Machine Learning and Artificial Intelligence - COS 402

Written Homework Assignment 4

Due Date: two classes from the announcement in class (slightly more than a week due to Thanksgiving), due in class

(1) Consulting other students from this course is allowed. In this case - clearly state whom you consulted with for each problem separately.

(2) Searching the internet or literature for solutions is NOT allowed.

(3) Submit your homework in separate pages for the different questions, each including your name and email address (this is to help the graders). Typing solutions up is strongly advised.

I (15 points)

Let p, q, r, s be boolean variables representing the following:

- p: He is hungry.
- q: He is in love.
- r: He knows it.
- s: He is happy.

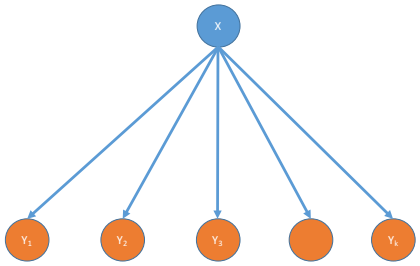
(a) Write the boolean formula representing the following: (A) If he is happy then he knows it. (B) If he is hungry then he is not happy. (C) If he is in love then he is happy.

(b) Let KB be the knowledge base consisting of the facts in the last part. Explain if the following sentences are entailed by KB, are contradicted by KB, or consistent with KB. Carefully explain you reasoning. (A) He is happy. (B) If he is in love then he knows it.

II (10 points)

Consider the following simple Bayes net – leader-follower. All the variables are binary. $P(X = 1) = \frac{1}{4}$ and $P(Y_i = 1|X = 1) = \frac{1}{8}, P(Y_i = 1|X = 0) = 0$.

FIGURE 1. Leader-follower



(a) Describe how you would generate samples from this description using a fair coin (a coin with the probability of HEADS turning up being $\frac{1}{2}$).

(b) Compute $P(X = 1|E)$ in the cases of E being the following events: (A) All $\{y_i\}$ are 1. (B) All $\{y_i\}$ are 0.

III (15 points)

Consider the Markov Chain over 4 states, with the following transition probabilities:

(a) From every state, the MC remains in the same state with probability $\frac{1}{4}$.

(b) States $2, 3$ move to all other states (including themselves) with probability $\frac{1}{4}$.

(c) State 1 moves to state 2 and 3 with equal probability.

(d) State 4 moves to state 3 with probability $\frac{3}{4}$.

Draw the transition graph of the Markov chain. Is this Markov chain ergodic? If so, compute its unique stationary distribution, and prove it is indeed stationary. If not, explain.