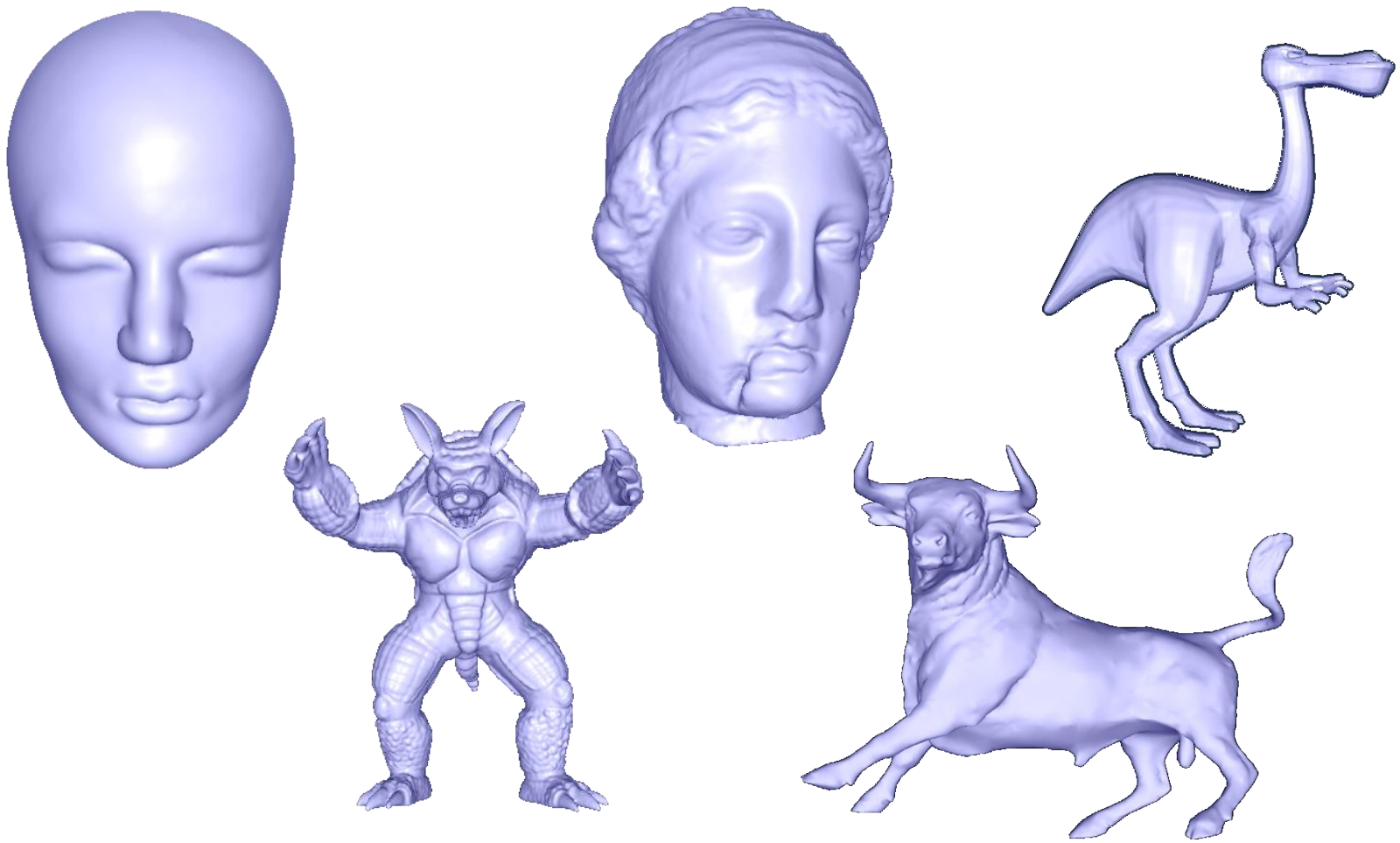
Symmetry Analysis and its Applications in Computer Graphics

Thomas Funkhouser
Alex Golovinskiy, Misha Kazhdan,
Vladimir Kim, Josh Podolak,
and Szymon Rusinkiewicz

Princeton University

Motivation

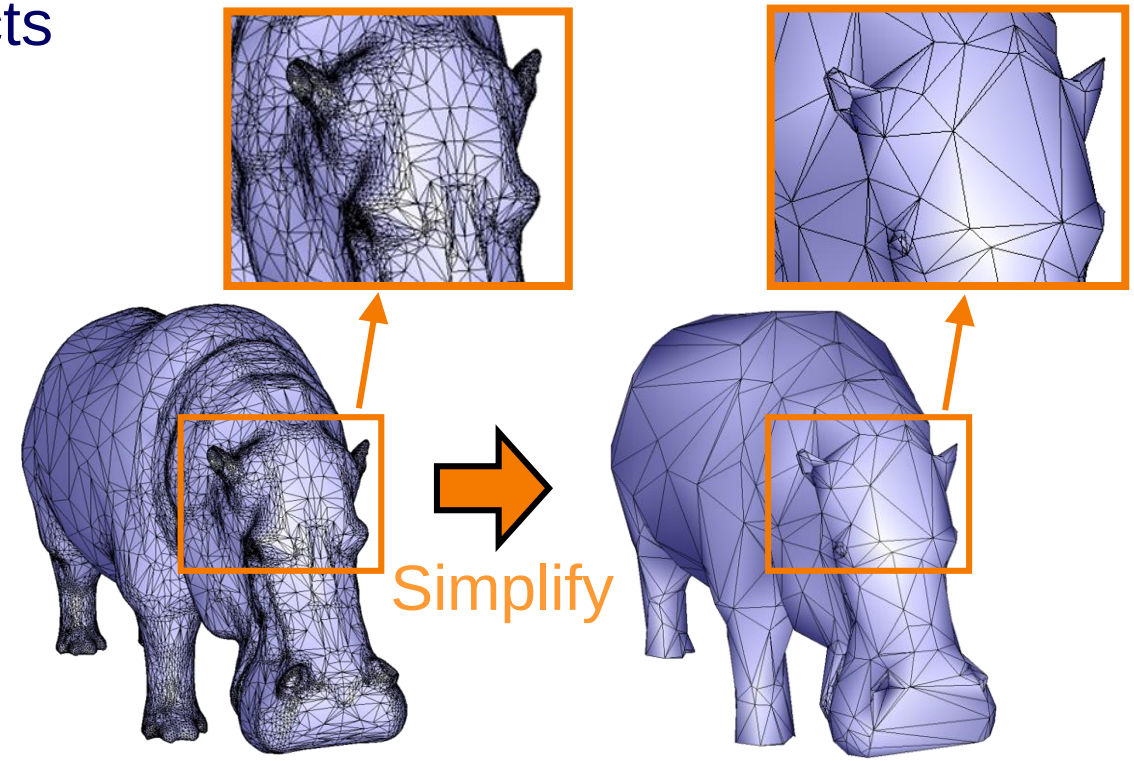
Symmetry is common in real-world objects



Problem

Most algorithms ignore (and sometimes destroy) symmetries when they process 3D models

- Rendering artifacts
- Simulation errors
- etc.



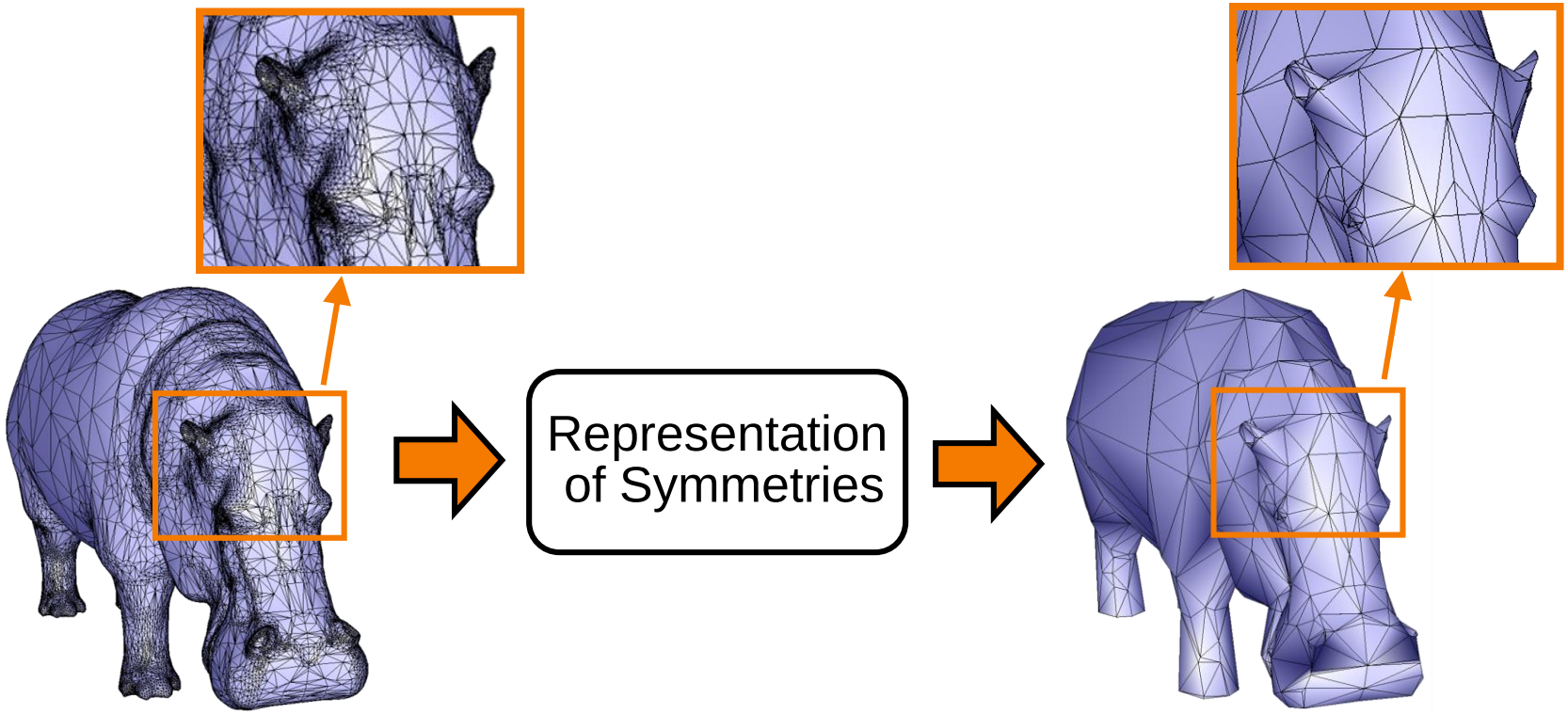
Symmetric Input Mesh

Asymmetric Output Mesh

[Cohen-Steiner04]

Goal

We need methods to detect, analyze, represent, and exploit symmetries in 3D models



Symmetric Input Mesh

Symmetric Output Mesh

Outline

Introduction

Background

Symmetry representations

- Symmetry descriptor
- Symmetry transform
- Principal symmetries

Applications

Conclusion

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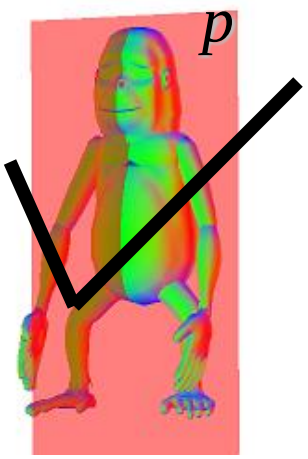
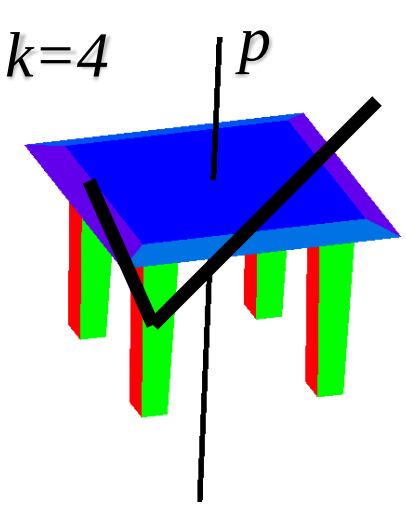
Symmetry

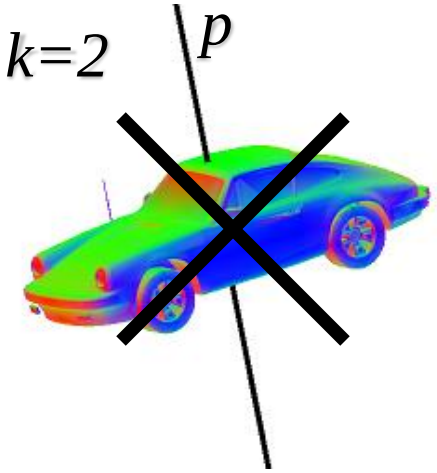
A shape has *reflective symmetry* w.r.t. some plane *p* if the reflection Ref_p through *p* fixes the collection



Symmetry

A shape has *rotational symmetry* of order k w.r.t. some axis p if the rotation Rot_p^k by an angle of $360^o/k$ about p fixes the collection.



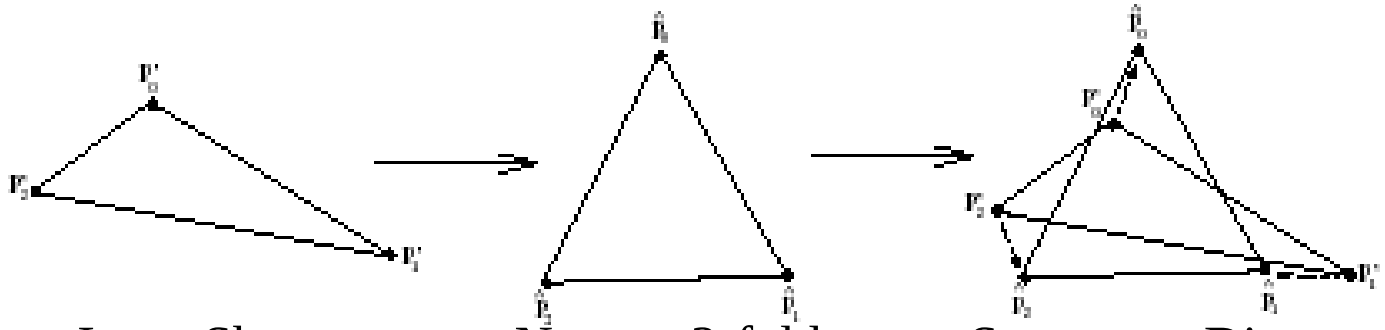


Approximate Symmetry

… but many shapes in computer graphics are not perfectly symmetric

Symmetry Distance

The *symmetry distance* of a boundary is the L^2 distance to the nearest symmetric boundary



Input Shape | Nearest 3-fold symmetric | Symmetry Distance

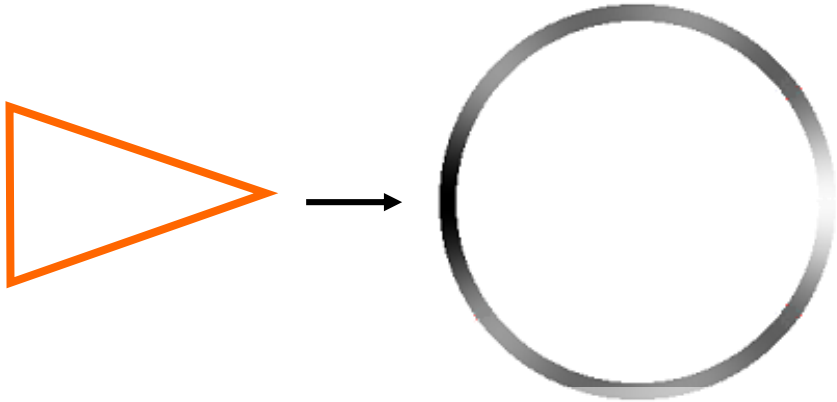
Requires point correspondences

IWVF, 1994. Zabrodsky et al.

IEEE, 1995. Zabrodsky et al.

Symmetry Distance

For circular functions, can replace distances between correspondences with correlation



2D Shape Function

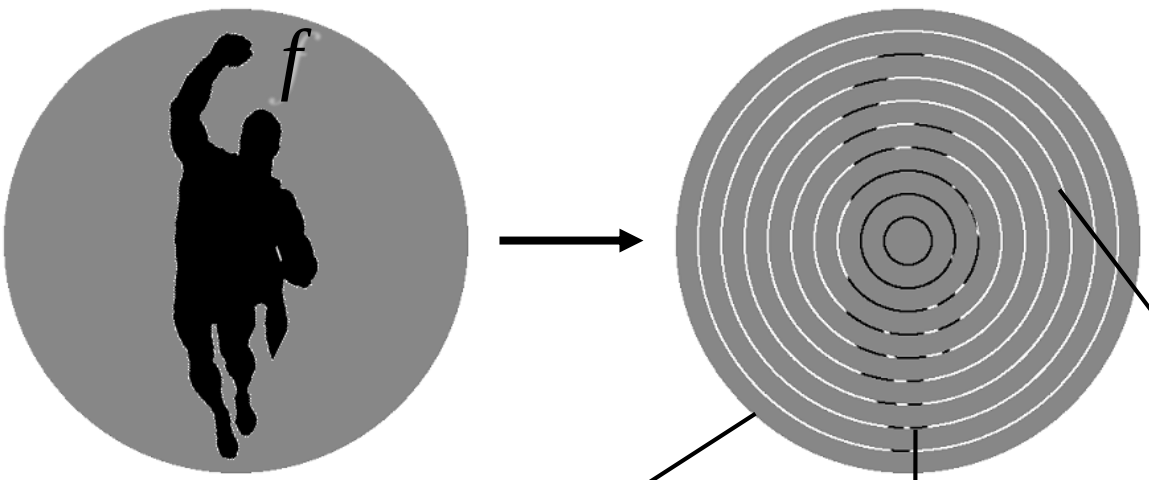
$$Sym(SD,\alpha) = \int_0^{2\pi} SD(t-\alpha)SD(t)dt$$

PRL, 1995. Sun

RTI, 1999. Sun et al.

Symmetry Distance

For images, can consider circular restrictions and compute correlations efficiently with Fourier transform



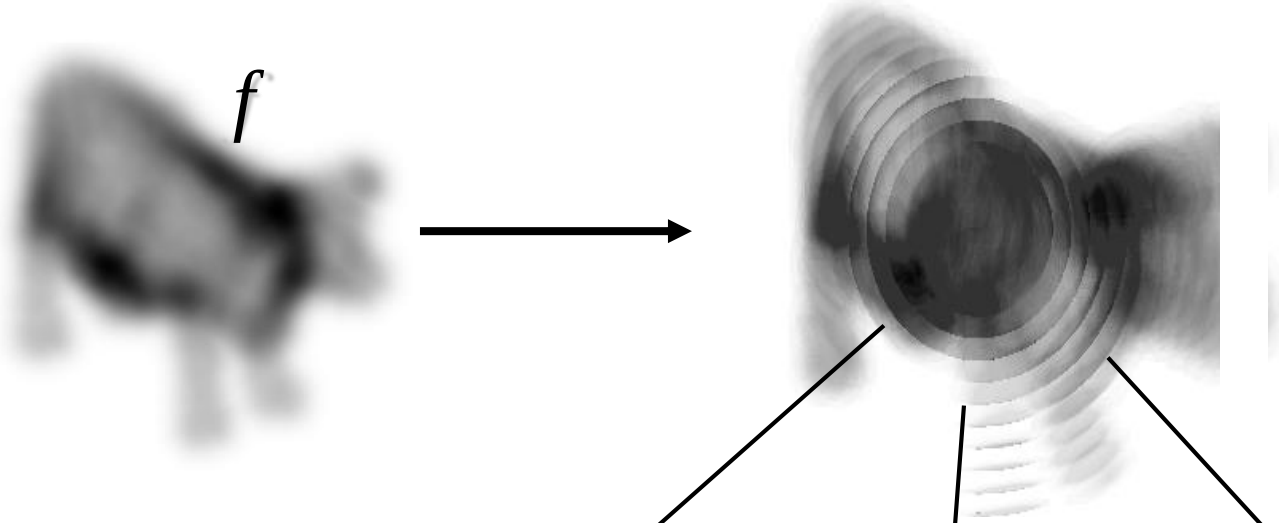
$$Sym(f,\gamma) = \overbrace{r_1}^{\text{scale factor}} \; Sym(\;,\gamma) + \overbrace{r_2}^{\text{scale factor}} \; Sym(\;,\gamma) + \overbrace{r_3}^{\text{scale factor}} \; Sym(\;,\gamma) + \ldots$$

PRL, 1995. Sun

RTI, 1999. Sun et al.

