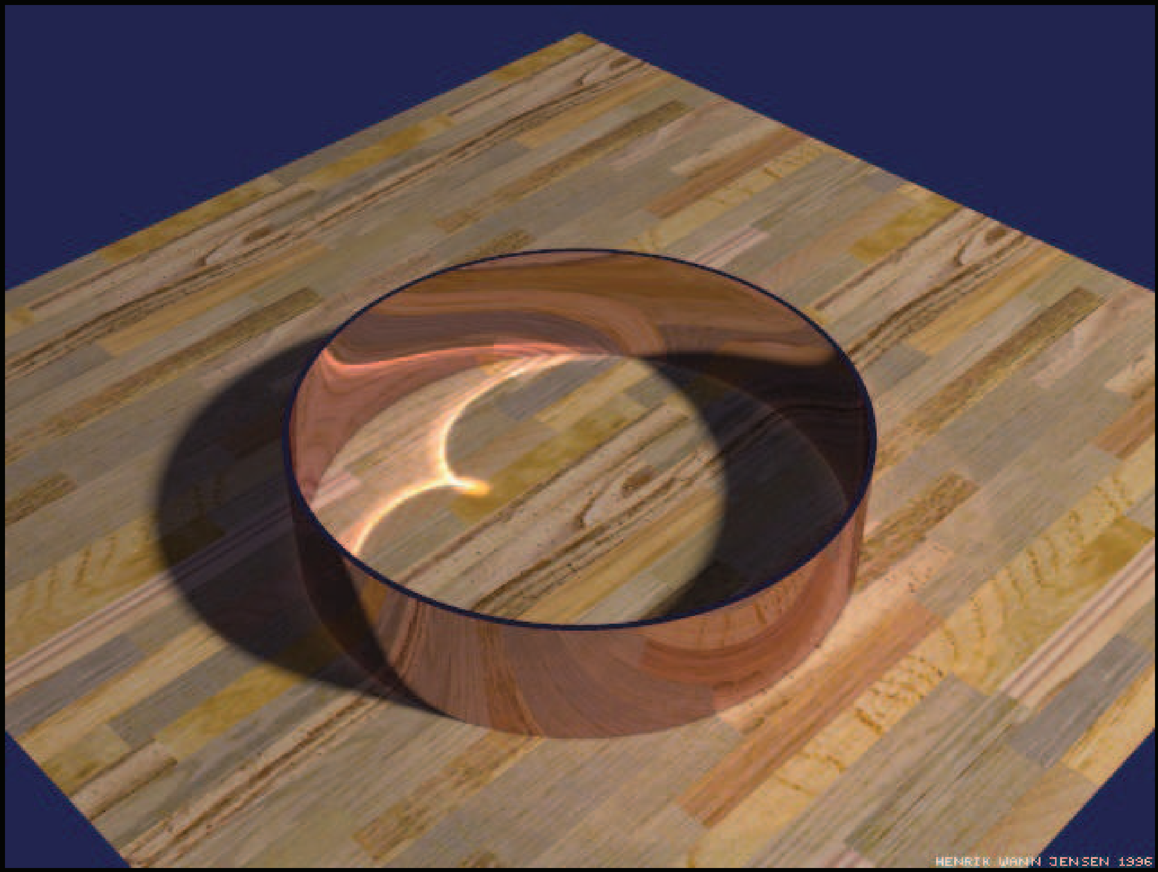
Also Gaussian filtering

# Caustic from a glass sphere




30000 photons / 50 photons in radiance estimate

# Metalring caustic



# Caustic on a glossy surface

340000 photons / $\approx$ 100 photons in radiance estimate

# Are we done?

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Works great for caustics :)

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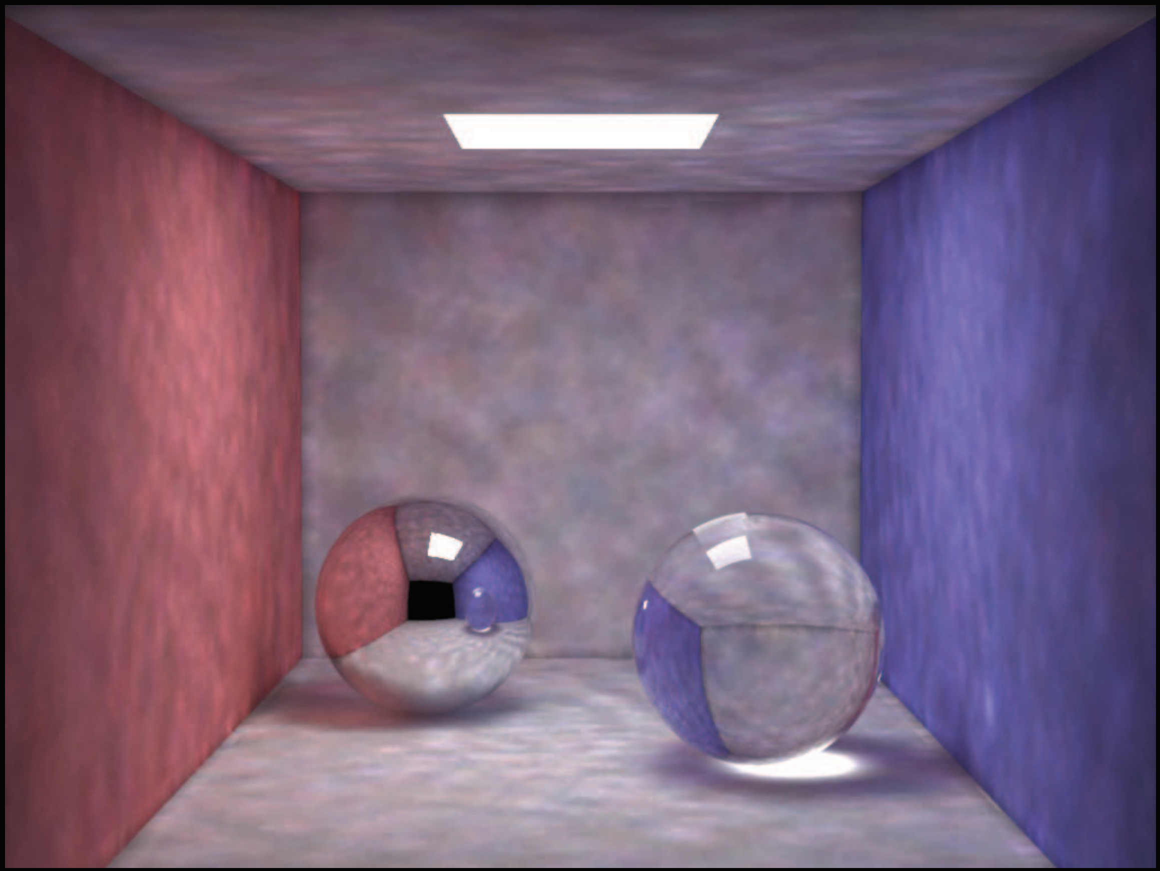
Works great for caustics :)

