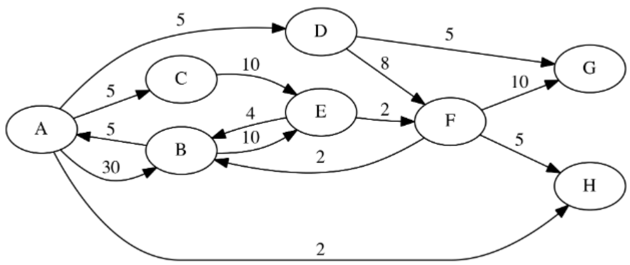
COS 226 Data Structures and Algorithms
Computer Science Department
Princeton University
Fall 2015

Week 9 Activity

1. Graph Algorithms

   Consider the weighted directed graph below.



(a) Complete the table distTo[ ] and edgeTo[ ] after all vertices have been relaxed during the execution of the Dijkstra's shortest path algorithm (starting with vertex A).

vertex	distTo [ ]	edgeTo [ ]
A		
B		
C		
D		
E		
F		
G		
H		

(b) The table below is created when applying Dijkstra's shortest path algorithm to a graph with 8 vertices 0,1..,7

v	distTo[]	edgeTo[]
0	∞	*null*
1	7.0	5
2	13.0	3
3	0.0	*null*
4	10.0	7
5	3.0	3
6	12.0	1
7	8.0	3

The table shows the shortest path tree (distTo and edgeTo) immediately after vertex 4 has been relaxed. Give the order in which the first five vertices were deleted from the minPQ.

2. Shortest Directed Cycle

   Given an edge-weighted directed graph and a vertex v, find the shortest simple cycle that starts from v and or report that graph is acyclic. The shortest cycle is a cycle that minimizes the total path length of the cycle.

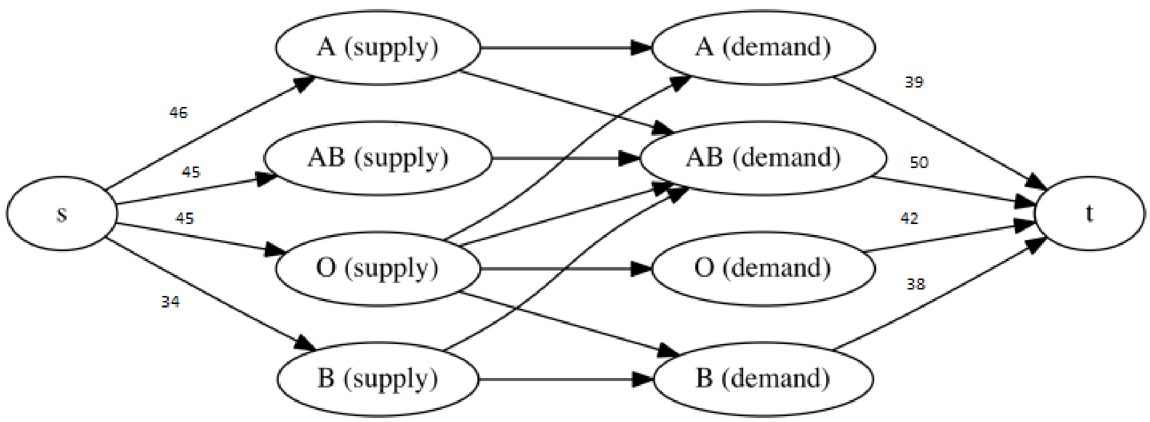
   (a) Discuss a possible algorithm for finding the shortest cycle.

   (b) What is the order of growth of the runtime of your algorithm in terms of V and E?

   (c) What is the order of growth of the space required to implement your algorithm in terms of V and E

3. Bonus Problem - Maxflow-Mincut algorithm
The following flow diagram shows the demand and supply of types A, B, AB and O blood types and various possibilities for each blood type donated.

For example, those with type A can donate blood to types A and AB, and type AB can receive blood from any type A, B, AB, or O types etc.
The following table shows the supply and demand numbers for each of the blood types.



The following table shows the total available blood units (170 units) and the total number of students who needs blood transfusion(169 units). Assume that each student requires only one unit of their blood type.

blood type	A	B	O	AB	sum
Supply	46	34	45	45	170
Demand	39	38	42	50	169

(a) identify edge capacities for each edge in the flow diagram. It is ok to have infinite edge capacities.

(b) Apply Ford-Fulkerson algorithm to find a max flow that can be allowed through the network. Mark all edges that are explored.

(c) What is the value of the max flow for the flow diagram?

(d) Which vertices are on the t side (target side!) of the min-cut? Note: These vertices should provide a concise description of why there is not enough blood for everyone. Just explain briefly.