Symmetry Distance

For 3D functions, can consider spherical restrictions and compute correlation efficiently with spherical harmonics and Wigner-D^{-1} transform

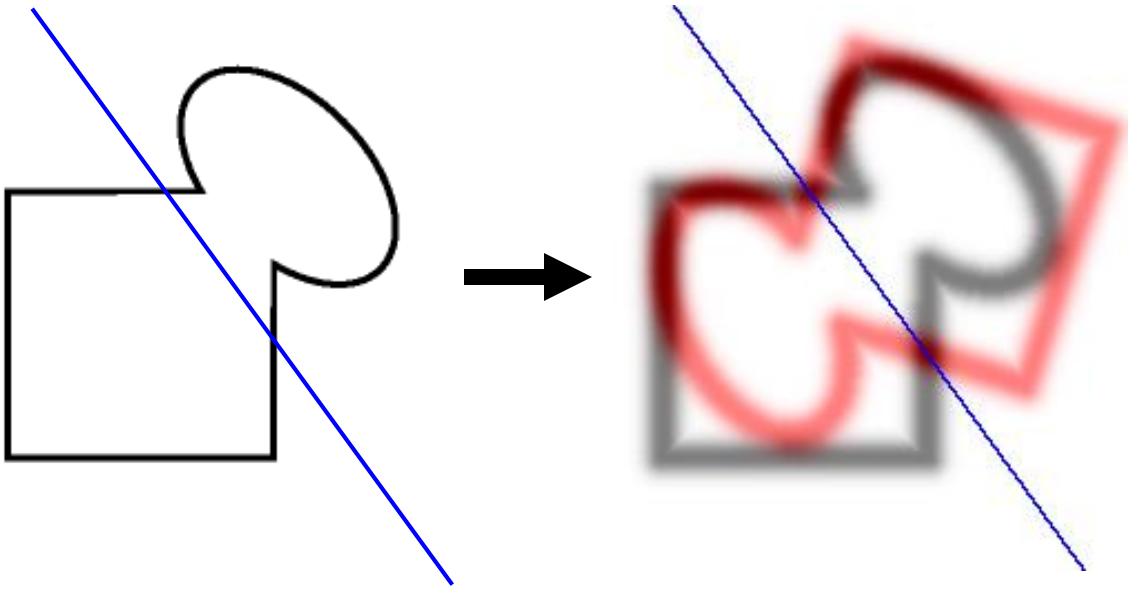


$$Sym(f,R) = \overbrace{r_1}^{\text{scale factor}} \; Sym(\;,R) + \overbrace{r_2}^{\text{scale factor}} \; Sym(\;,R) + \overbrace{r_3}^{\text{scale factor}} \; Sym(\;,R) + \ldots$$

ECCV, 2002. Kazhdan et al.

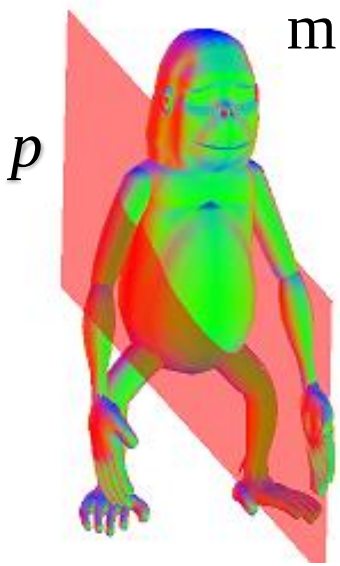
Symmetry Distance

For boundaries, can approximate symmetry distance by converting to a function and then computing correlation



Symmetry Distance

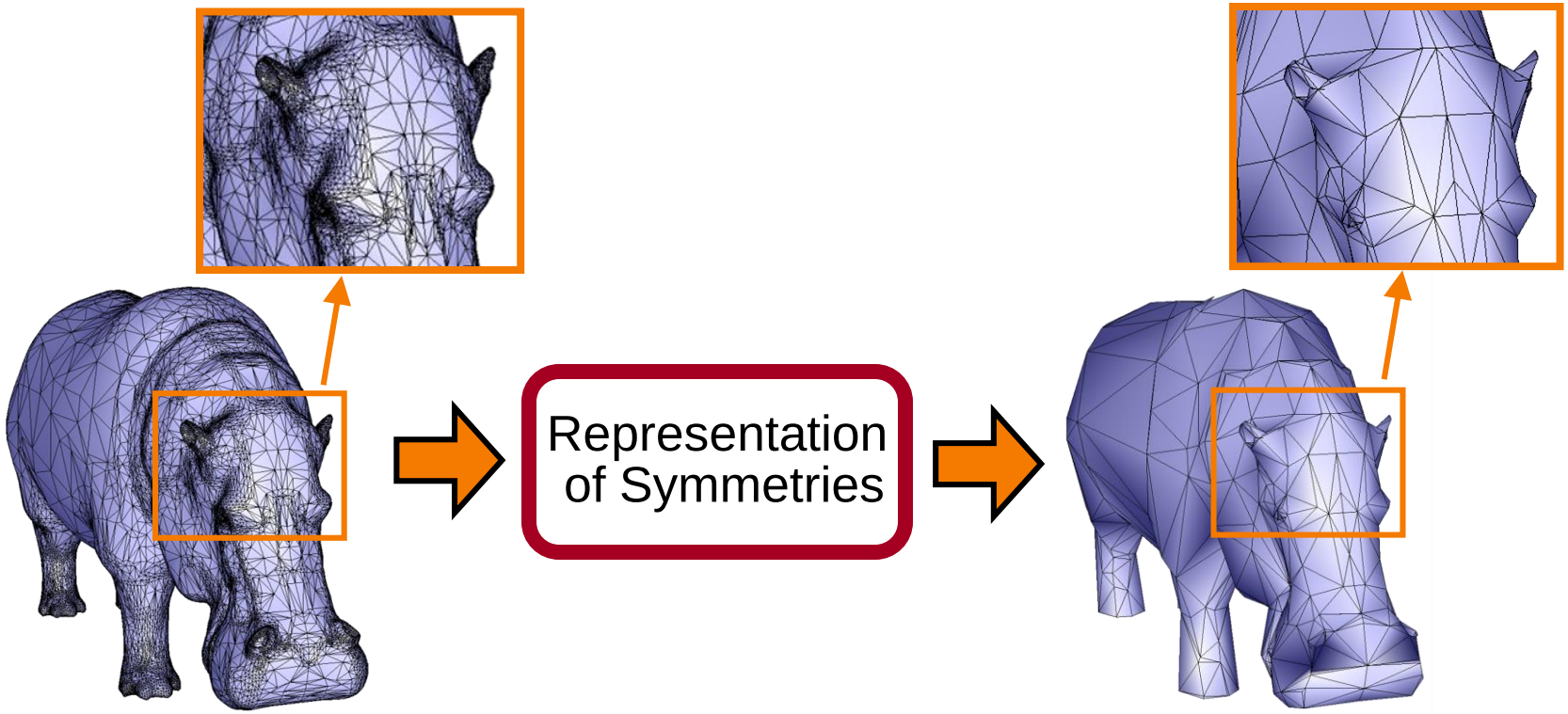
Can use this method to compute symmetry distance for any 3D mesh (without finding correspondences)



Sym(m,p) = 0.3

Back to the Goal …

We need methods to detect, analyze, **represent**, and exploit symmetries in 3D models



Symmetric Input Mesh

Symmetric Output Mesh

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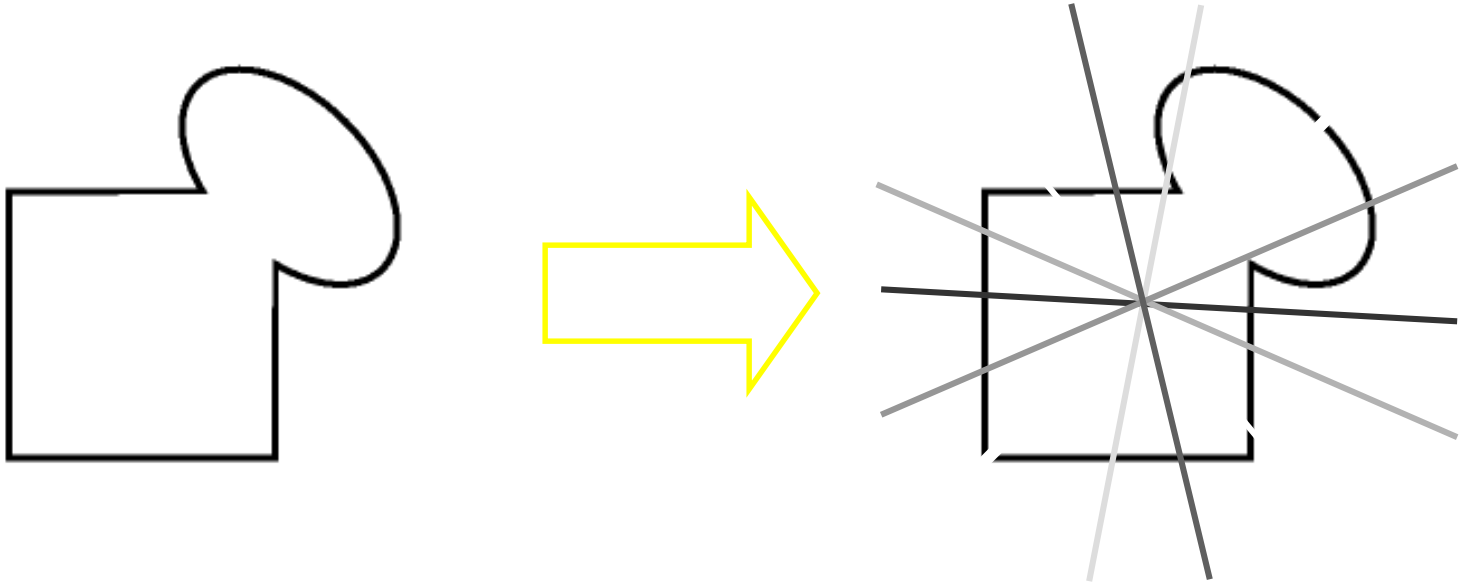
Symmetry Descriptor

Measure the symmetry of an object with respect to every transformation through its center



Input Shape

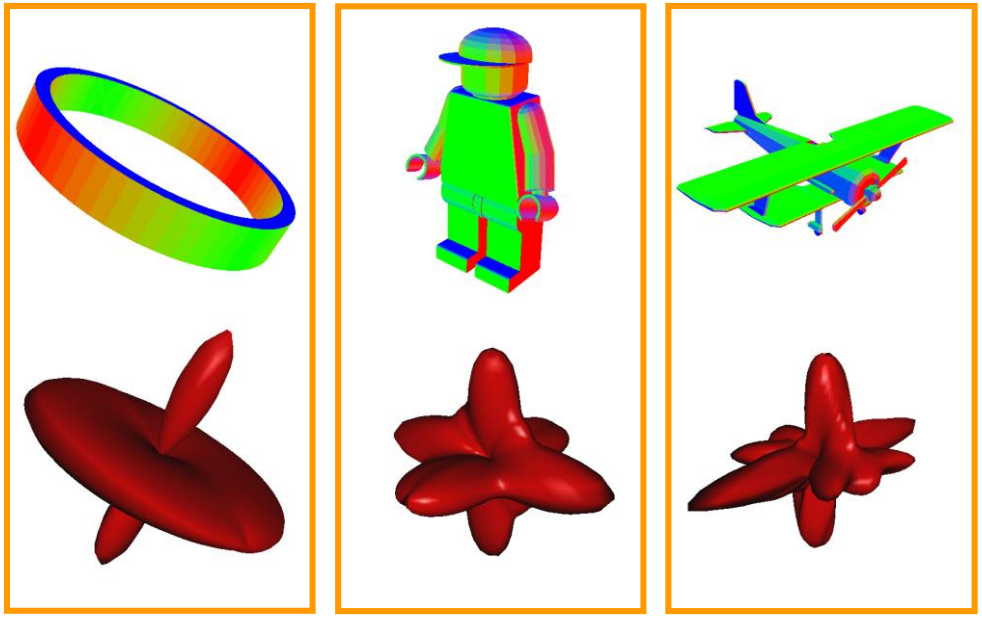
Planar Reflective Symmetry Descriptor

Symmetry Descriptor

Planar reflective symmetry descriptors:

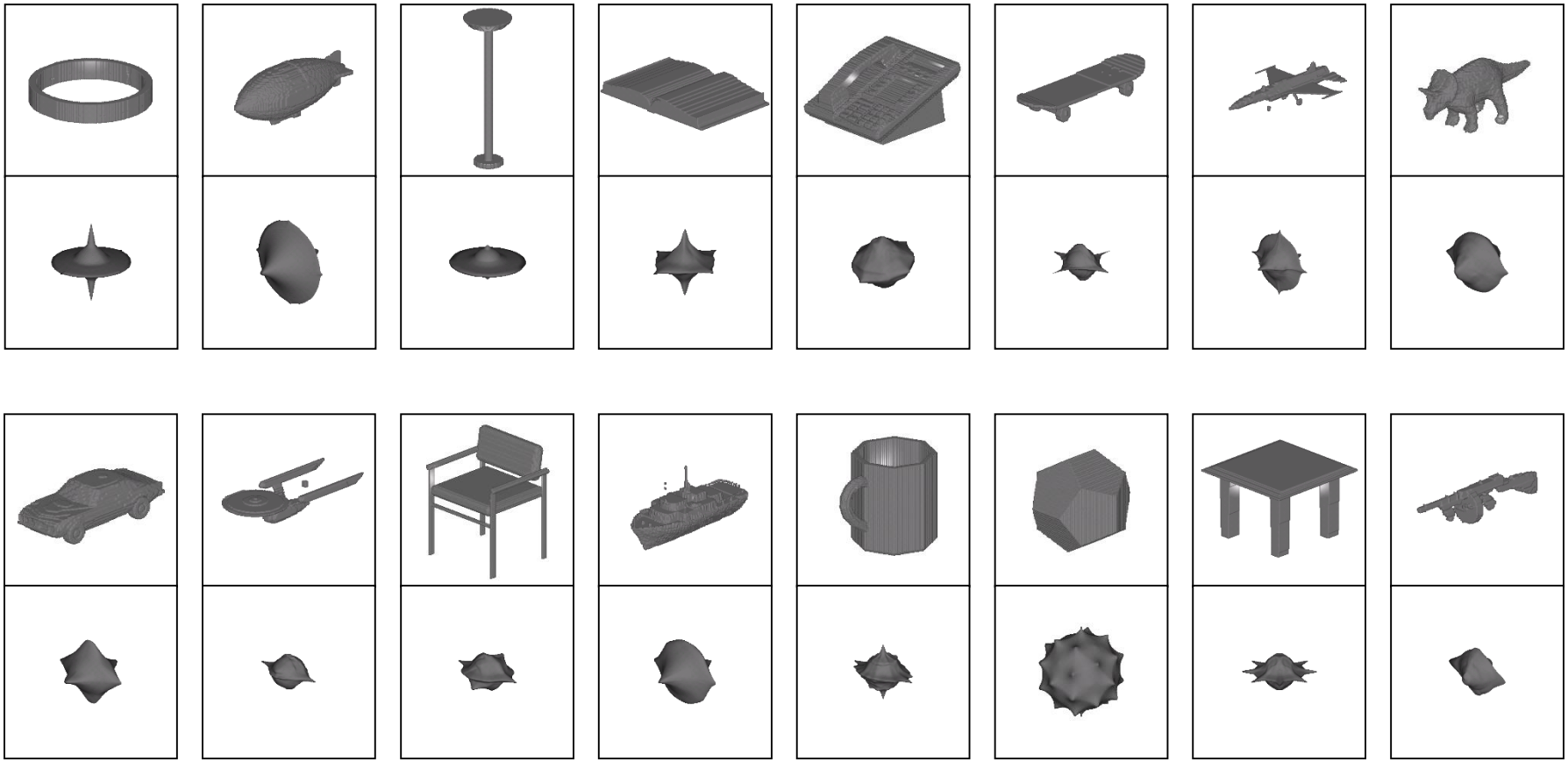
Input Mesh

Planar Reflective Symmetry Descriptor



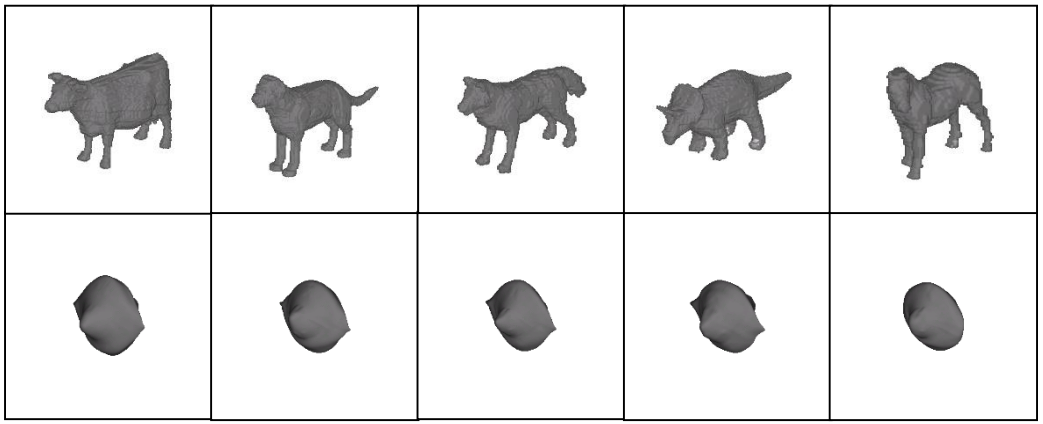
Symmetry Descriptor

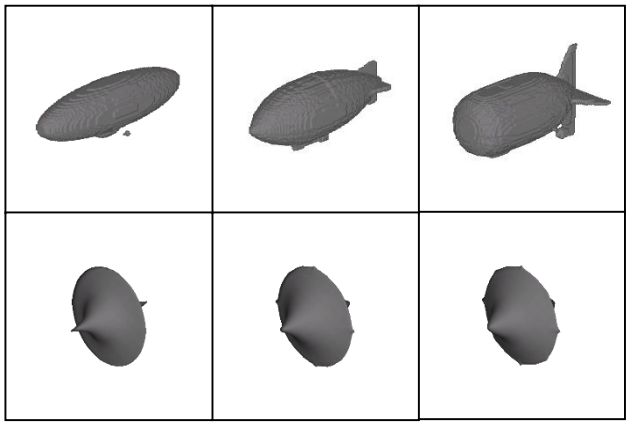
Planar reflective symmetry descriptors:

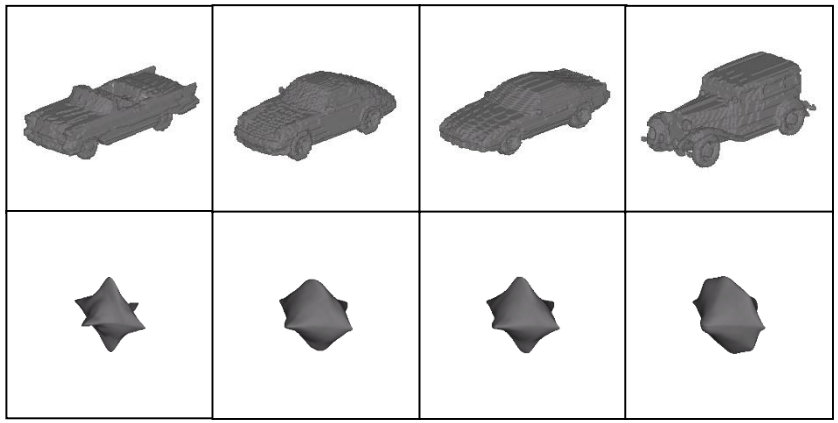


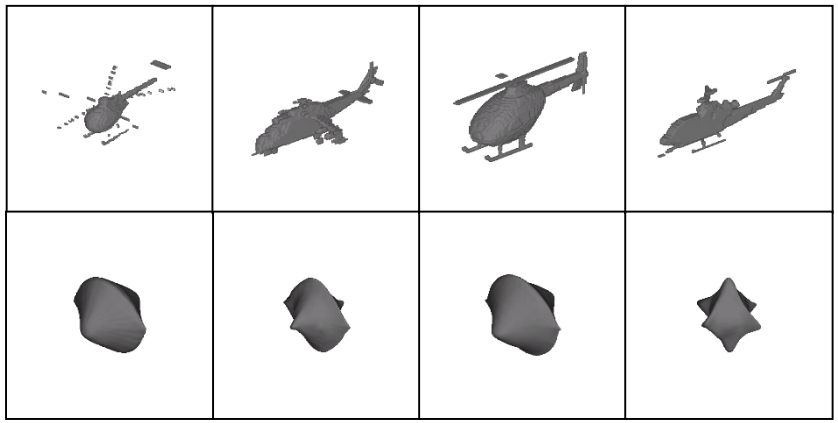
Symmetry Descriptor

Planar reflective symmetry descriptors:



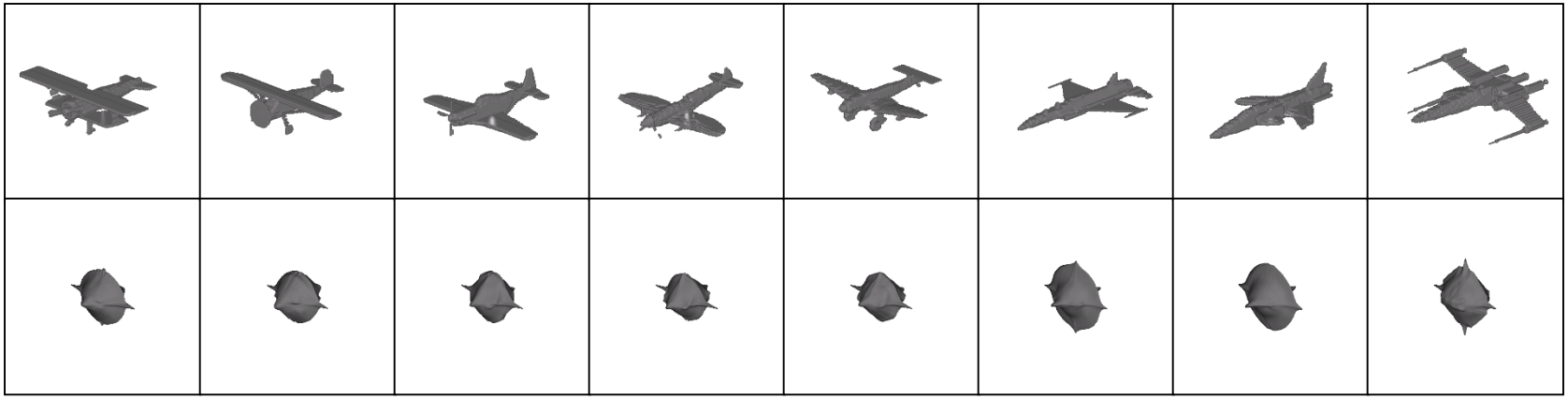


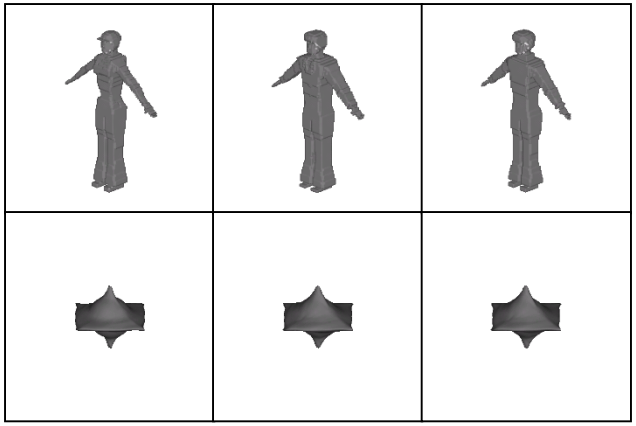


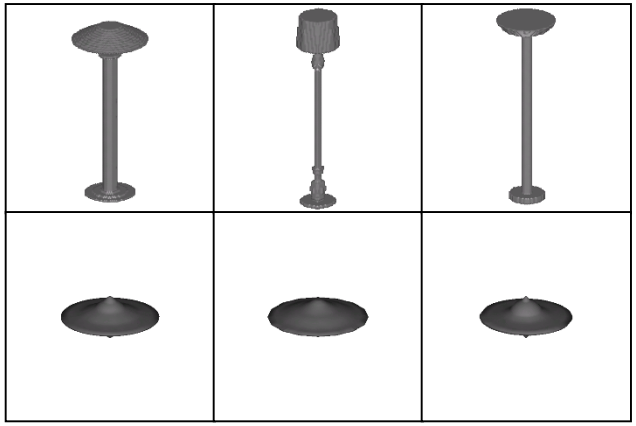


Symmetry Descriptor

Planar reflective symmetry descriptors:

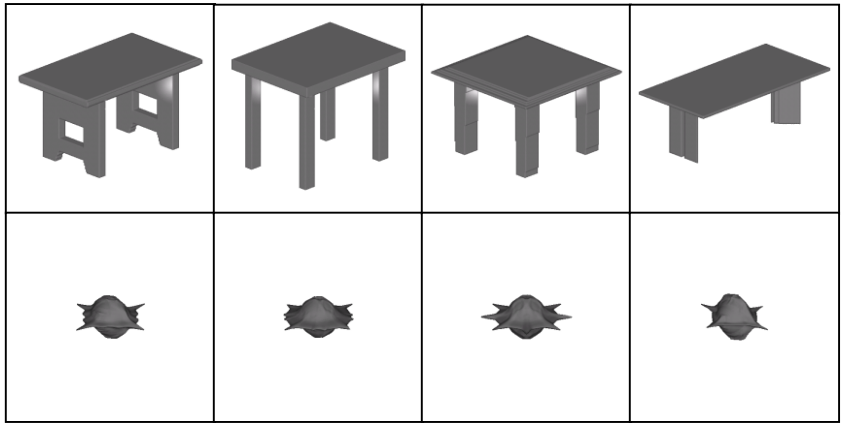


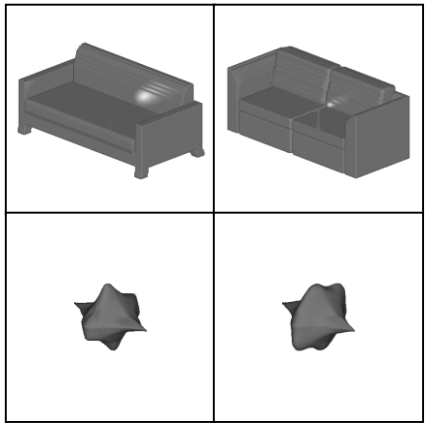




Symmetry Descriptor

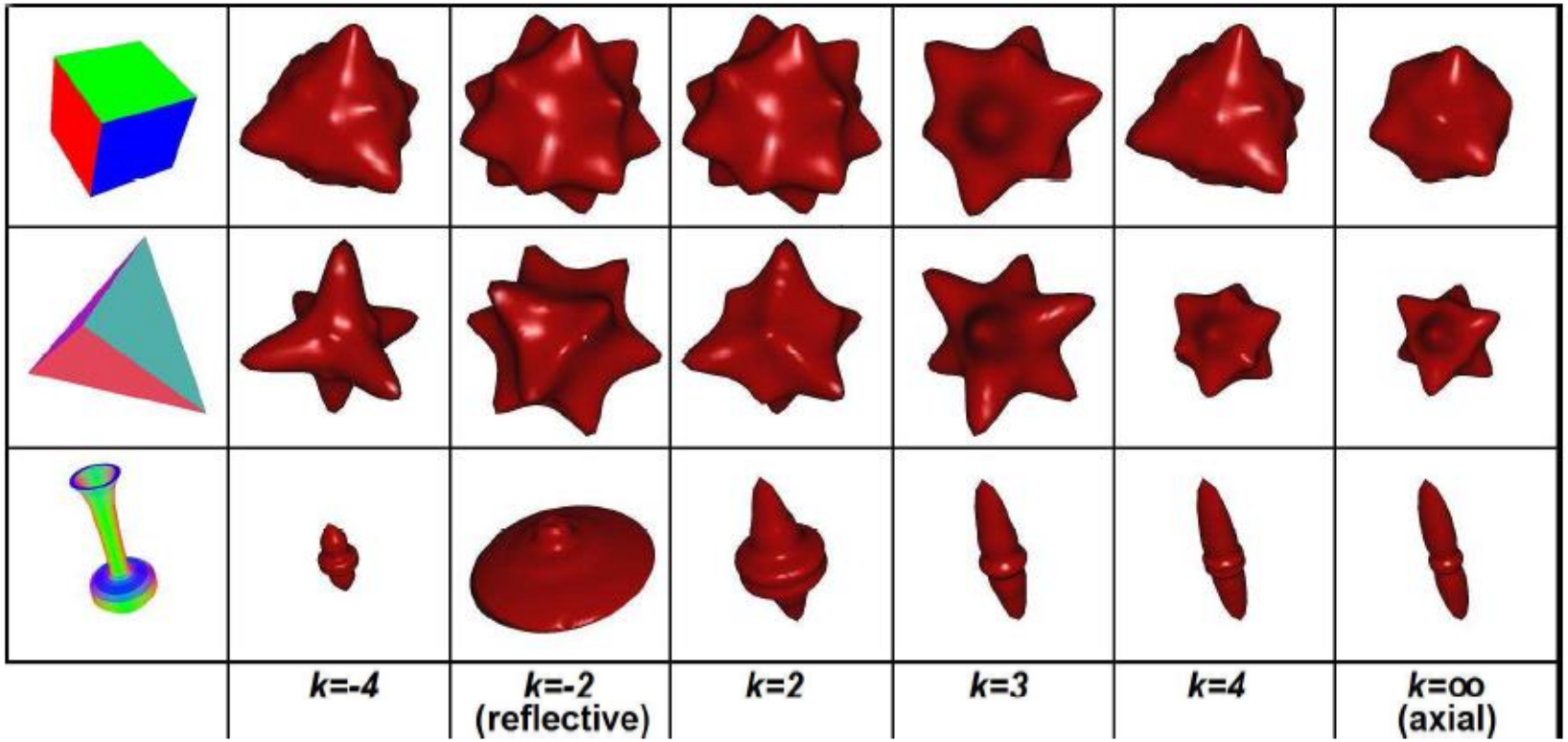
Planar reflective symmetry descriptors:





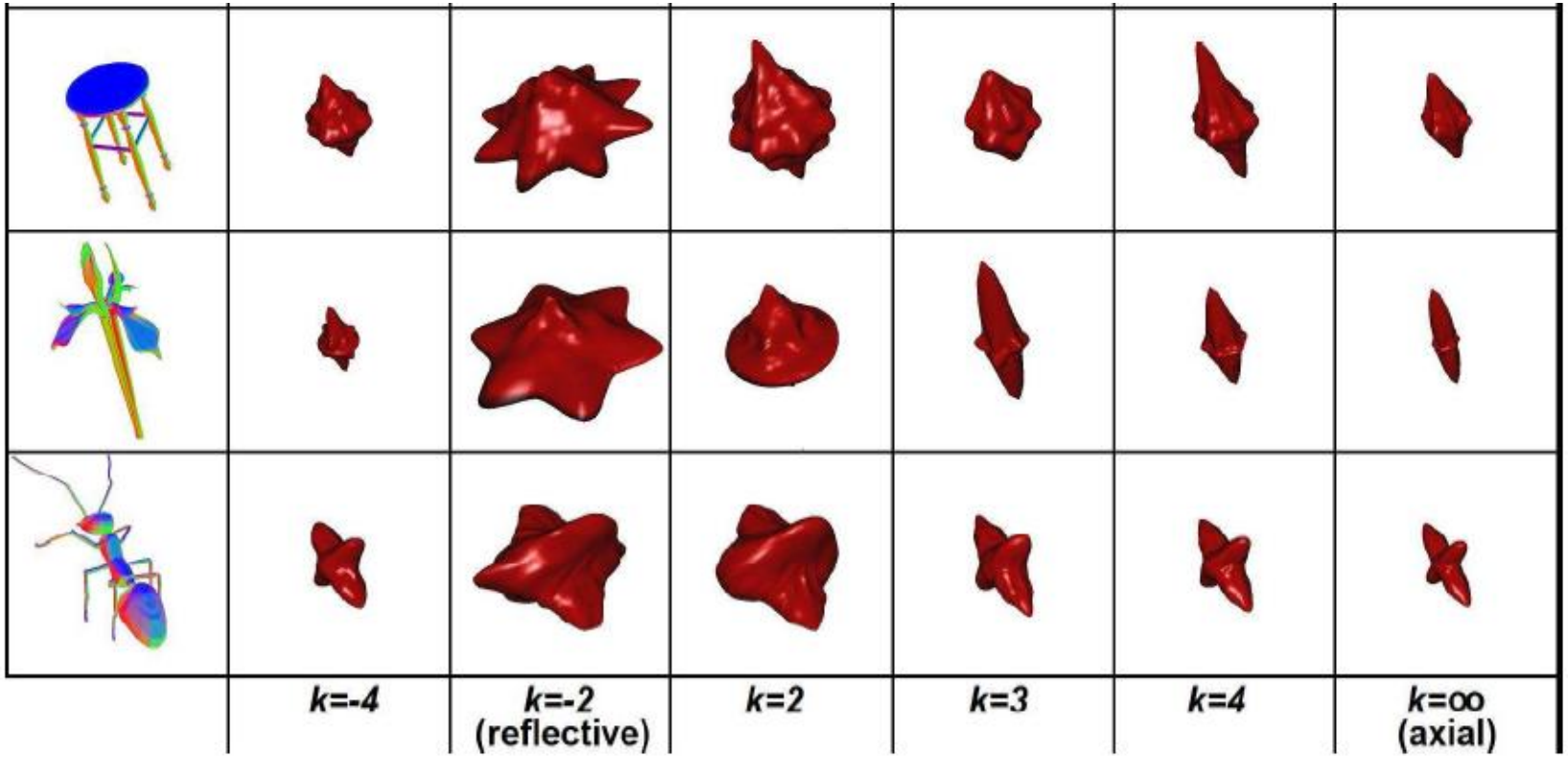
Symmetry Descriptor

Rotational symmetry descriptors:



Symmetry Descriptor

Rotational symmetry descriptors:



Symmetry Descriptor

Properties:

- Canonical parameterization
  - Parameterized over the (projective) sphere
- Insensitive to noise:
  - Integration scales down high frequency Fourier coefficients
- Global:
  - For functions *f* and *g*, and any reflection *r:*

$$\left| Sym(f,r) - Sym(g,r) \right| \leq \left\| f - g \right\|_2$$

Outline

Introduction

Background

Symmetry representations

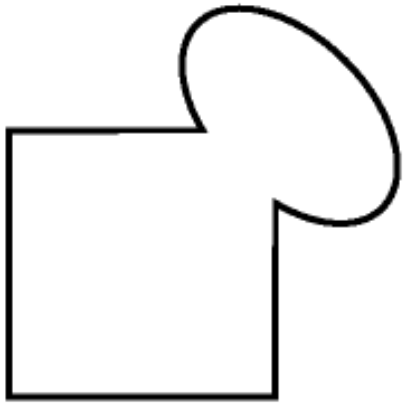
- Symmetry descriptor
- Symmetry transform ←
- Principal symmetries

Applications

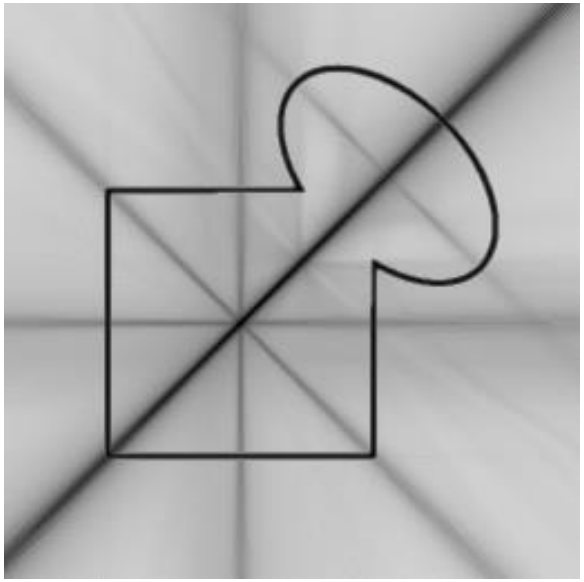
Conclusion

Symmetry Transform

Measure the symmetry of an object with respect to all transformations



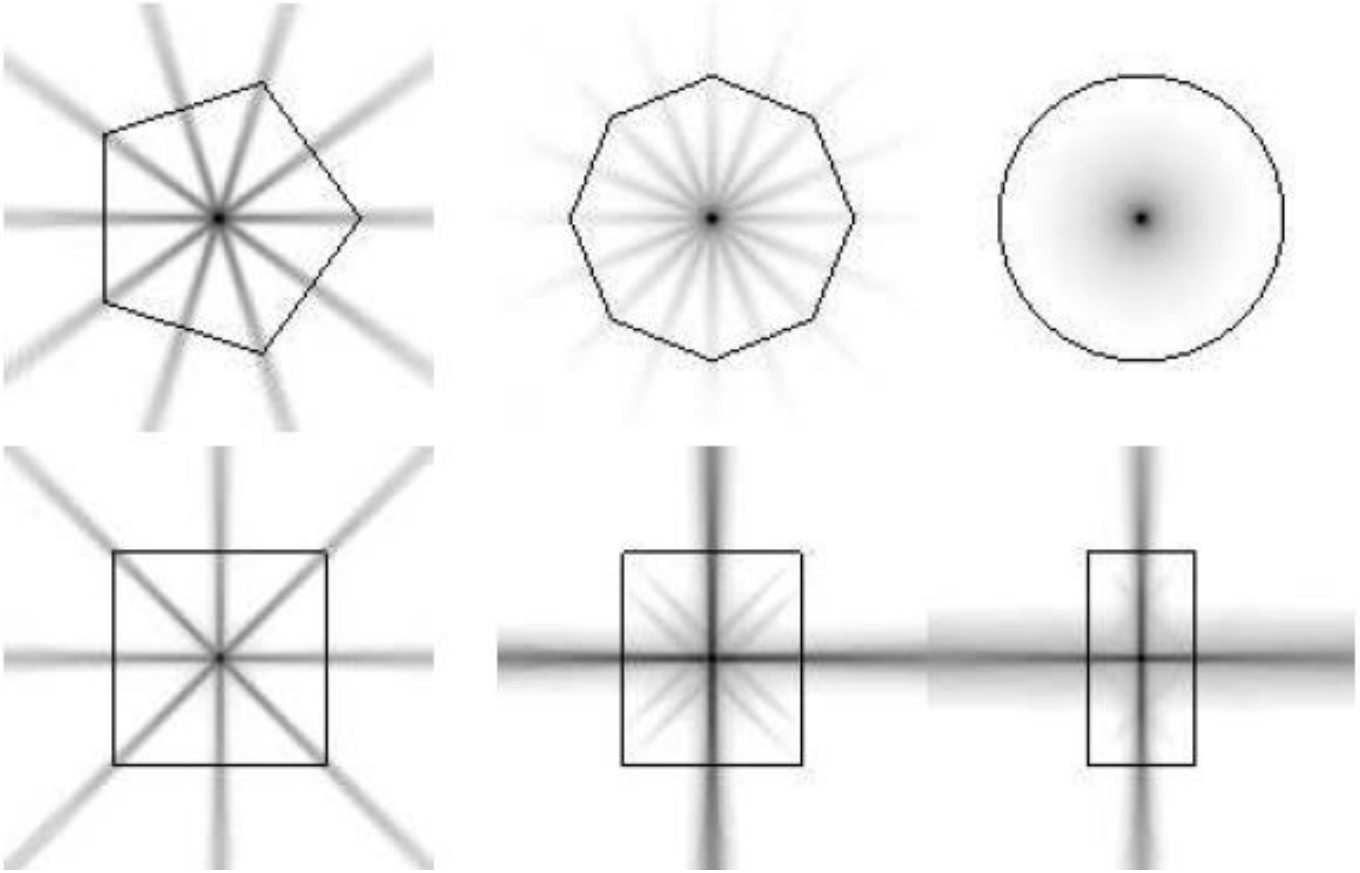
Input Shape



Planar Reflective Symmetry Transform

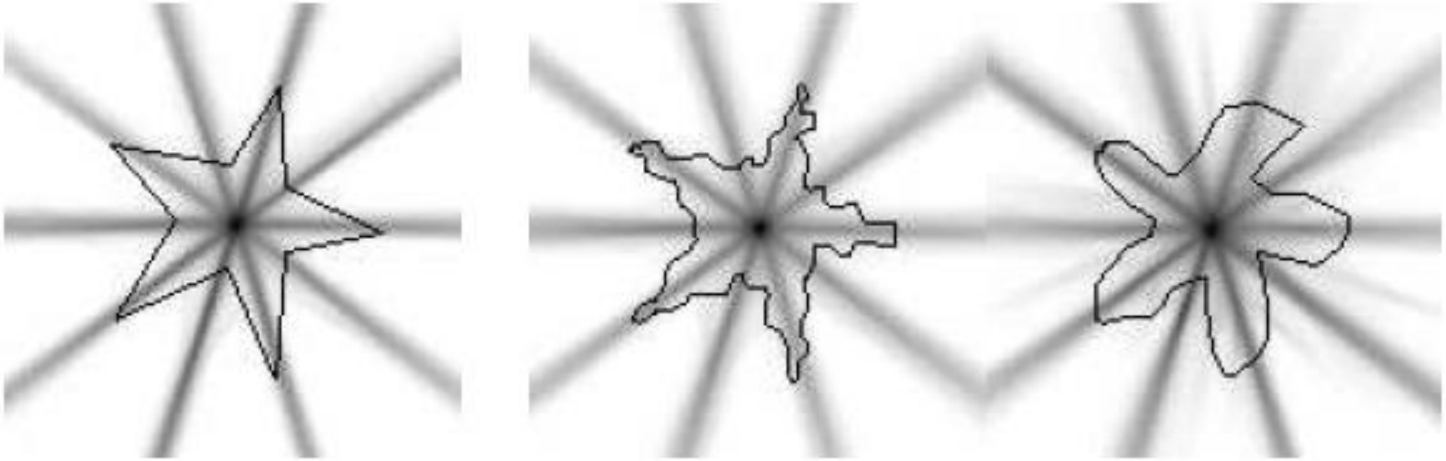
Symmetry Transform

Planar reflective symmetry transform:



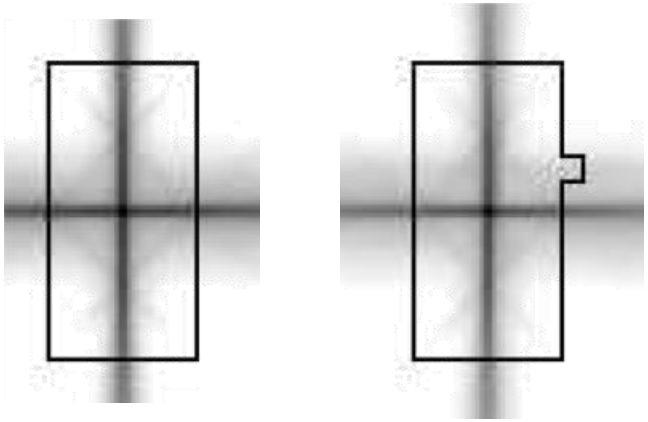
Symmetry Transform

Stability with noise:

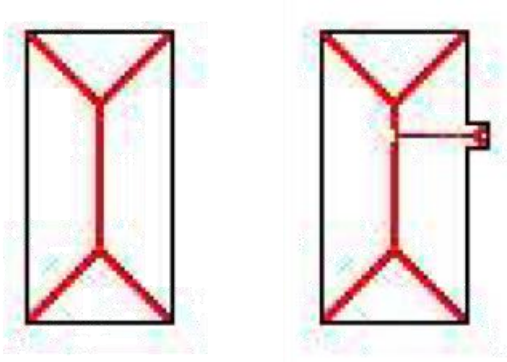


Symmetry Transform

Stability with small extra features:



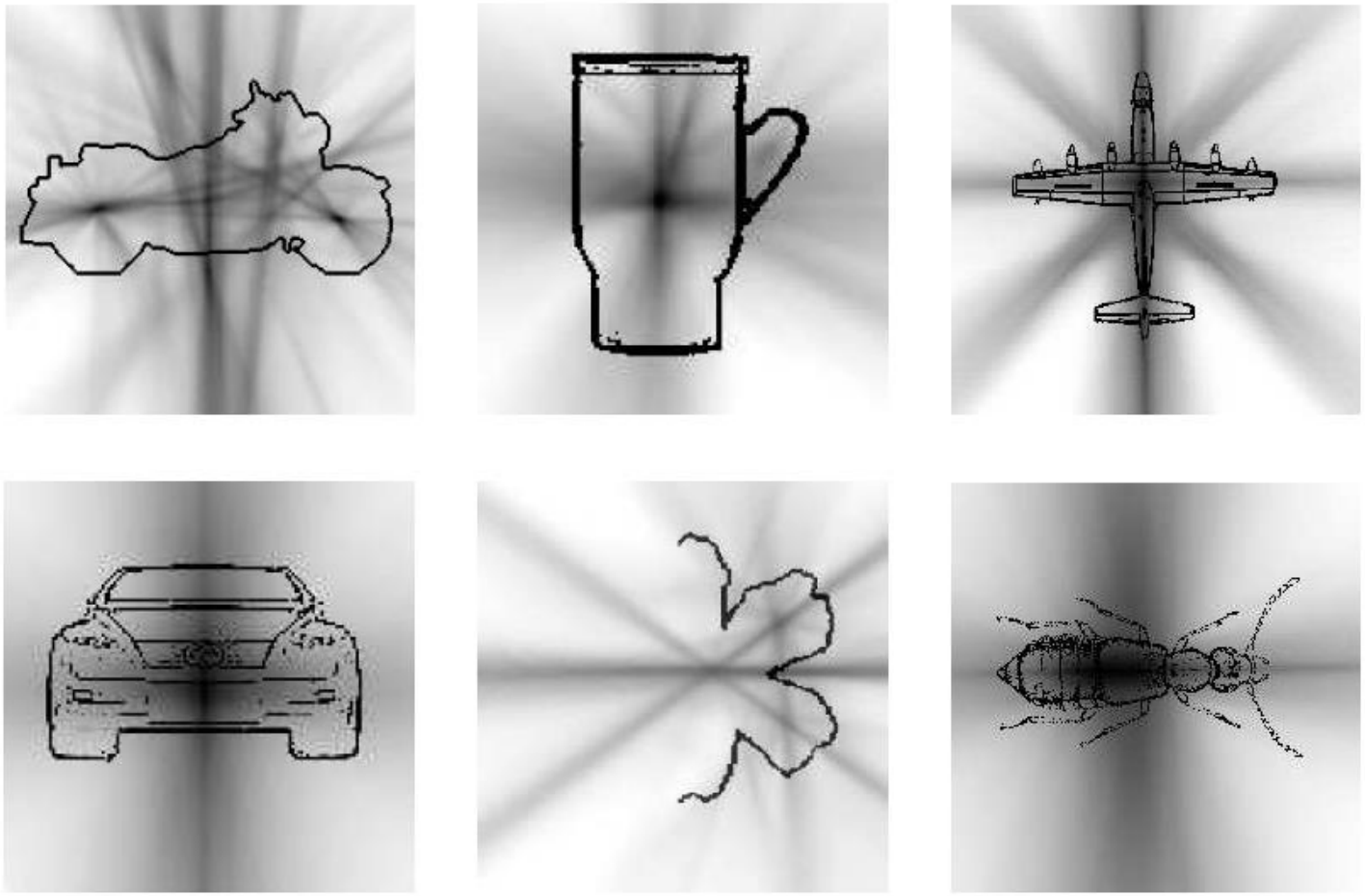
Planar Reflective Symmetry Transform



Medial Axis Transform

Symmetry Transform

Highlights large symmetric features of shape:



Symmetry Transform

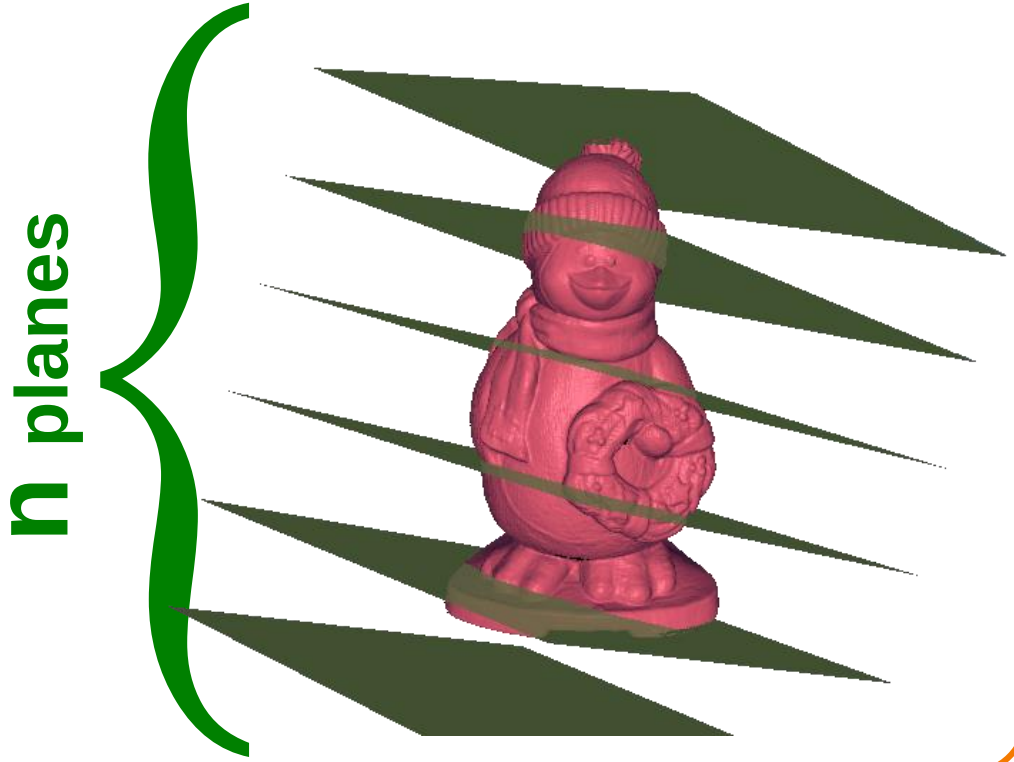
Computation:

- Brute Force: $O(n^6)$

$O(n^3)$ planes

X = $O(n^6)$

$O(n^3)$ correlation



Symmetry Transform

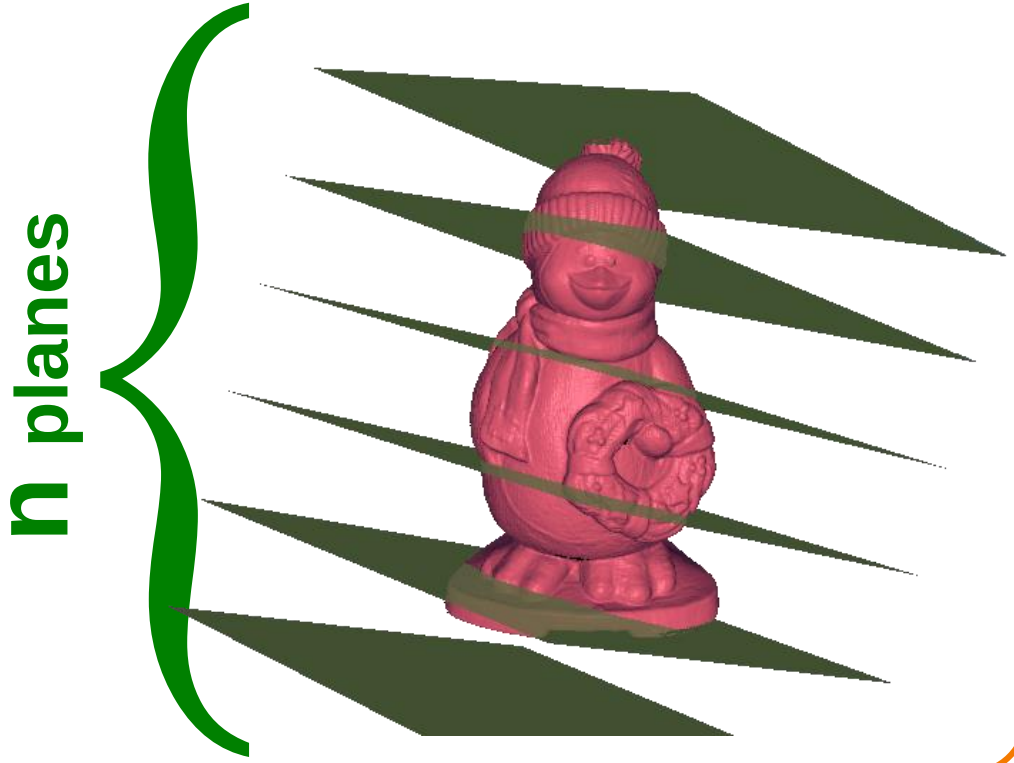
Computation:

- Brute Force: $O(n^6)$
- ➢ Convolution: $O(n^5 \log n)$

$O(n^2)$ normal directions

X = $O(n^5 \log n)$

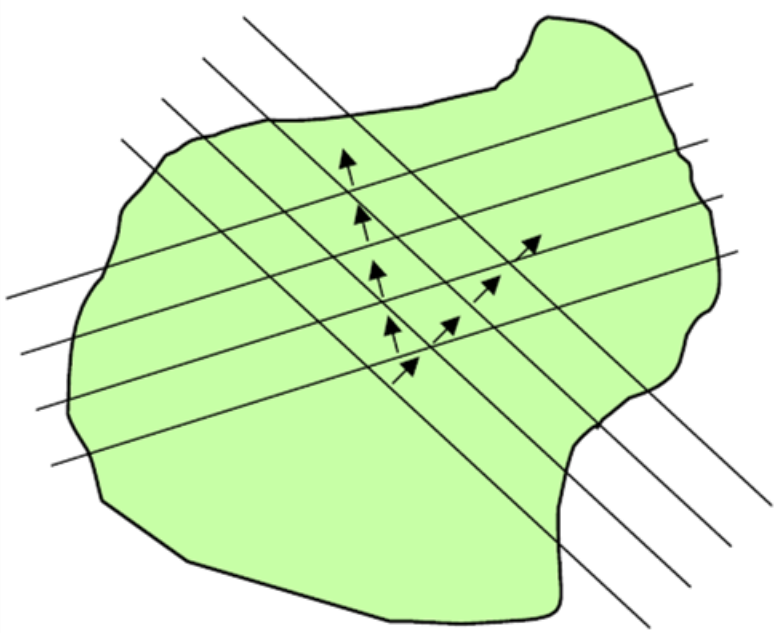
$O(n^3 \log n)$ per direction



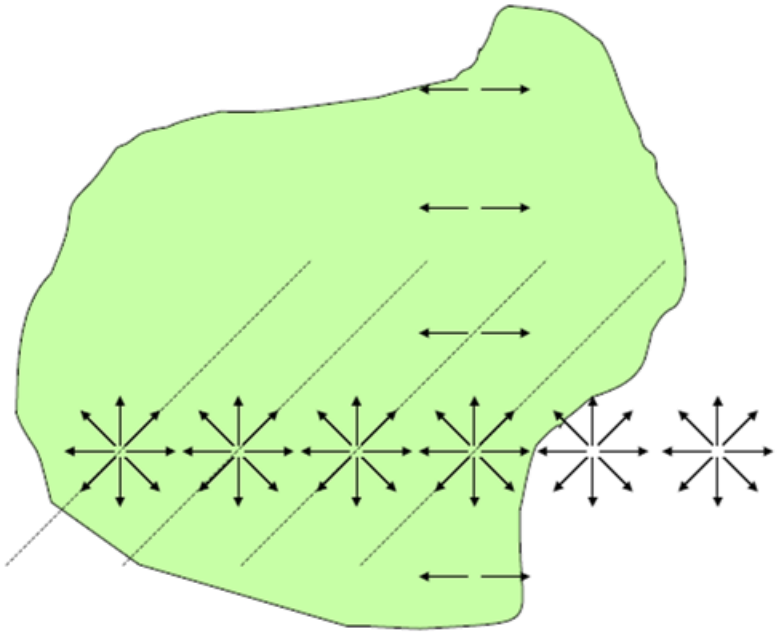
Symmetry Transform

Computation:

- Brute Force: $O(n^6)$
- ➢ Convolution: $O(n^5 \log n)$



Translate planes



Rotate planes

Symmetry Transform

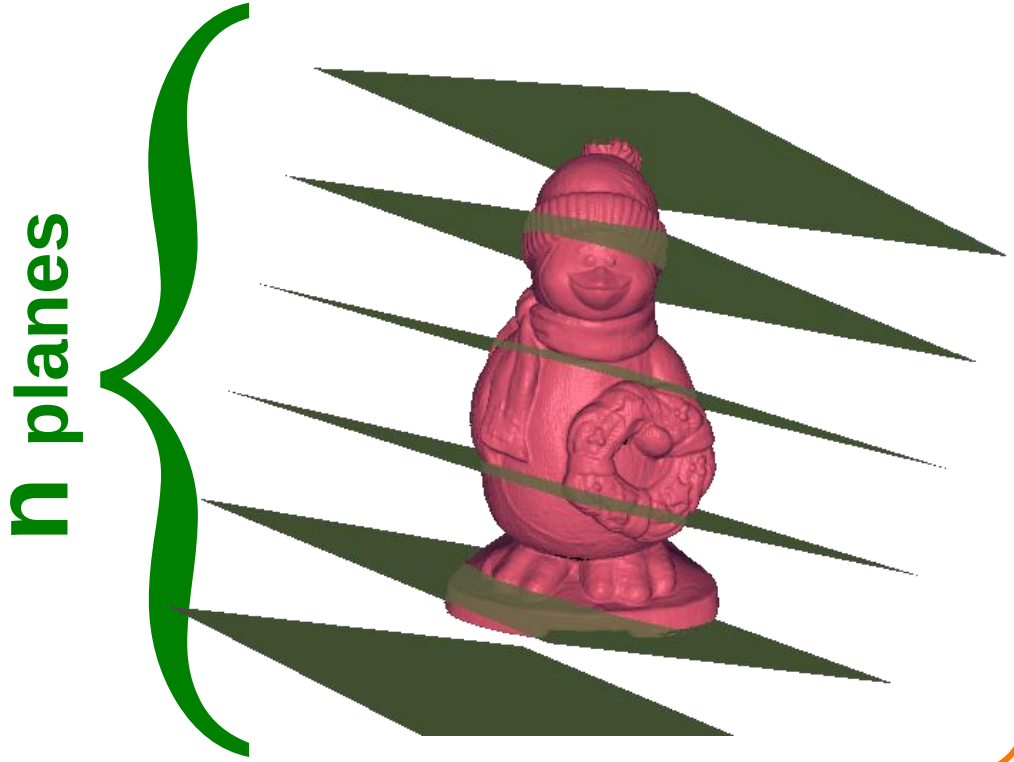
Computation:

- Brute Force: $O(n^6)$
- Convolution: $O(n^5 \log n)$
- ➢ Monte Carlo: $O(n^4)$ for surfaces