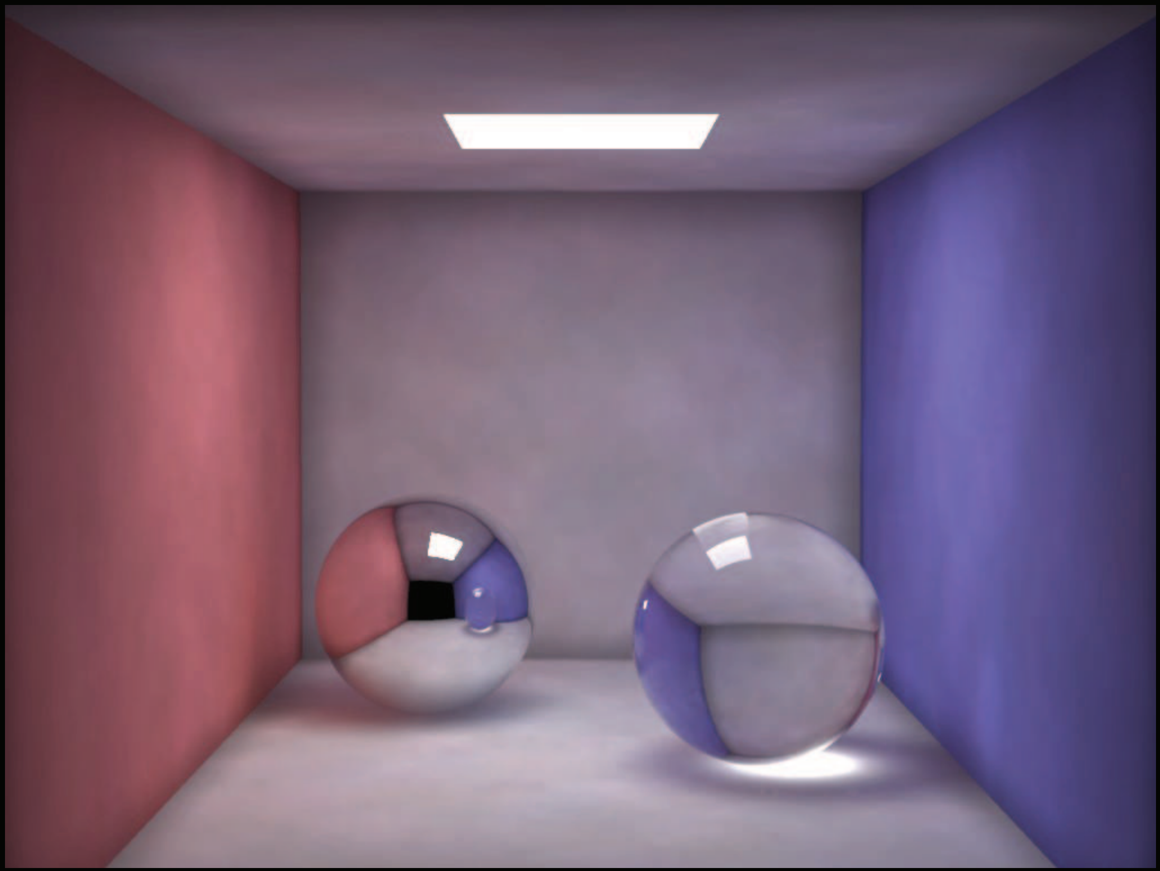
But what about other global illumination effects such as color bleeding?

