

COS126 Number Systems Activity — Booksite 5.1

Binary and Decimal

1. What is the binary integer 101, represented in decimal? $4 + 1 = 5$
2. What is the binary integer 1010, represented in decimal? $8 + 2 = 10$.
(How is this related to the previous answer?) **Twice as much as 101**
3. What is the binary integer 10100, represented in decimal? **20**.
(What is the pattern?) **Again twice as much since all ones became twice as valuable**
4. What is the binary integer 101001, represented in decimal? **41. Twice as much plus one**.
(Could you write a program to use this approach?) **Yes, and it is useful in LFSR!**
5. What is the decimal integer 116, represented in binary?

Right to left: see “Converting from decimal to base b” on booksite §5.1. 116 is even, so ends in a 0, preceded by representation of $116/2 = 58$. 58 even so it ends in a 0, etc. \Rightarrow **1110100**
Left to right: biggest power of 2 that fits (≤ 116) is 64, leaving $116-64 = 52$. Biggest power of 2 in this remainder is 32. Keep going with remainders, $116=64+32+16+4 =$ binary **1110100**.

Binary and Hex

6. What are the hexadecimal numbers C, D, and E, expressed in binary? **These are twelve, thirteen, fourteen, which are 1100, 1101, 1110.**
7. Express the hexadecimal number CODE as a sum of 4 terms corresponding to the 4 digits. What is the value of this expression when converted to binary? **Note that $16 = 2^4, 16^3 = 2^{12}$ and $\times 2$ shifts us left by one position. CODE is $12 \times 16^3 + 0 \times 16^2 + 13 \times 16^1 + 14 \times 16^0 = 1100\ 0000\ 1101\ 1110$ (C 0 D E)**
8. What is the binary number 100100110, represented in hexadecimal? (Avoid using decimal.) **Reverse the previous process. 1 0010 0110 and converting each 4 bits to a hex digit, 126**

Bitwise Operators (In Q9 thru Q13, all numbers are in binary)

9. What is the binary value of $1010 \mid 110$? **1110**
10. What is the binary value of $1010 \& 110$? **10**
11. What is the binary value of $1010 \ll 10$? **101000**
12. What is the binary value of $1010 \gg 10$? **10**
13. What is the binary value of $1010 \wedge 110$? **1100**
14. What is the value, expressed in hexadecimal, of $C05126 \wedge CBE245 \wedge C05126$? (What is the trick?)
Since the order of inputs to xor doesn't matter, this equals $CBE245 \wedge C05126 \wedge C05126$. Since anything xor'ed with itself is 0, this is $CBE245 \wedge 0 = \mathbf{CBE245}$

16-bit Two's-Complement Representations

15. What is the complement of 0101 0000 1100 1111? **1010 1111 0011 0000**
16. Give the **16-bit two's-complement** binary representation of the decimal integer 116
(Use question 5) **0000 0000 0111 0100**
17. Give the 16-bit two's-complement binary representation of the decimal integer -116 **First complement the bits of $+116$, then add one, giving 1111 1111 1000 1100**
18. What is the 16-bit two's-complement **hexadecimal** representation of the decimal integer -116 ?
Like Q8 (converting each 4 bits to a hex digit) **FF8C**
19. What is the decimal representation of the 16-bit two's-complement hexadecimal number FFFE?
Since the first bit is 1, this number is negative. Call this negative number X . Then the binary representation of the positive number $-X$ is obtained by flipping bits (0000 0000 0000 0001) and adding one (0000 0000 0000 0010). So $-X$ is 2, i.e. X is **-2**.

Challenges (Read Booksite §5.1)

20. What should the binary numbers 0.1 and 0.01 represent? **In decimal these are 10^{-1} and 10^{-2} . In binary these are likewise $2^{-1} = 1/2$ and $2^{-2} = 1/4$**
21. Booksite exercises 5.1.18, 5.1.23, 5.1.25, Booksite creative exercises 5.1.6, 5.1.29