$O(n^2)$ surface points

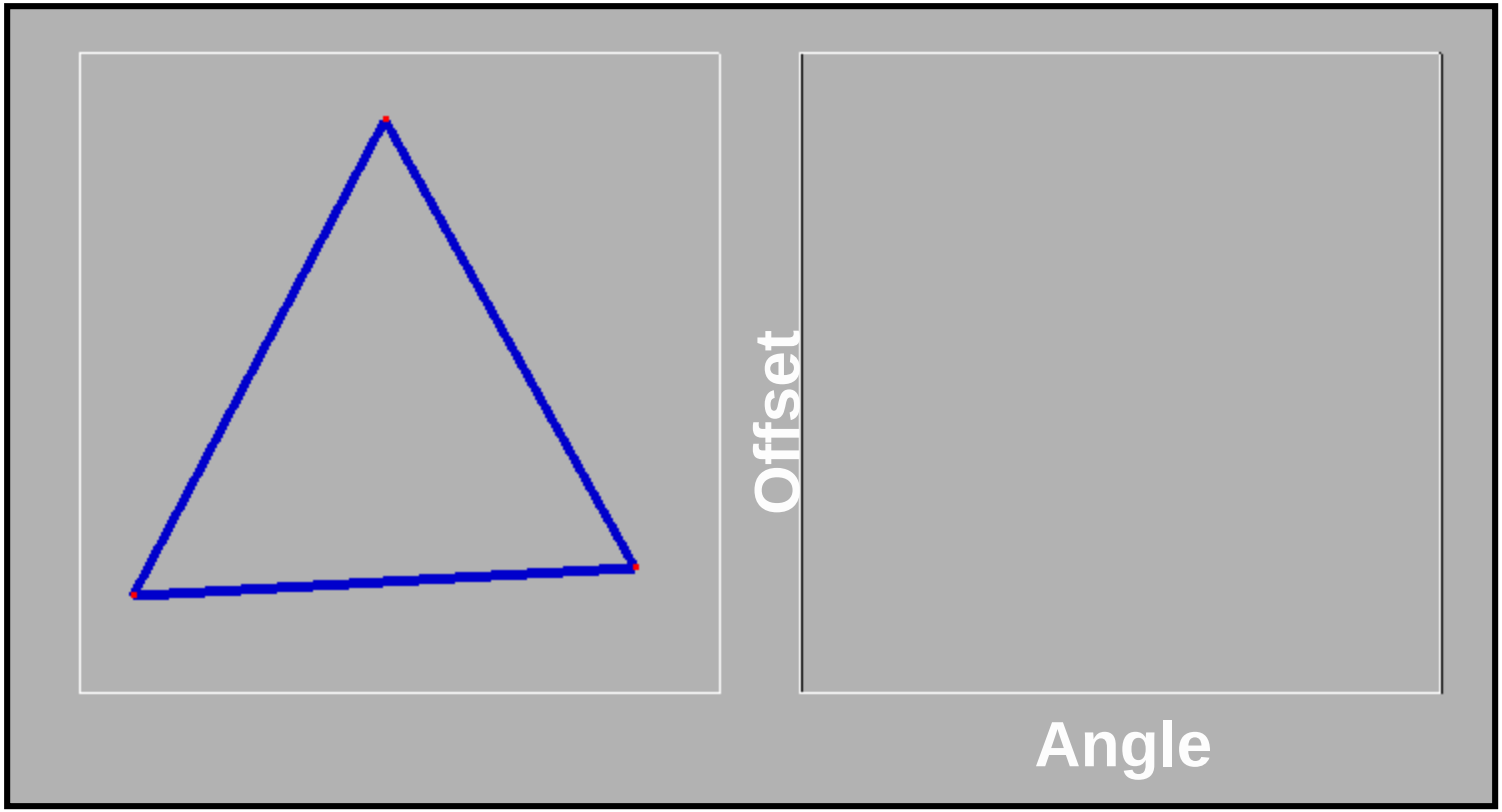
X = $O(n^4)$

$O(n^2)$ pairs



Symmetry Transform

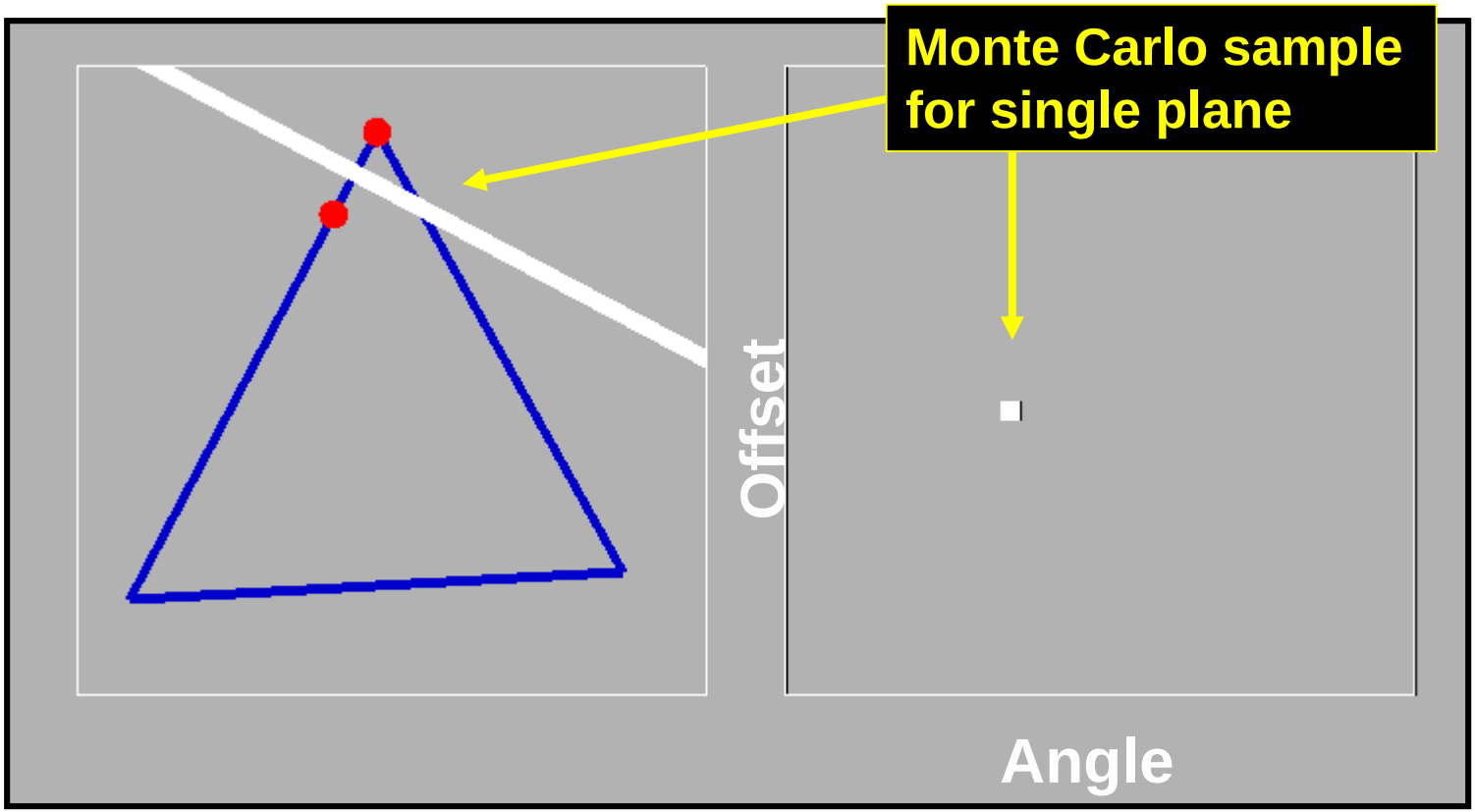
Monte Carlo algorithm:



Input Model **Symmetry Transform**

Symmetry Transform

Monte Carlo algorithm:



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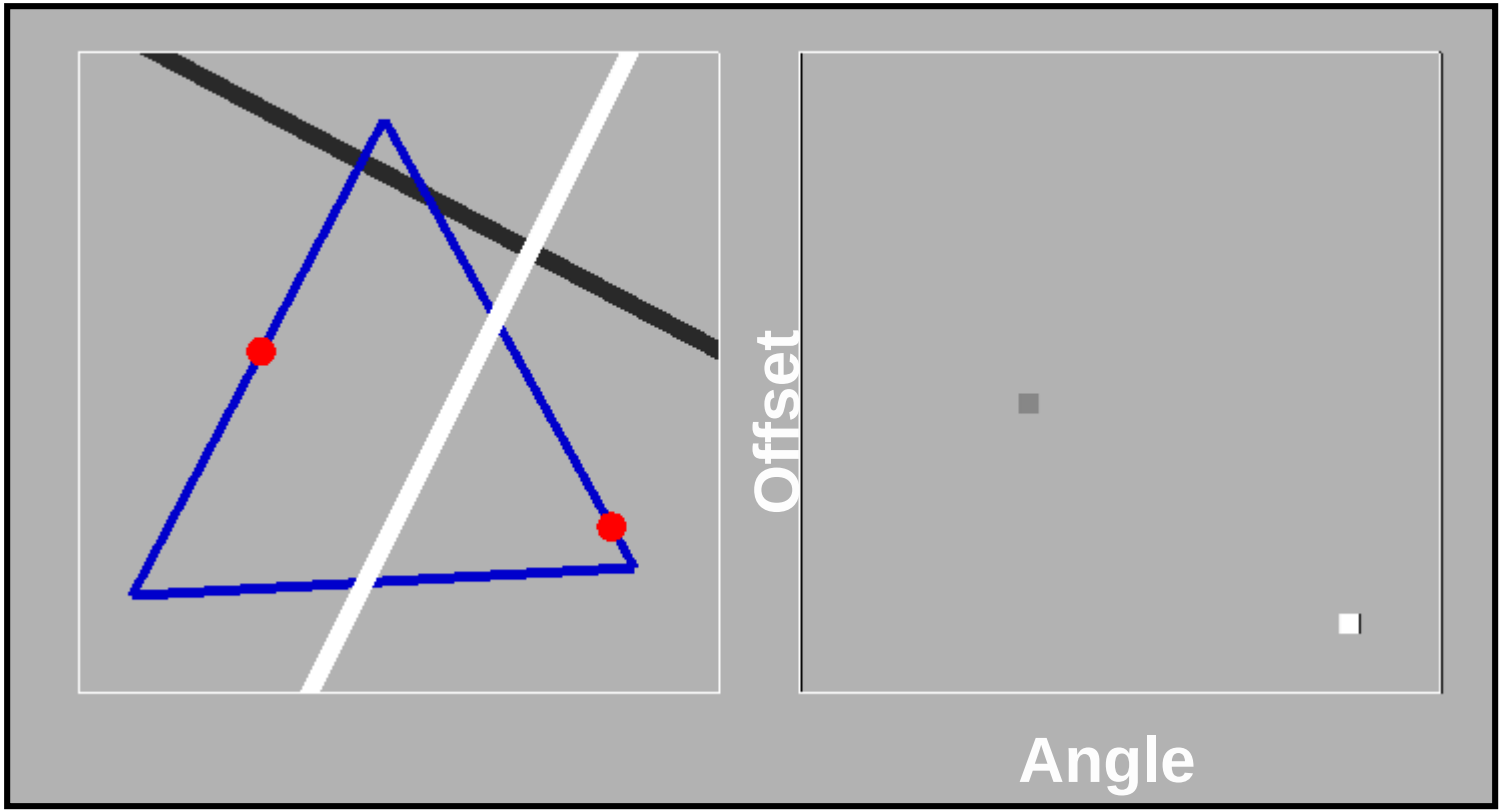

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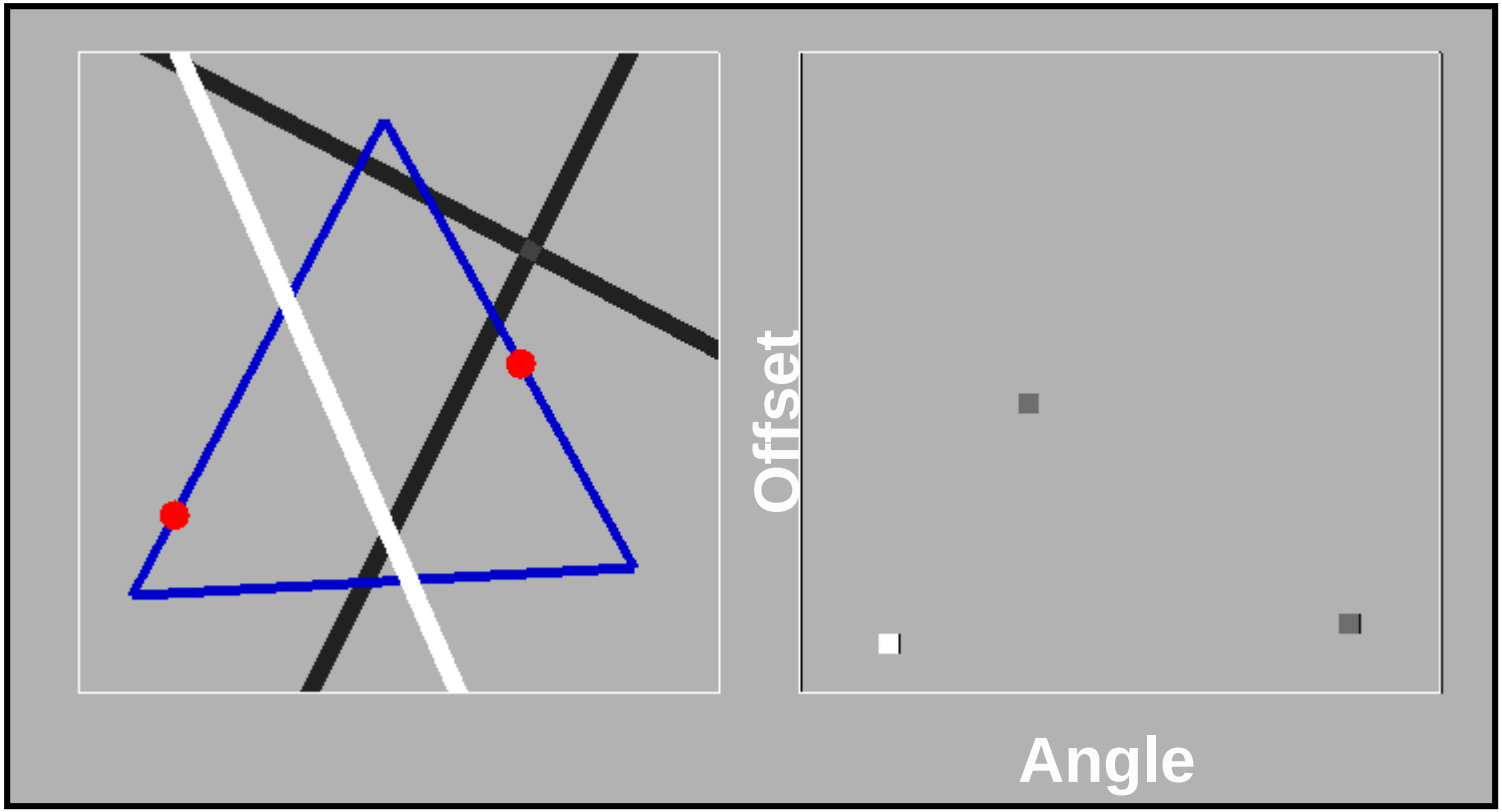
Monte Carlo algorithm:



Input Model **Symmetry Transform**

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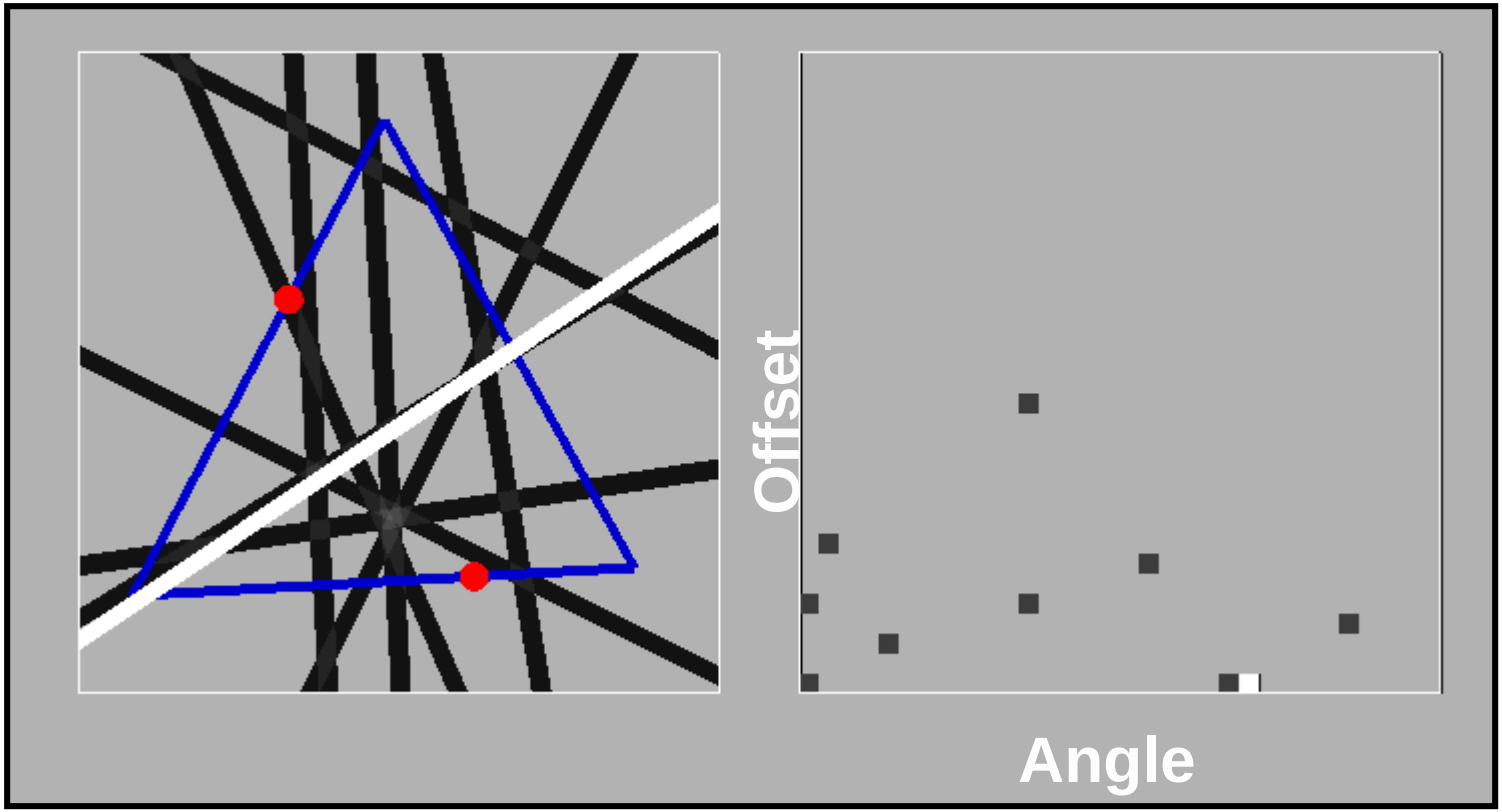
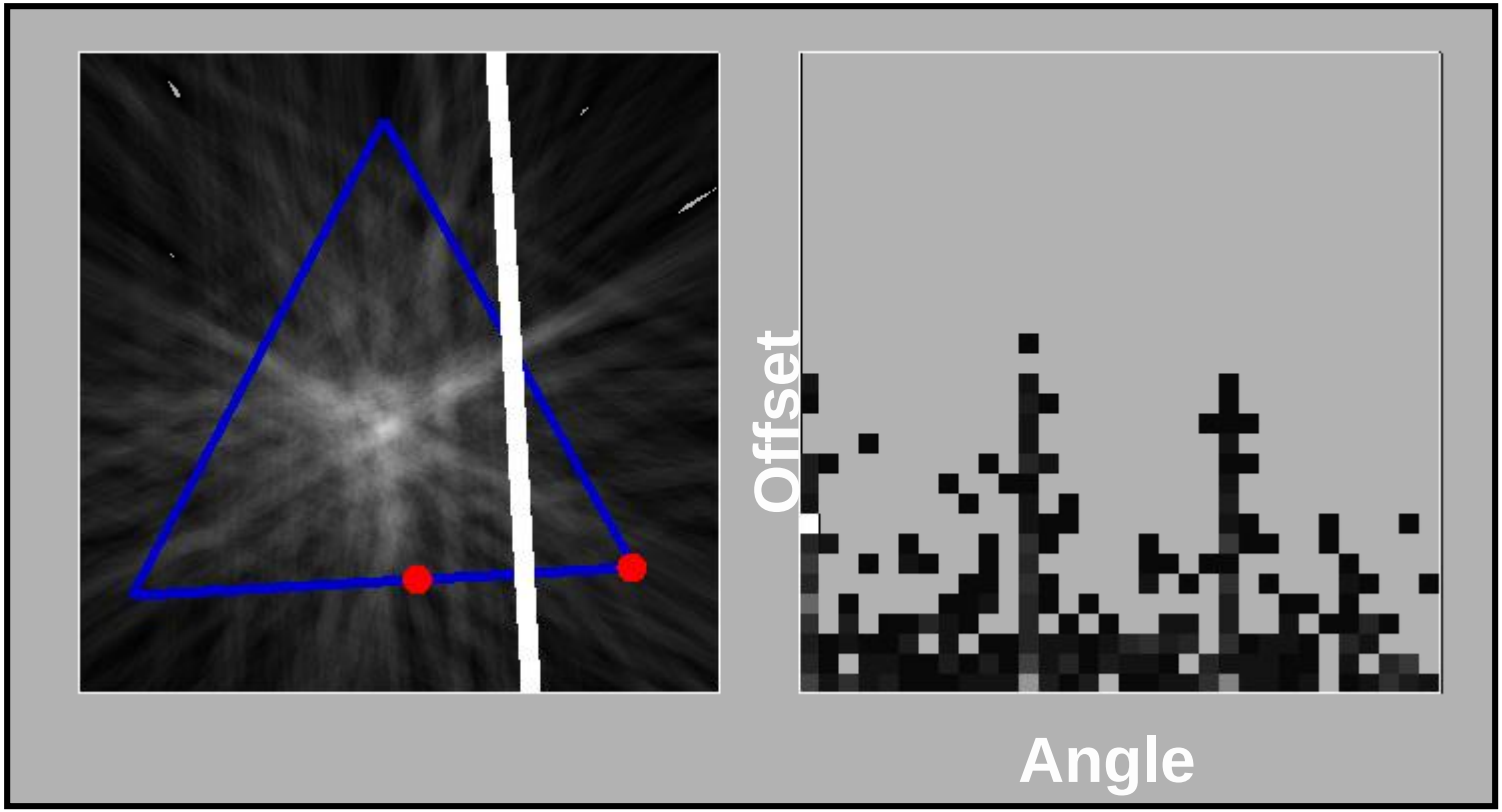
Monte Carlo algorithm:



Input Model **Symmetry Transform**

Symmetry Transform

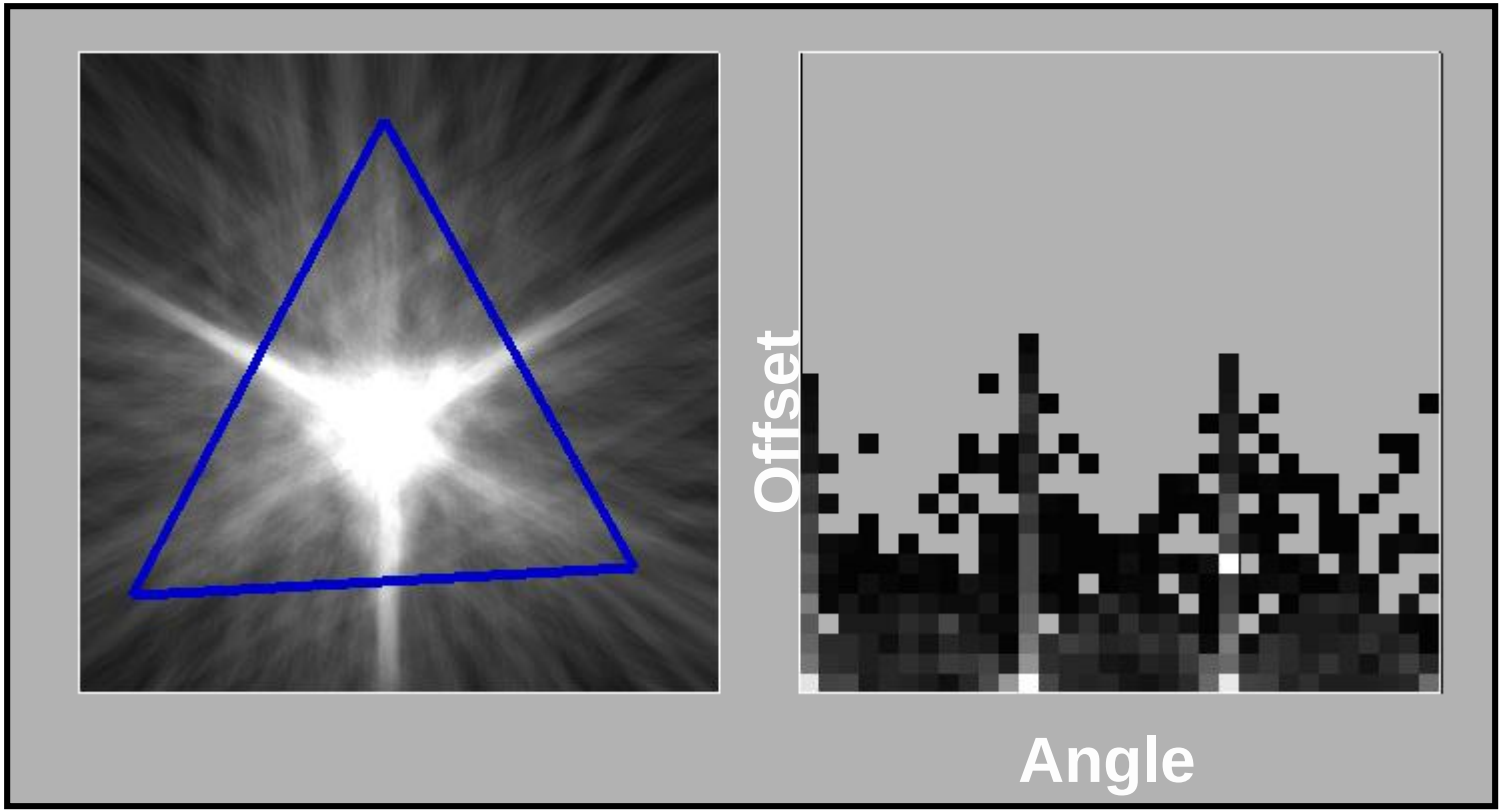
Monte Carlo algorithm:



Input Model **Symmetry Transform**

Symmetry Transform

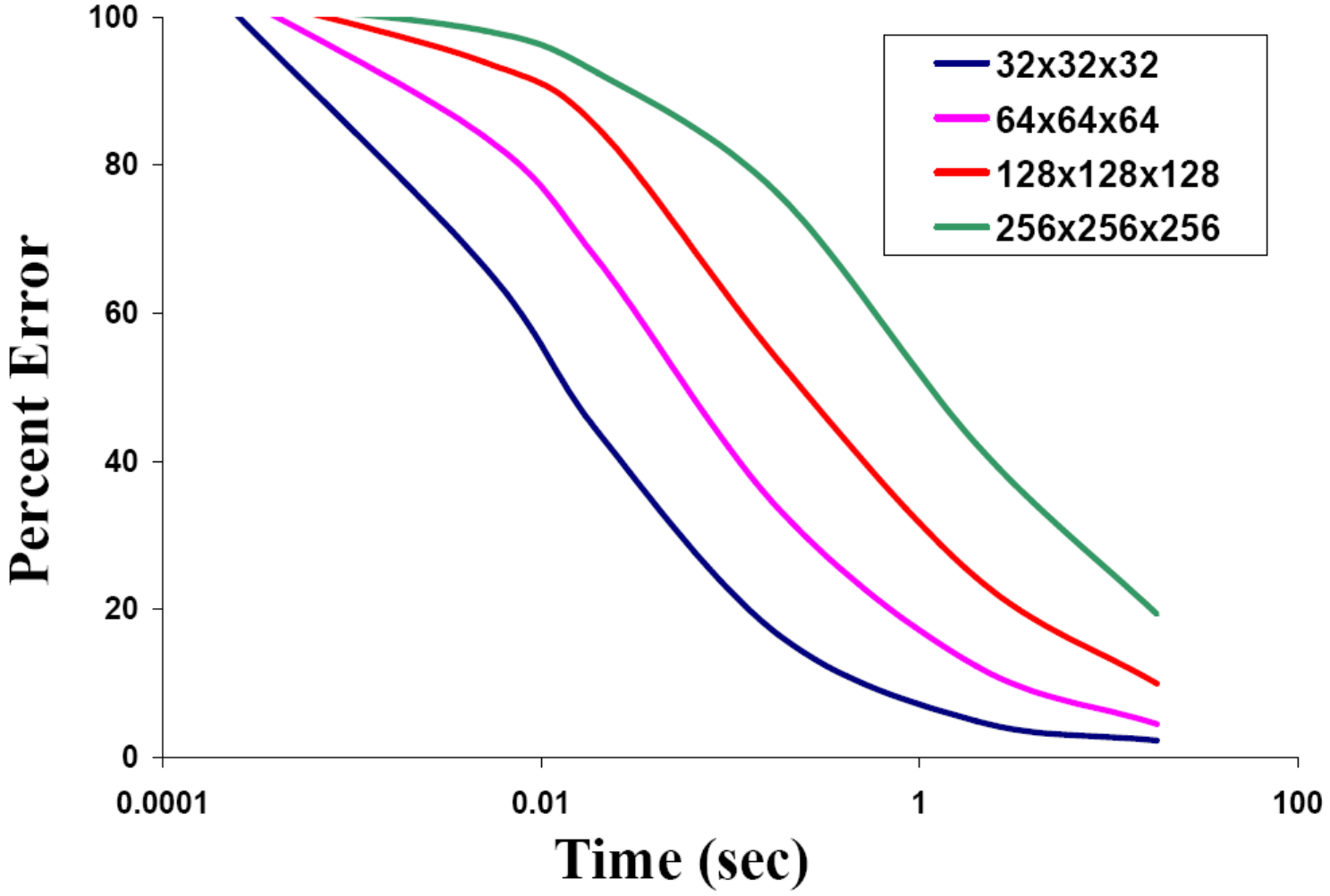
Monte Carlo algorithm:



Input Model **Symmetry Transform**

Symmetry Transform

Compute time:



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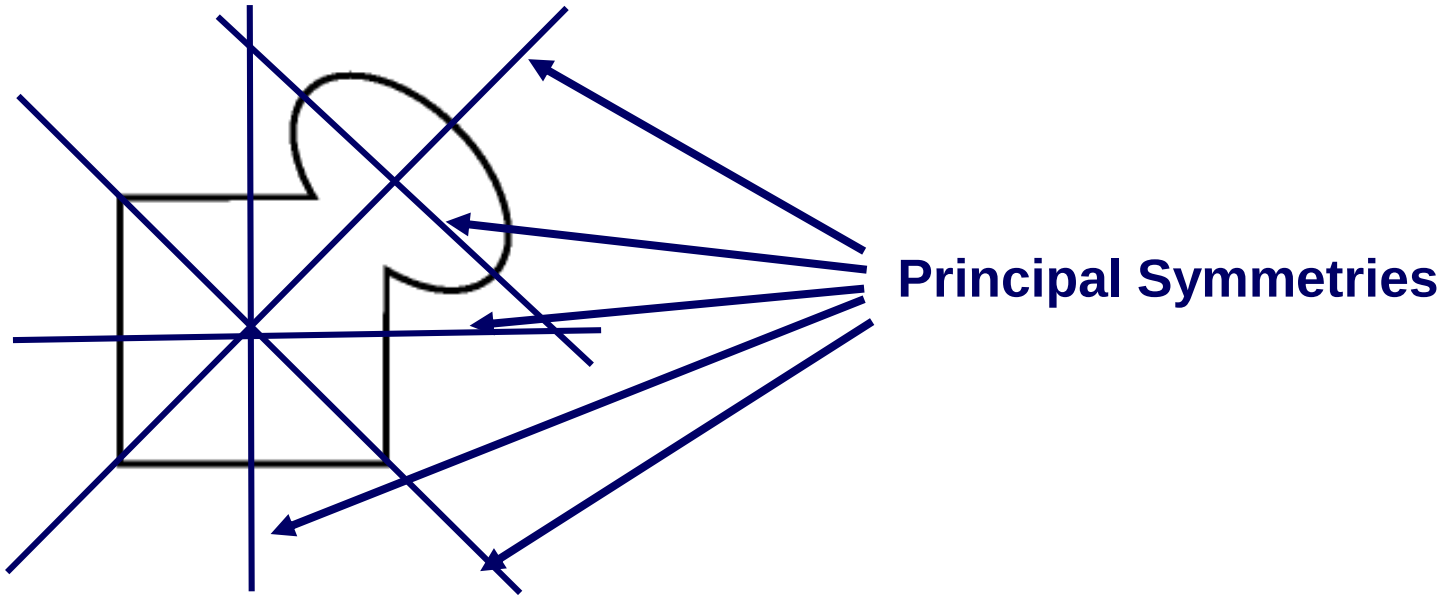
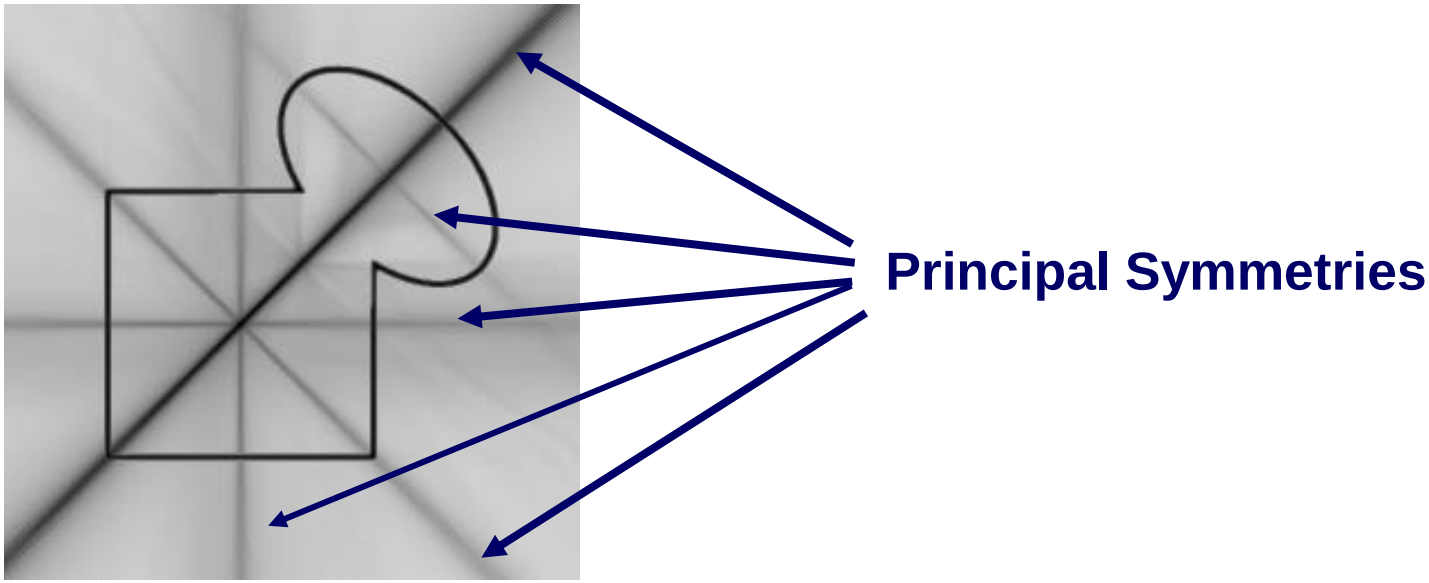
Principal Symmetries

Motivation: finding and representing significant symmetries is sufficient for many applications



Principal Symmetries

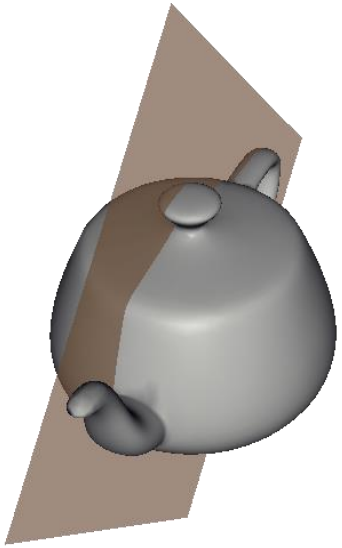
Observation: significant symmetries are usually local maxima of symmetry transformation



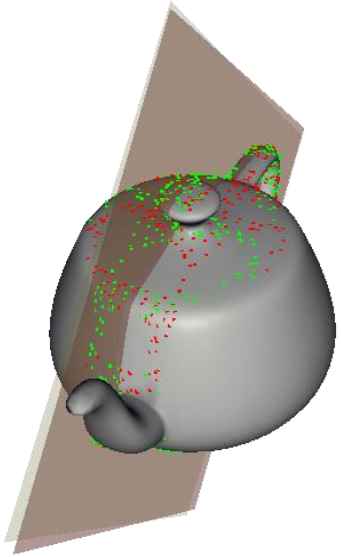
Principal Symmetries

Computing local maxima precisely:

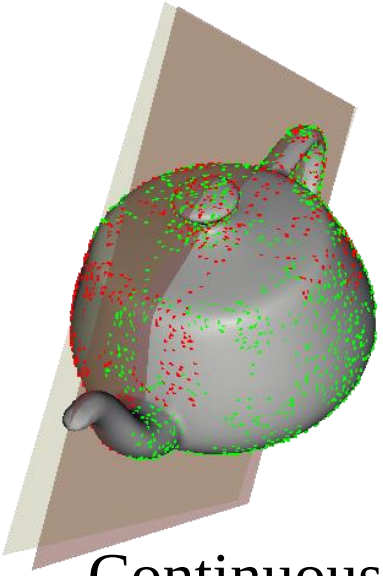
- Start from local maxima of discrete transform
- Establish closest point correspondences
- Refine transformation
- Iterate until find local maxima



Discrete
Local Maximum



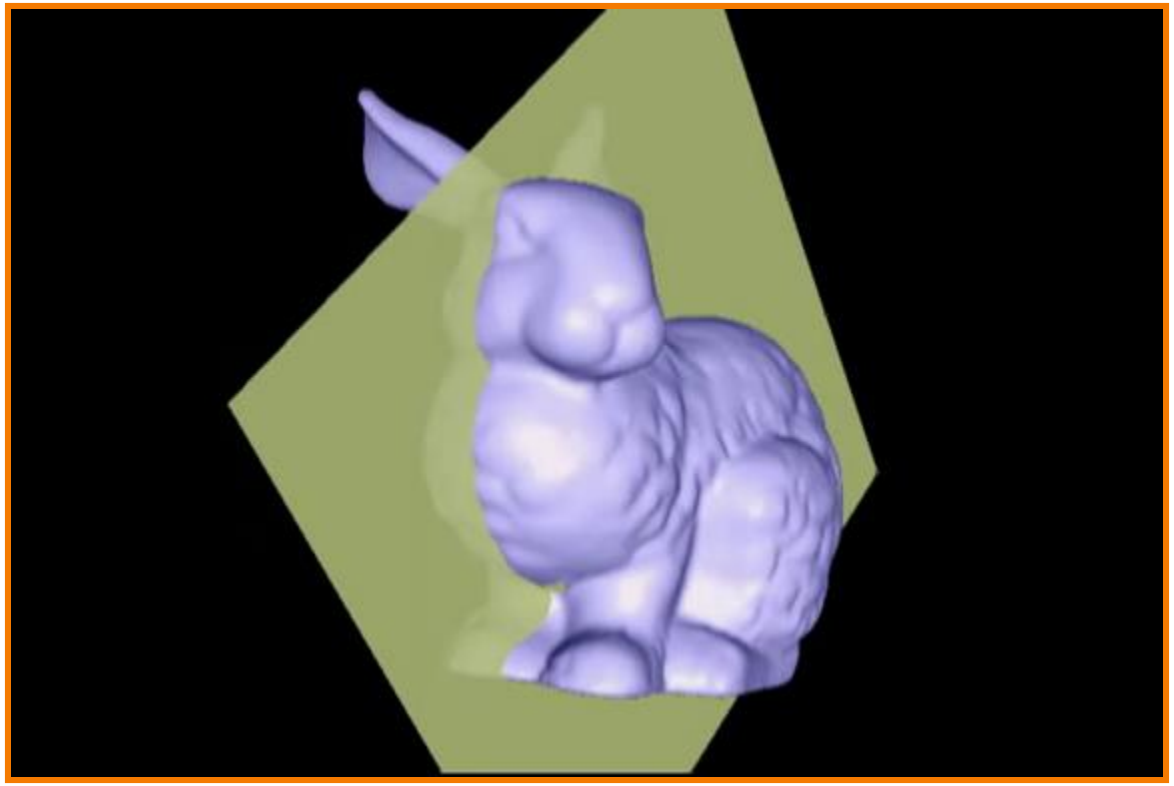
Closest Point
Correspondences



Continuous
Local Maximum

Principal Symmetries

Computing local maxima precisely:



Outline

Introduction

Background

Symmetry representations

- Symmetry descriptor
- Symmetry transform
- Principal symmetries

Applications ←

Conclusion

Related Work

Exploiting symmetry in geometric processing

- Completion – Zabrodsky93, Thrun et al. 2005
- Alignment – Zabrodsky et al. 1995, Kazhdan et al. 2002
- Symmetrization - Zabrodsky et al. 1997, Mitra et al. 2007
- Feature detection – Reisfeld et al. 1995
- Reverse engineering - Mills et al. 2001
- Instancing - Martinet et al. 2005
- Matching – Kazhdan et al. 2002, Gal et al. 2005
- Compression - Simari et al. 2006
- Segmentation - Mitra et al. 2006, Podolak et al. 2006
- Viewpoint selection – Podolak et al. 2006
- Editing – Mitra et al. 2006
- Simplification – Podolak et al. 2007