# ct visualization ofthe radiance estin



200000 photons / 50 photons in radiance estimate

# ct visualization ofthe radiance estin



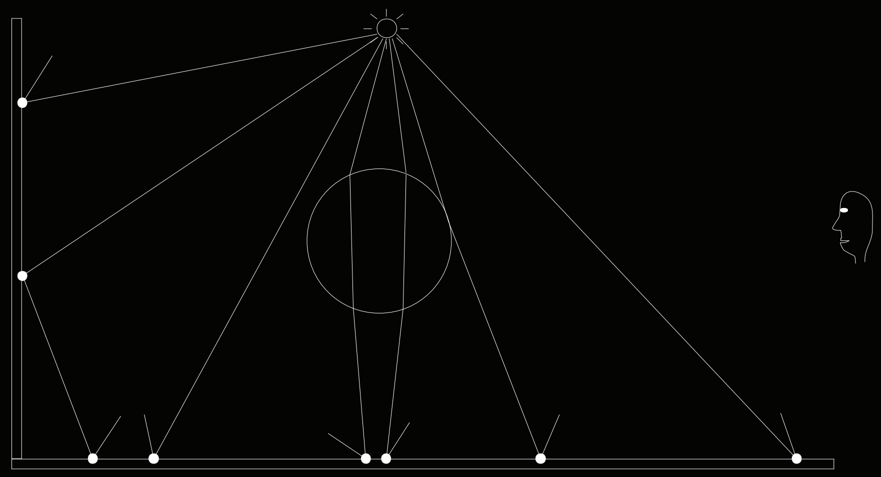
200000 photons / 500 photons in radiance estimate

# A practical two-pass method

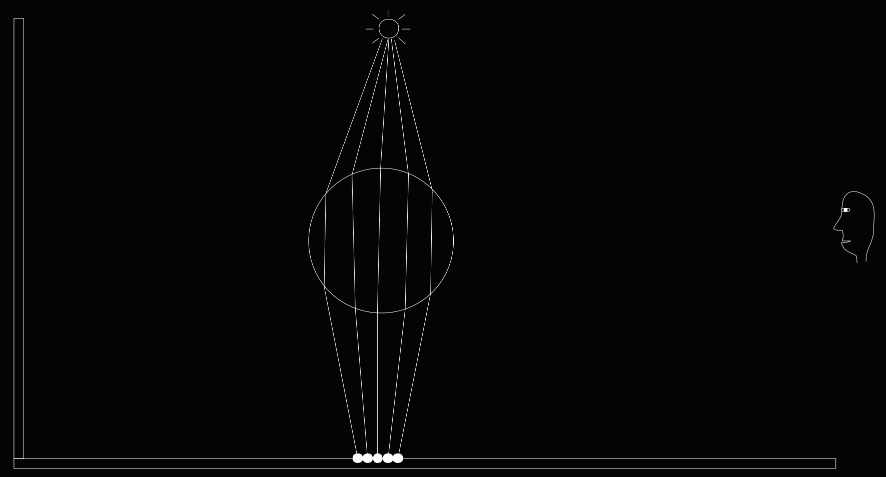
Observations:

- Caustics are difficult to sample (from the eye)
- Indirect illumination requires many photons

# Two photon maps



global photon map



caustics photon map

# The Rendering Equation

$$L_r(x,\vec{\omega}) = \int_{\Omega_x} f_r(x,\vec{\omega}',\vec{\omega}) L_i(x,\vec{\omega}') \cos\theta' \, d\omega'$$

# The Rendering Equation

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Split incoming radiance:

$$L_i = \underbrace{L_{i,l}}_{\text{direct}} + \underbrace{L_{i,c}}_{\text{caustics}} + \underbrace{L_{i,d}}_{\text{soft indirect}}$$

# The Rendering Equation

$$L_r(x,\vec{\omega}) = \int_{\Omega_x} f_r(x,\vec{\omega}',\vec{\omega}) L_i(x,\vec{\omega}') \cos\theta' \, d\omega'$$

Split incoming radiance:

$$L_i = \underbrace{L_{i,l}}_{\text{direct}} + \underbrace{L_{i,c}}_{\text{caustics}} + \underbrace{L_{i,d}}_{\text{soft indirect}}$$

Split the BRDF

$$f_r = \underbrace{f_{r,d}}_{\text{diffuse}} + \underbrace{f_{r,s}}_{\text{specular}}$$

# The Rendering Equation

$$L_r = \int_{\Omega_x} f_r \, L_i \, \cos\theta' \, d\omega'$$

# The Rendering Equation

$$
\begin{aligned}
L_r &= \int_{\Omega_x} f_r \, L_i \, \cos\theta' \, d\omega' \\
&= \int_{\Omega_x} f_r \, L_l \, \cos\theta' \, d\omega' +
\end{aligned}
$$

direct

# The Rendering Equation

$$
\begin{aligned}
L_r &= \int_{\Omega_x} f_r \, L_i \, \cos\theta' \, d\omega' \\
&= \int_{\Omega_x} f_r \, L_l \, \cos\theta' \, d\omega' + && \text{direct} \\
&\quad \int_{\Omega_x} f_{r,s} \, (L_{i,c} + L_d) \, \cos\theta' \, d\omega' + && \text{specular}
\end{aligned}
$$

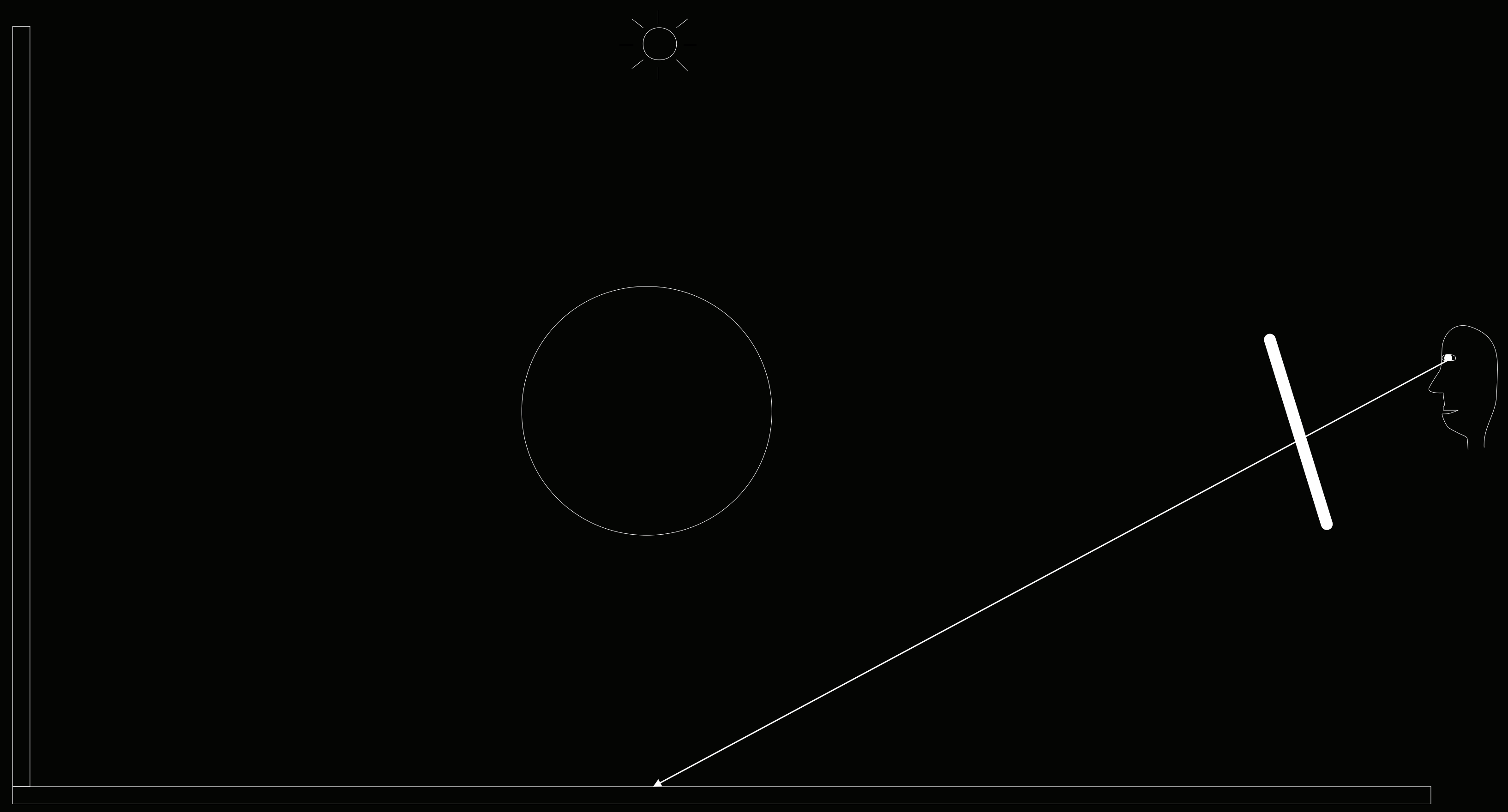
# The Rendering Equation

$$
\begin{aligned}
L_r &= \int_{\Omega_x} f_r \, L_i \, \cos\theta' \, d\omega' \\
&= \int_{\Omega_x} f_r \, L_l \, \cos\theta' \, d\omega' + && \text{direct} \\
&\quad \int_{\Omega_x} f_{r,s} \, (L_{i,c} + L_d) \, \cos\theta' \, d\omega' + && \text{specular} \\
&\quad \int_{\Omega_x} f_{r,d} \, L_c \, \cos\theta' \, d\omega' + && \text{caustics}
\end{aligned}
$$

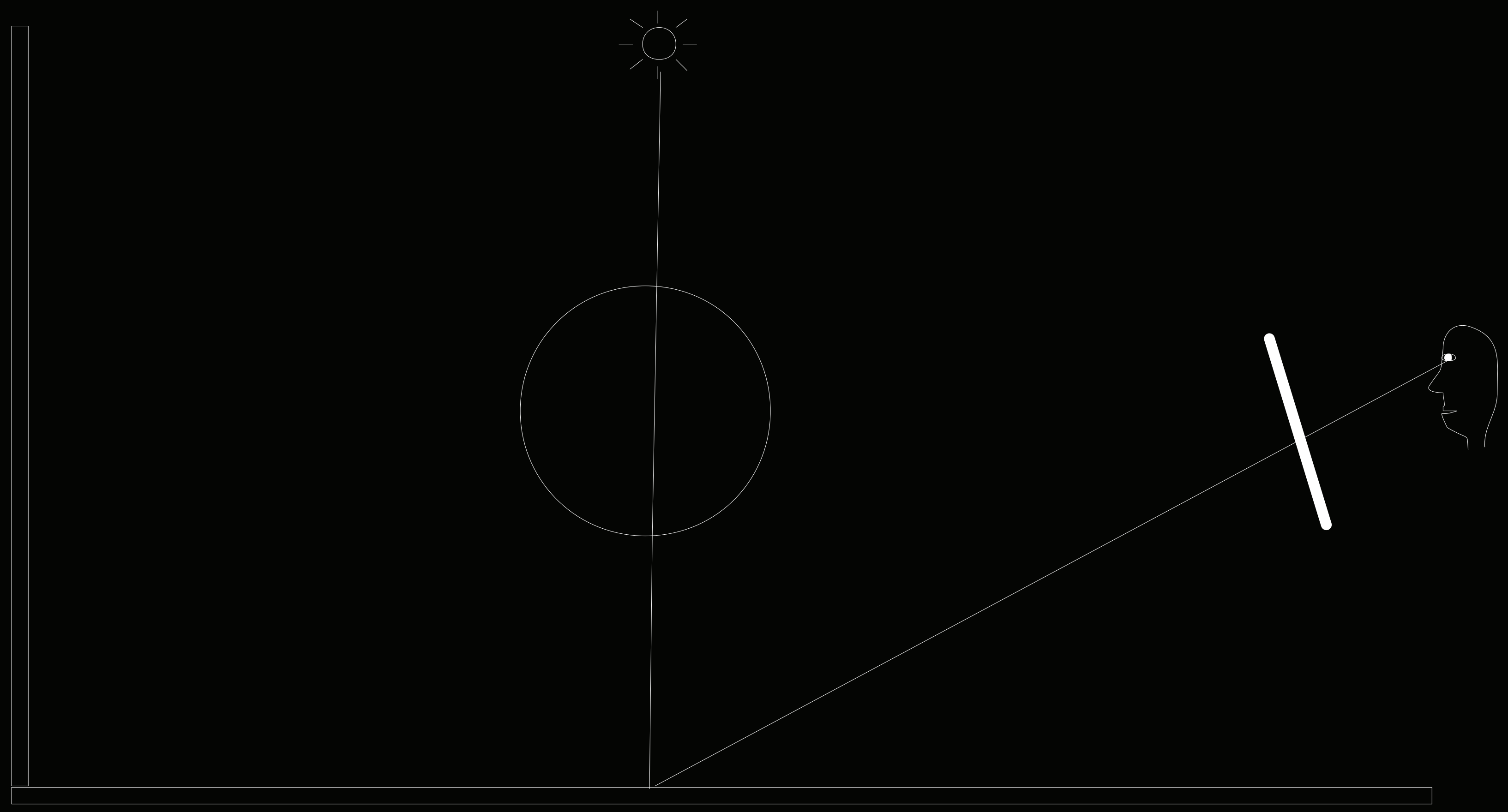
# The Rendering Equation

$$
\begin{aligned}
L_r &= \int_{\Omega_x} f_r \, L_i \, \cos\theta' \, d\omega' \\
&= \int_{\Omega_x} f_r \, L_l \, \cos\theta' \, d\omega' + && \text{direct} \\
&\quad \int_{\Omega_x} f_{r,s} \, (L_{i,c} + L_d) \, \cos\theta' \, d\omega' + && \text{specular} \\
&\quad \int_{\Omega_x} f_{r,d} \, L_c \, \cos\theta' \, d\omega' + && \text{caustics} \\
&\quad \int_{\Omega_x} f_{r,d} \, L_d \, \cos\theta' \, d\omega' && \text{soft indirect}
\end{aligned}
$$

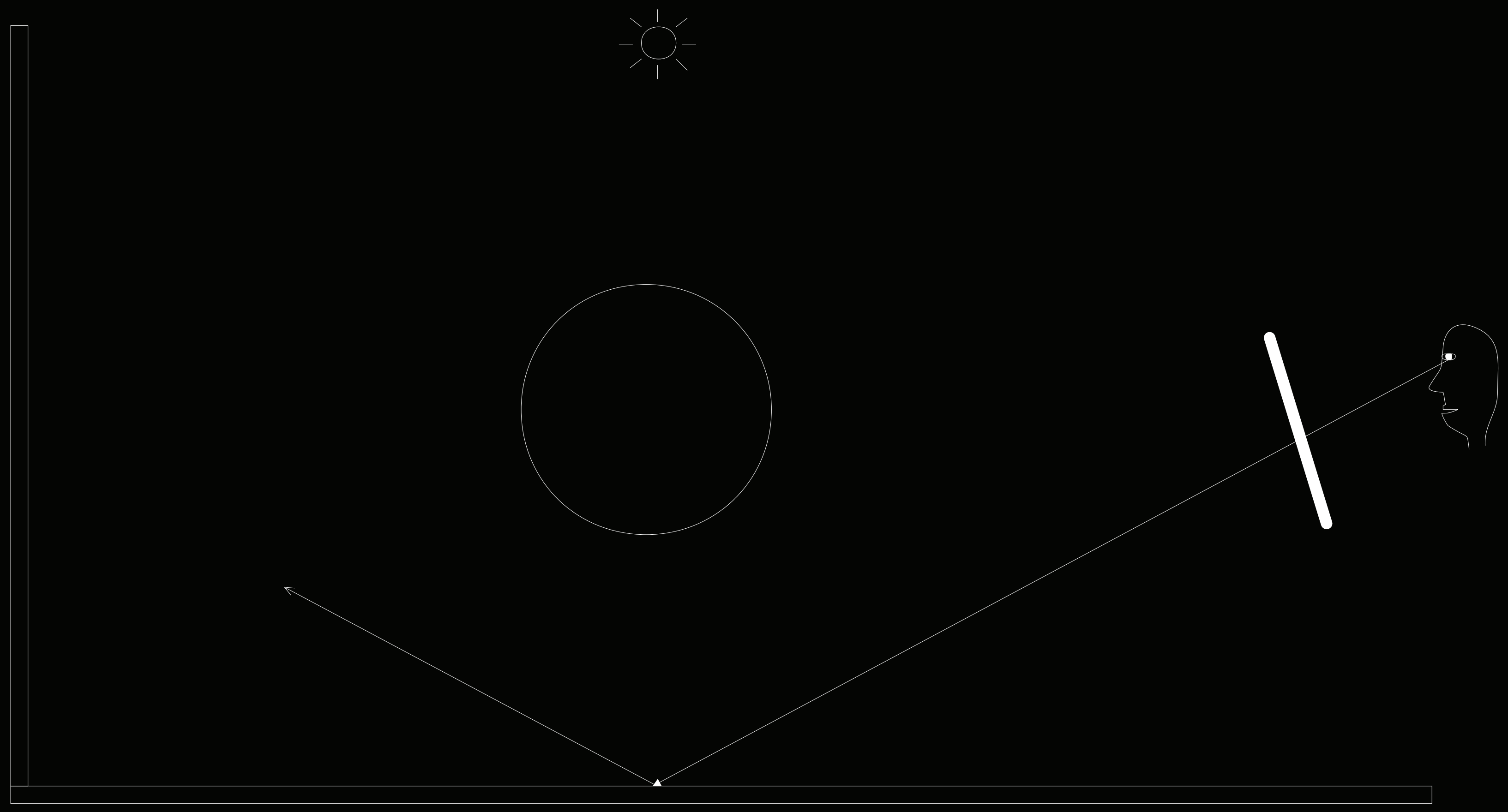
# Rendering



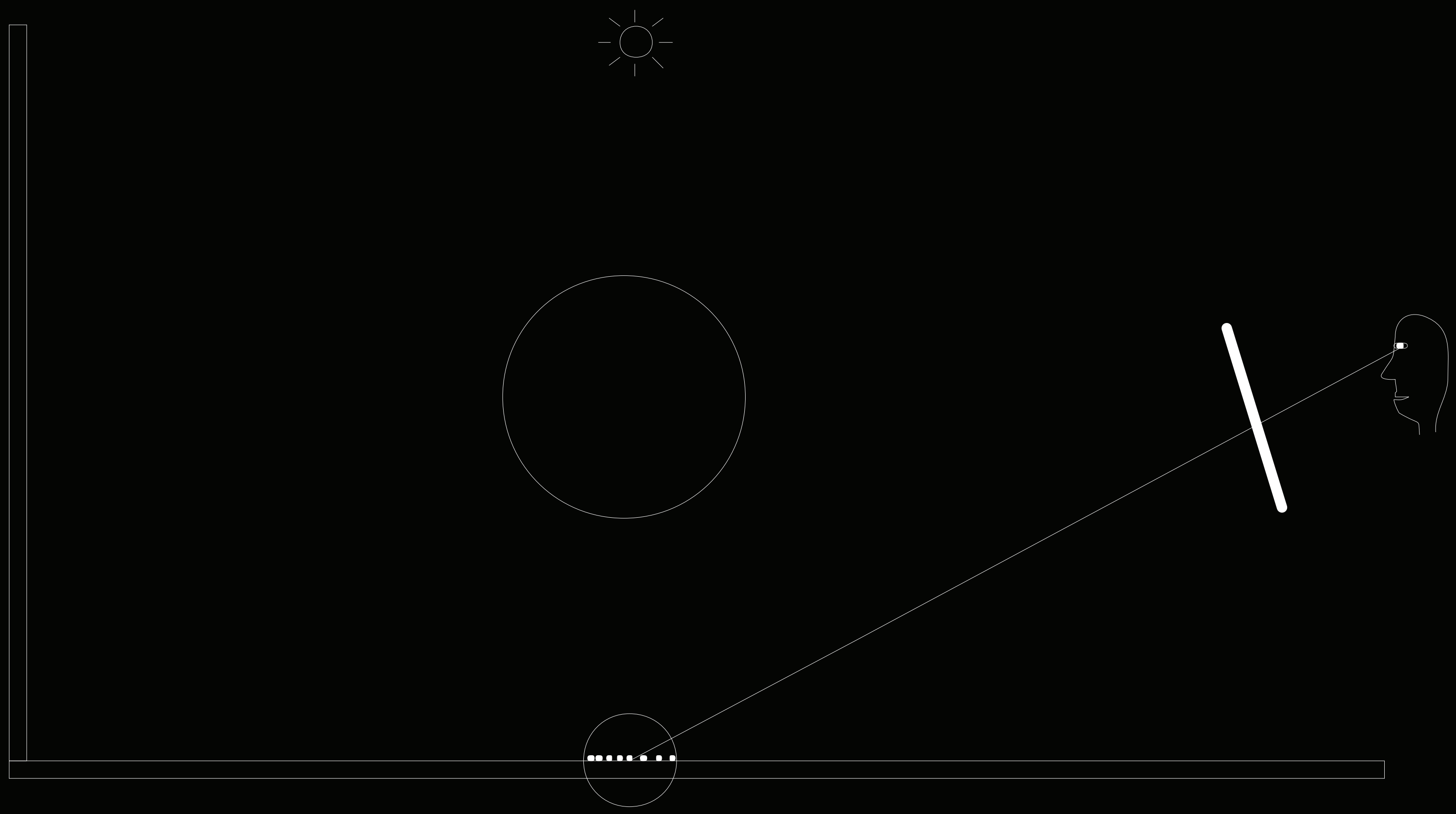
# Rendering: direct illumination



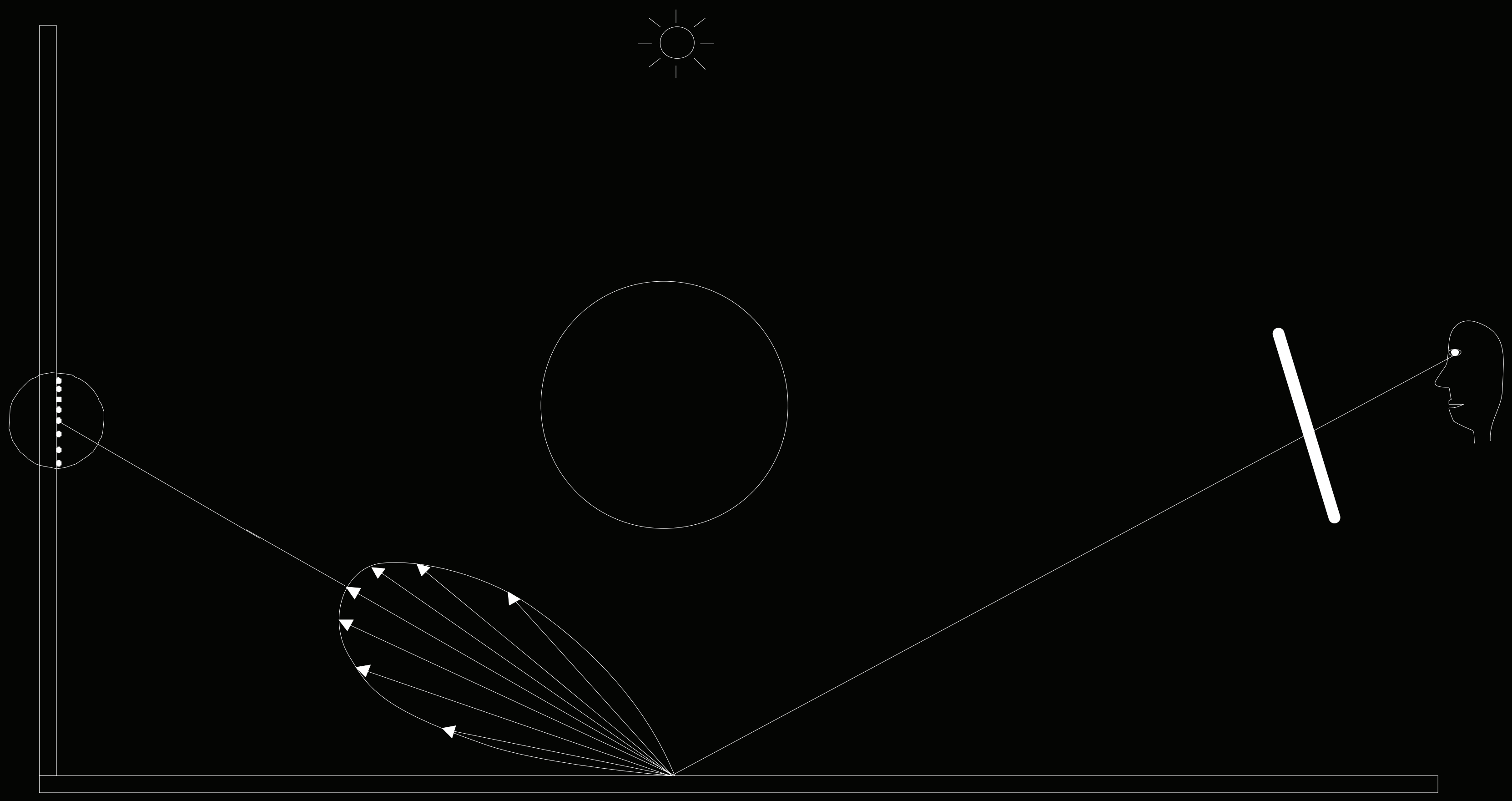
# Rendering: specular reflection



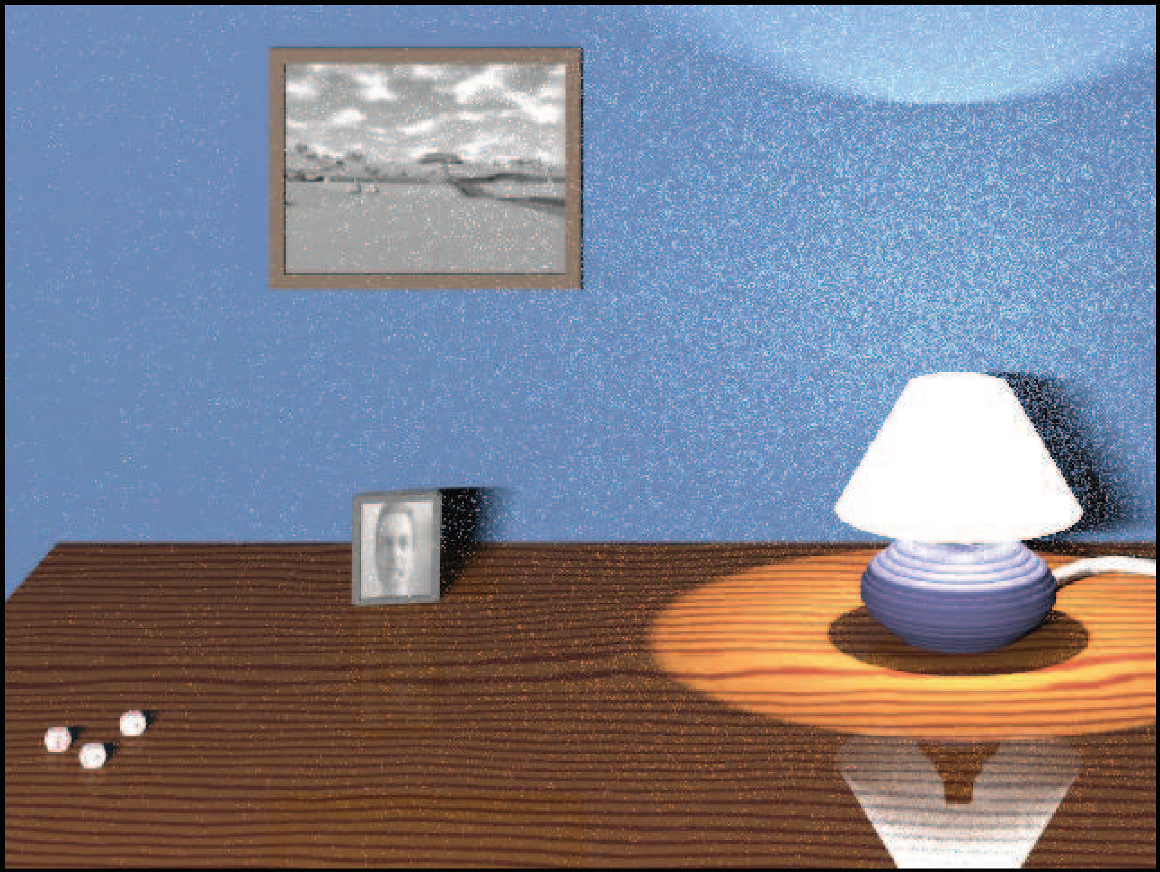
# Rendering: caustics



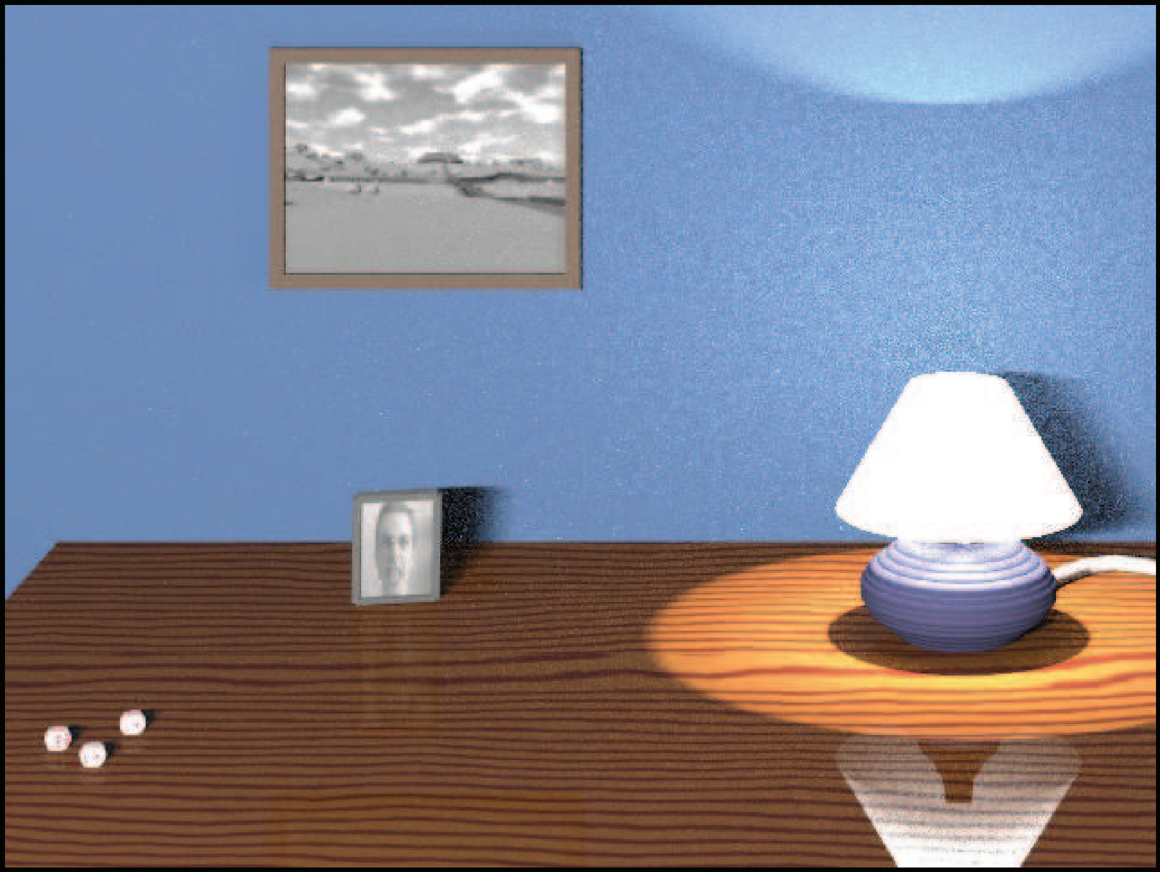
# Rendering: indirect illumination



# No Importance Sampling



# Importance Sampling



(Using the 50 nearest photons)

# Lightning