Applications

Alignment

Matching

Segmentation

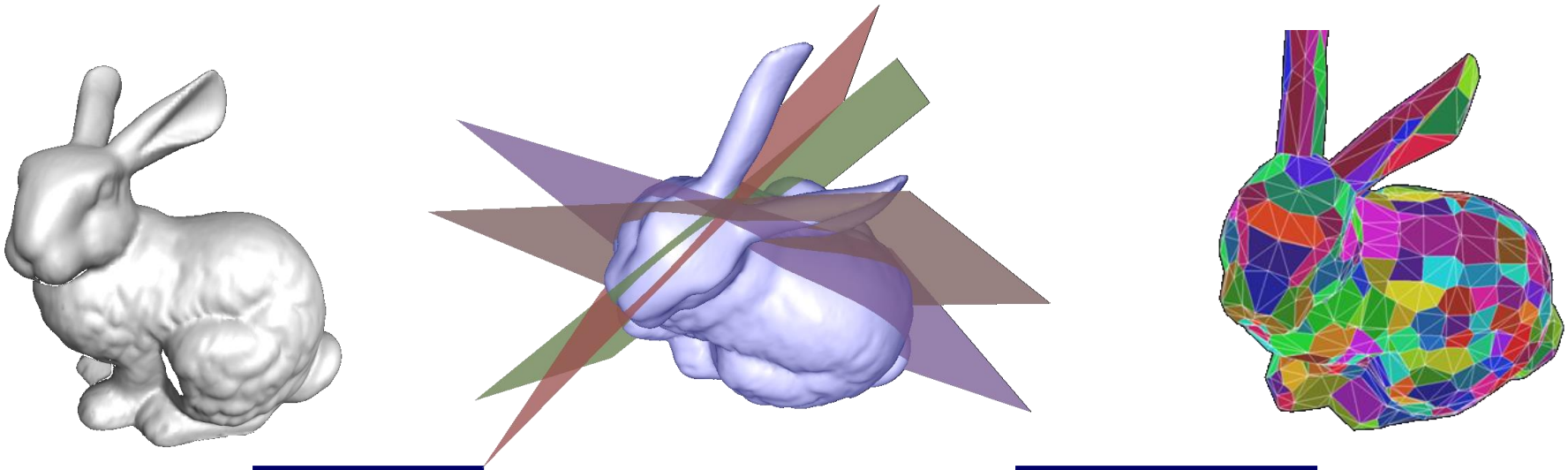
Viewpoint selection

Simplification

Beautification

Texture synthesis

Symmetry-Aware Processing



Input Mesh → Symmetry Analysis → *Principal Symmetry Transformations + Regions of Support* → Symmetry Aware Processing → Output Mesh

Applications

Alignment ←

Matching

Segmentation

Viewpoint selection

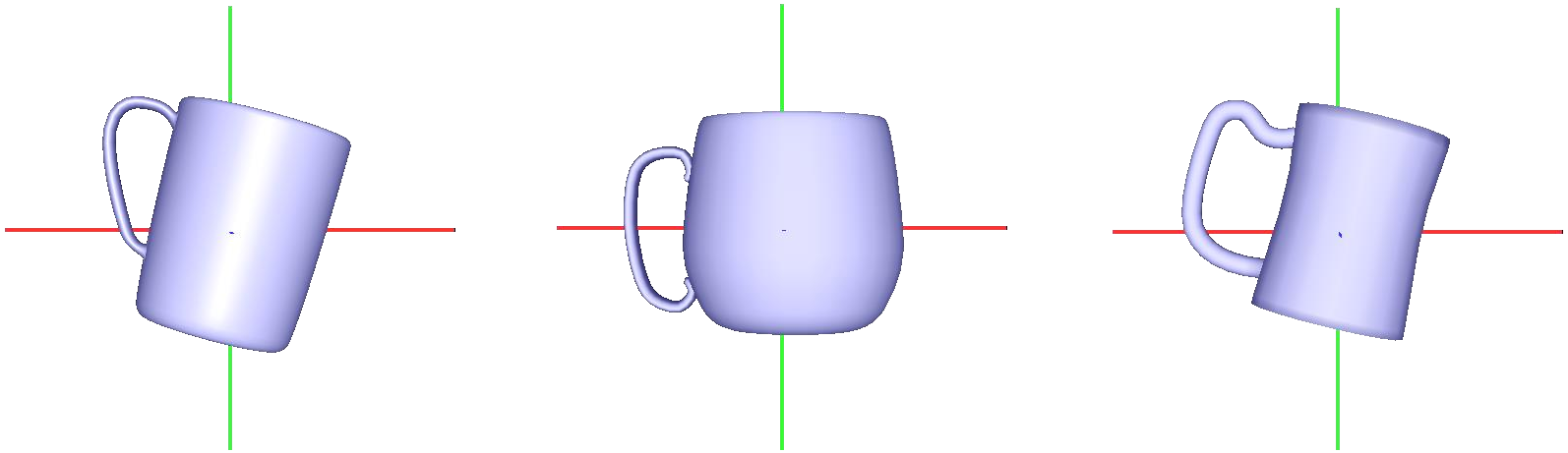
Simplification

Beautification

Texture synthesis

Application: Alignment

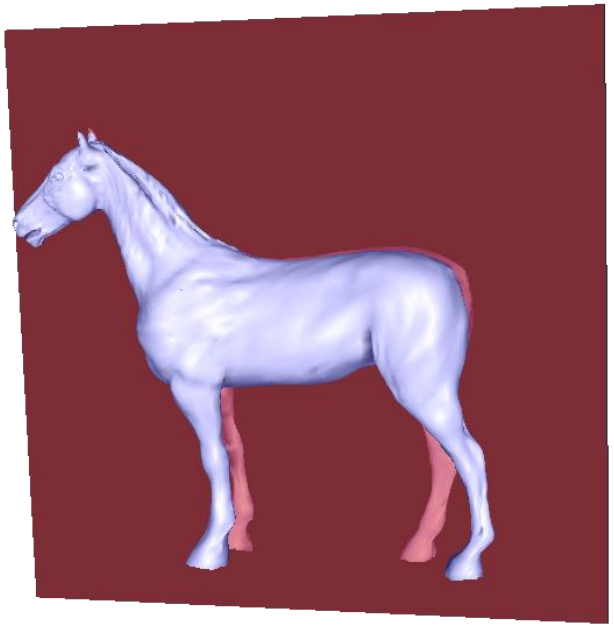
Motivation: registration, modeling, etc.



PCA Alignment

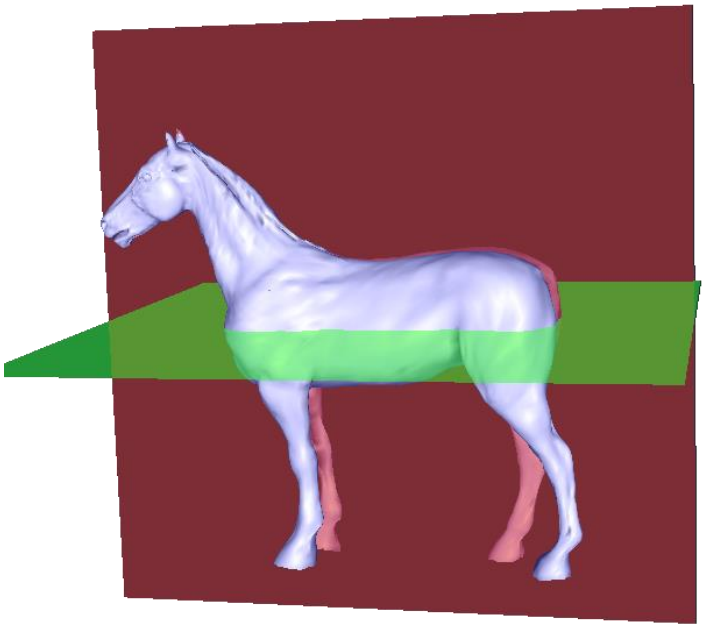
Application: Alignment

Approach: align planes with highest symmetries



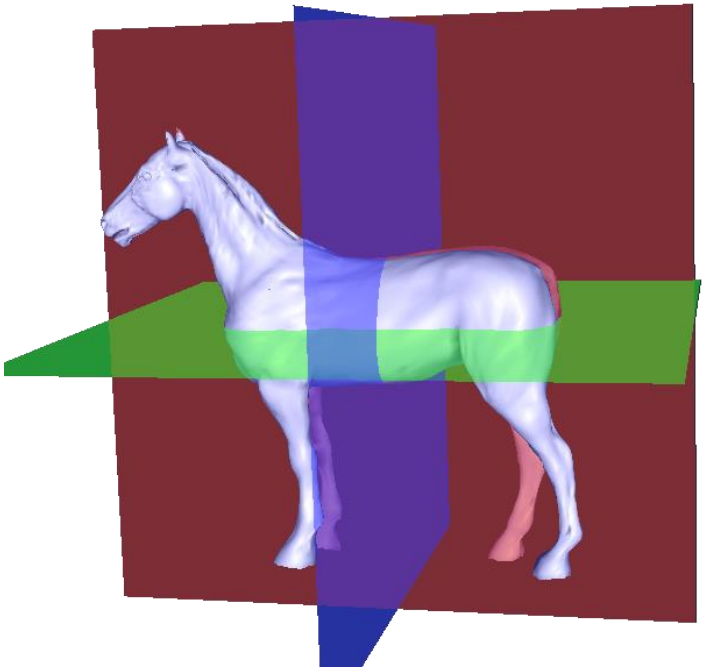
Application: Alignment

Approach: align planes with highest symmetries



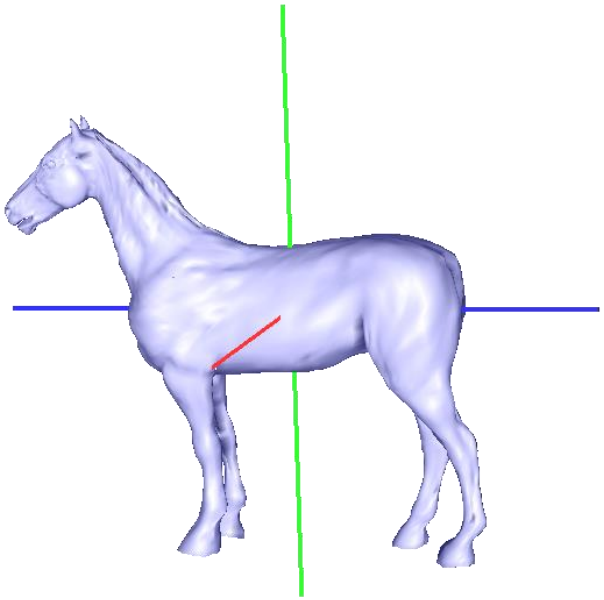
Application: Alignment

Approach: align planes with highest symmetries



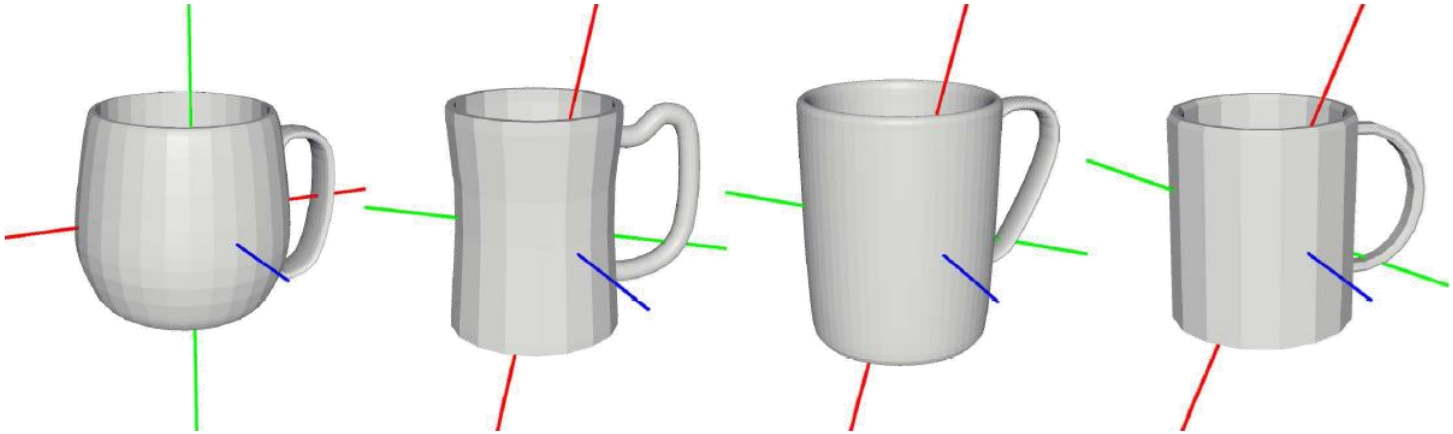
Application: Alignment

Approach: align planes with highest symmetries

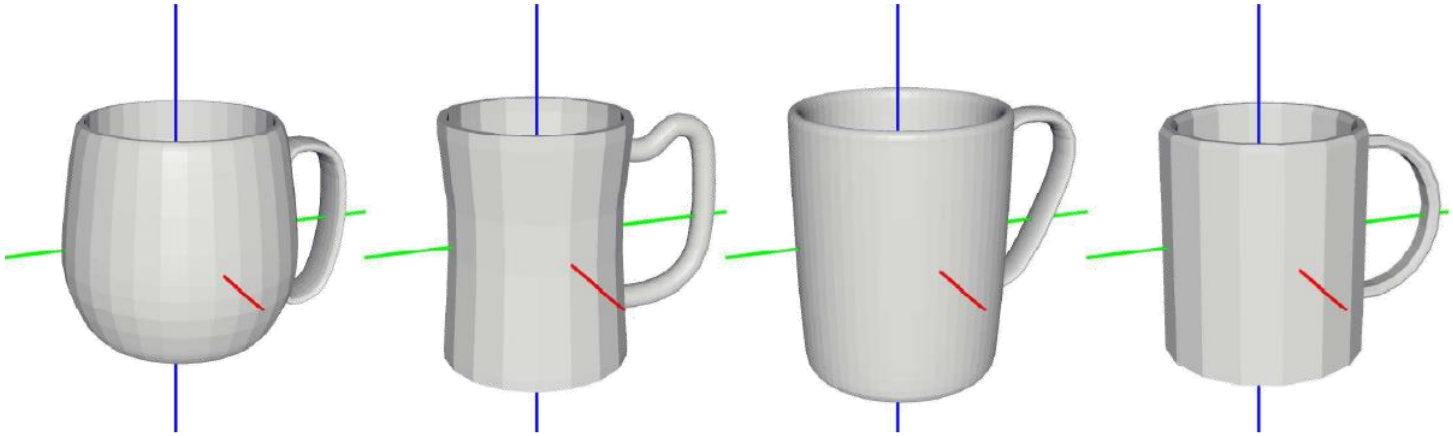


Application: Alignment

Results:



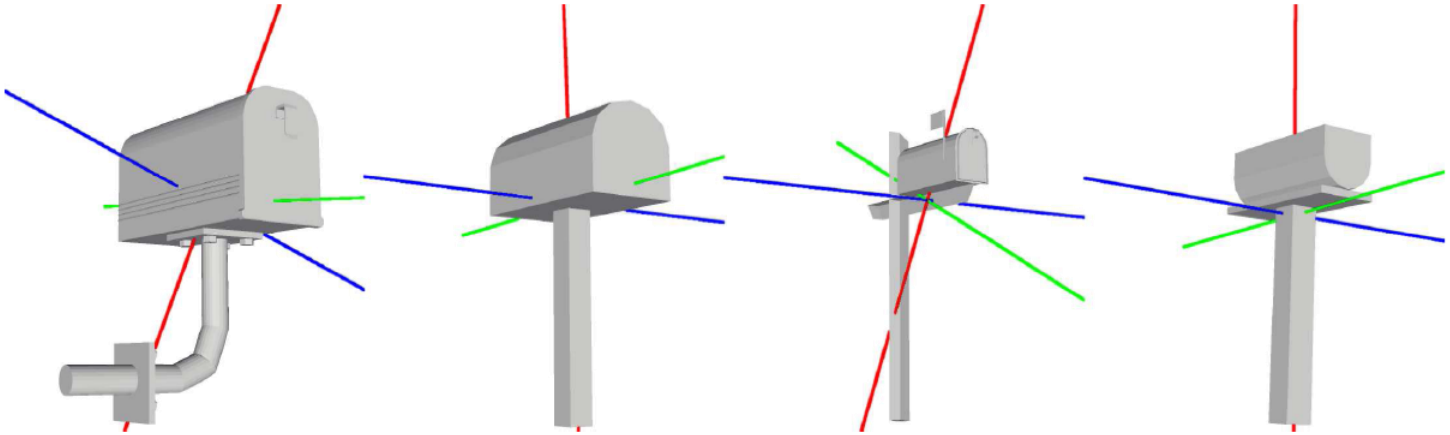
Center of mass and PCA



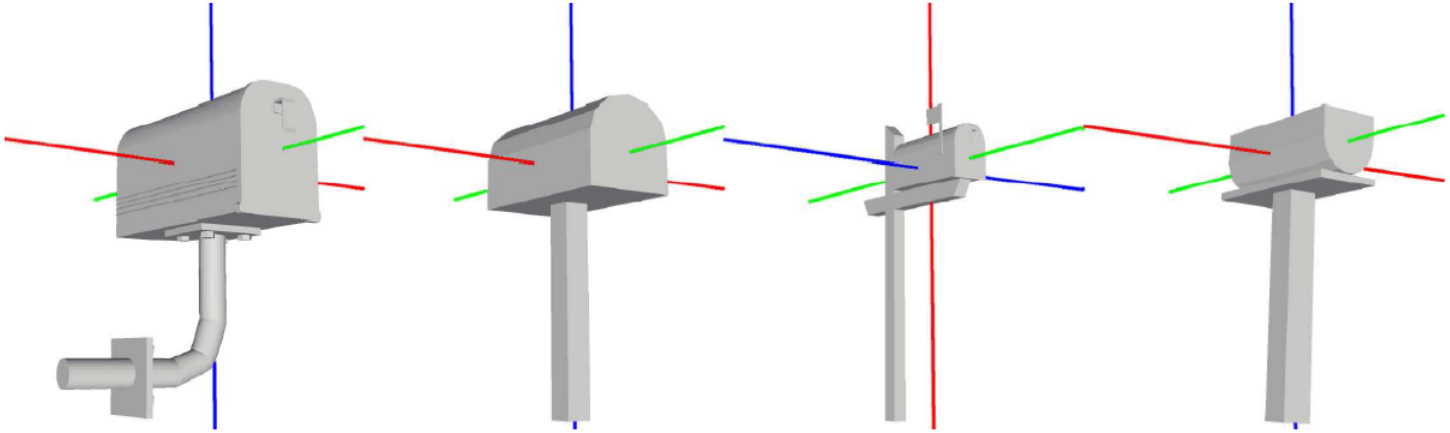
Center of symmetry and principal symmetry axes

Application: Alignment

Results:



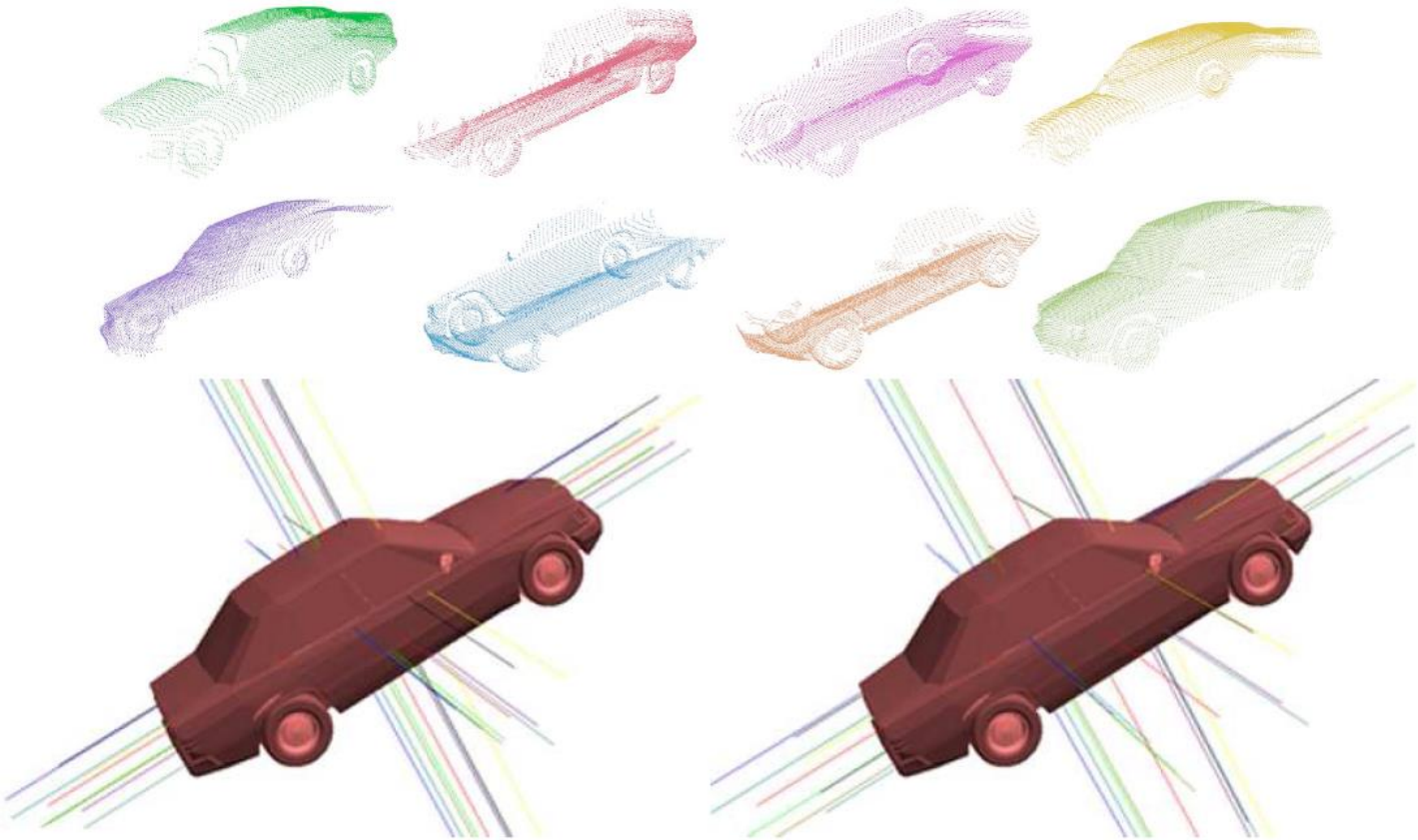
Center of mass and PCA



Center of symmetry and principal symmetry axes

Application: Alignment

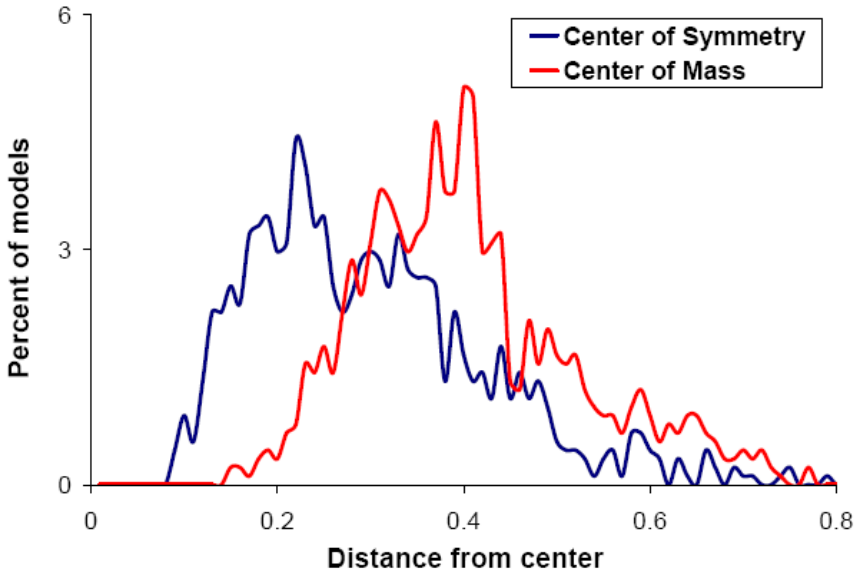
Results:

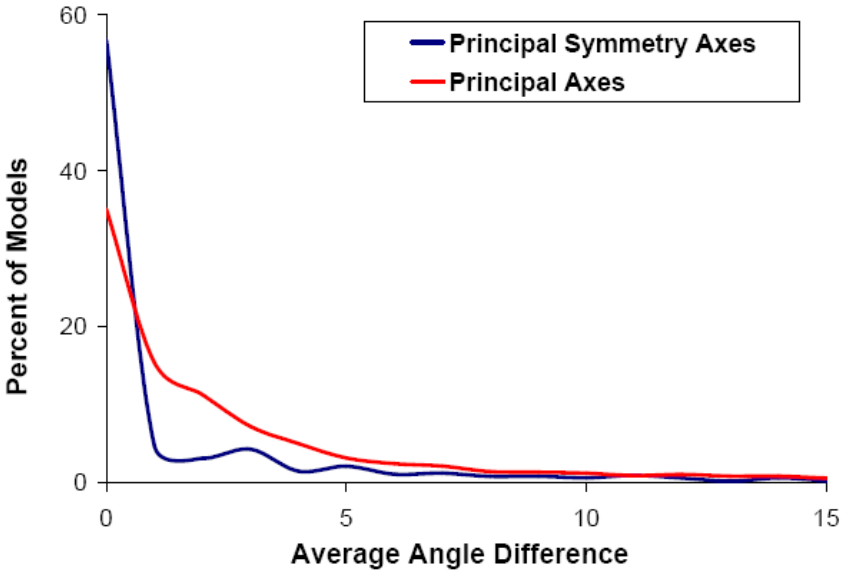


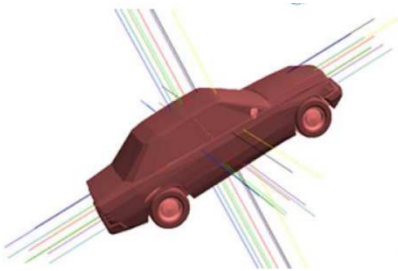
(Center of Symmetry) (Center of Mass)

Application: Alignment

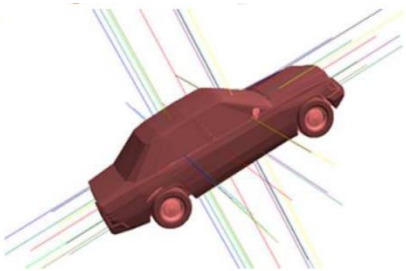
Results:







(Center of Symmetry)



(Center of Mass)

Applications

Alignment

Matching ←

Segmentation

Viewpoint selection

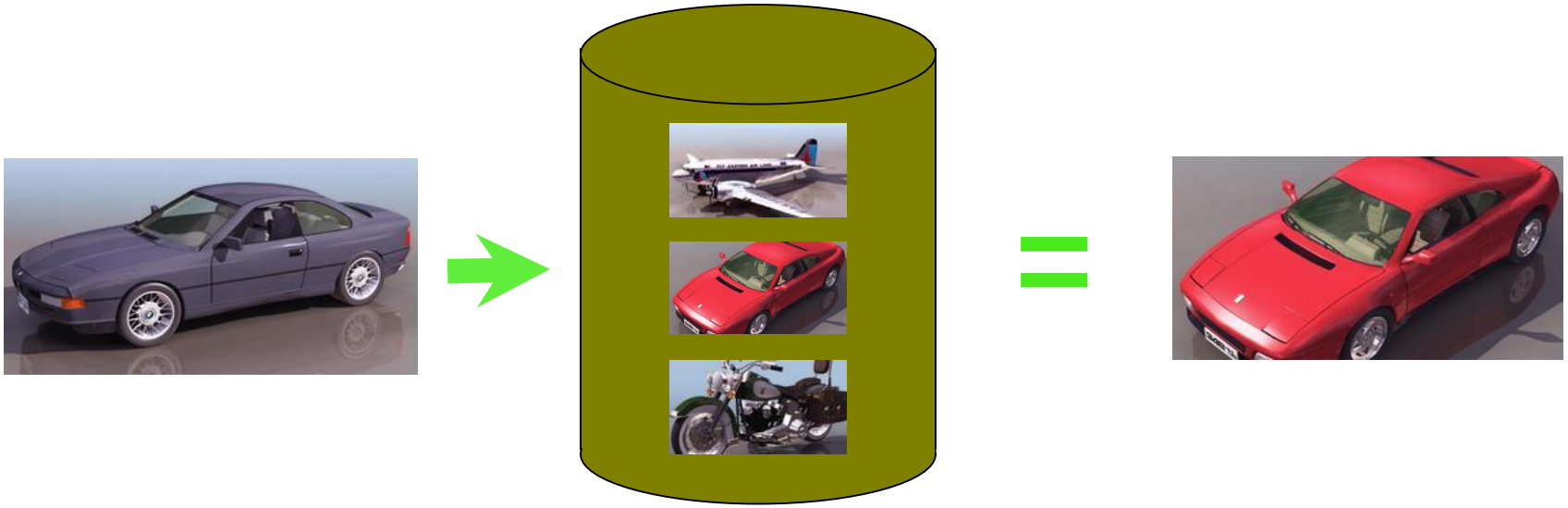
Simplification

Beautification

Texture synthesis

Application: Matching

Motivation: similarity search of database



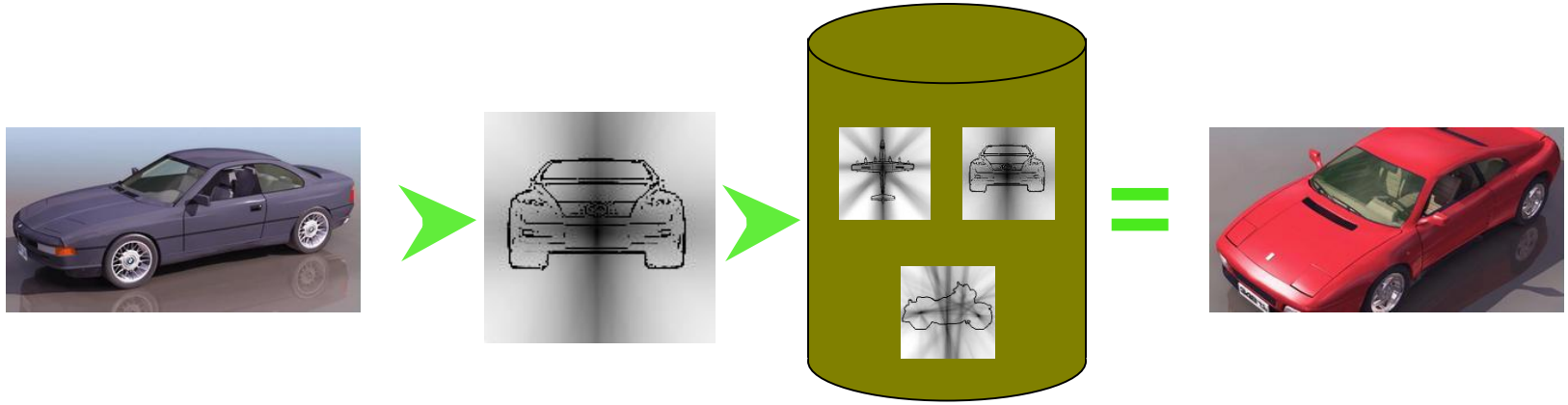
Query Database Best Match

Application: Matching

Observation: symmetry is more consistent than shape for some object classes

Application: Matching

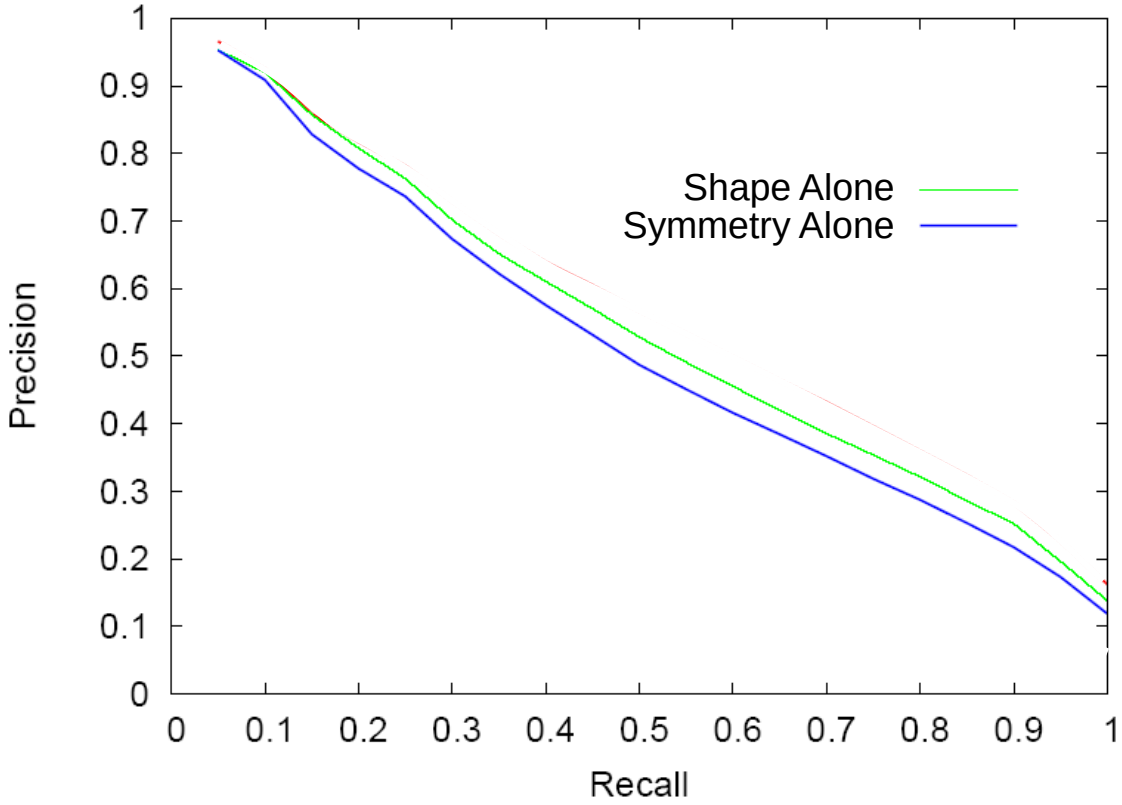
Approach: use symmetry transform (or descriptor) as shape descriptor



Query | Symmetry Transform | Database | Best Match

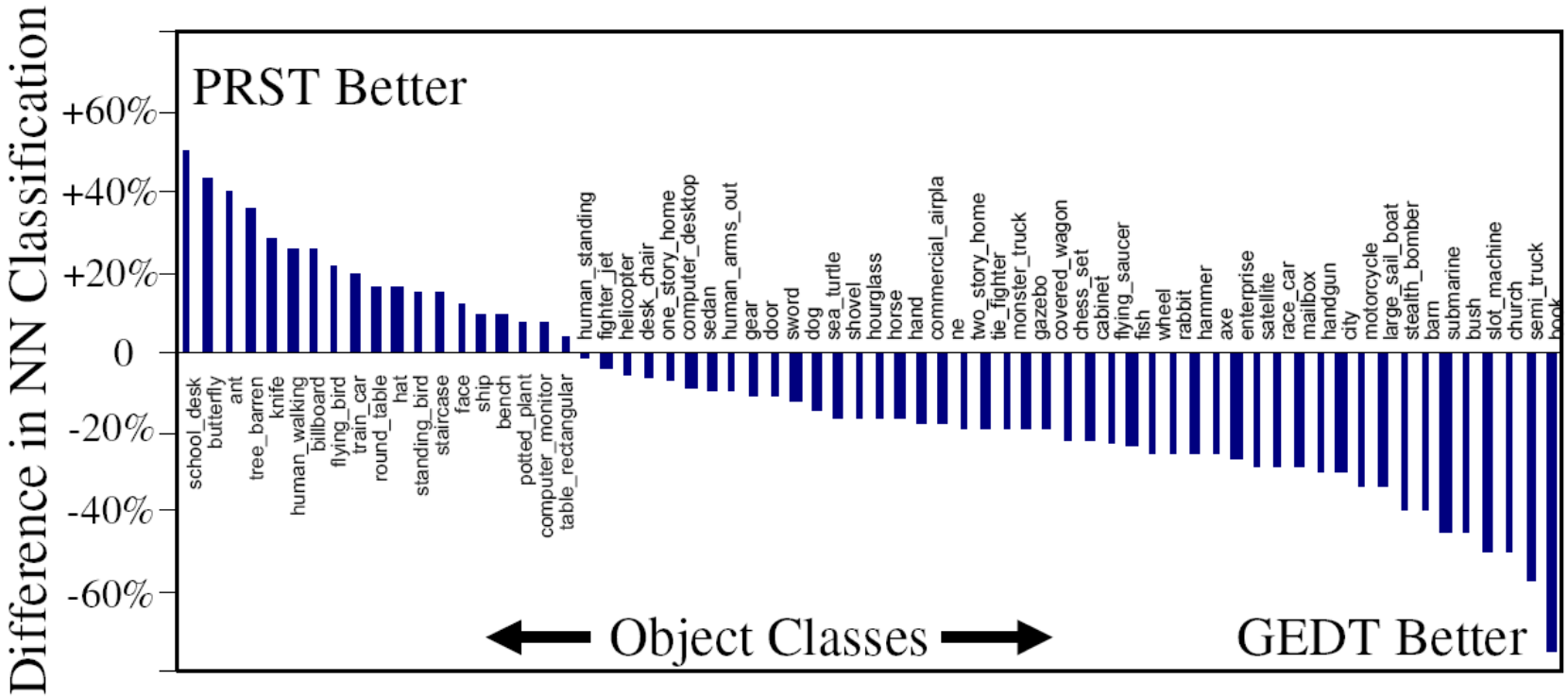
Application: Matching

Results: symmetry is not as discriminating as shape



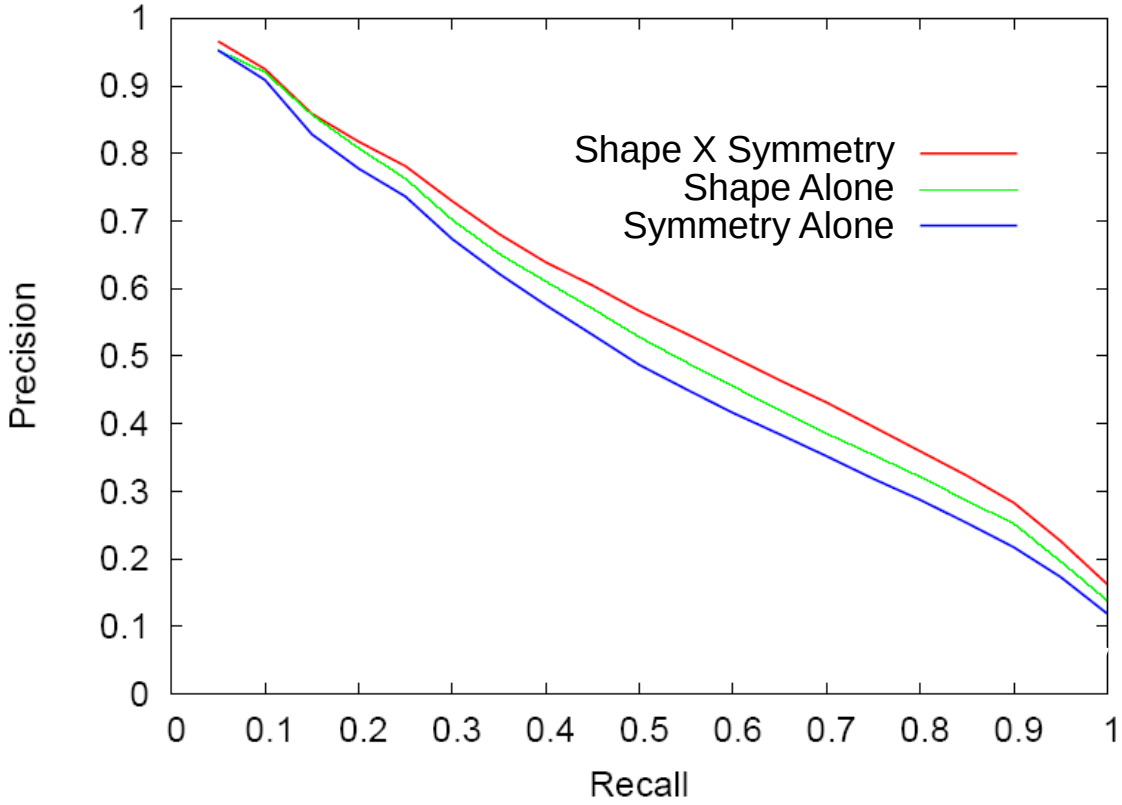
Application: Matching

Results: symmetry is not as discriminating as shape, but it is better for some classes



Application: Matching

Results: symmetry is not as discriminating as shape, but it is better for some classes, and so the two together are better than either alone



Applications

Alignment

Matching

Segmentation ←

Viewpoint selection

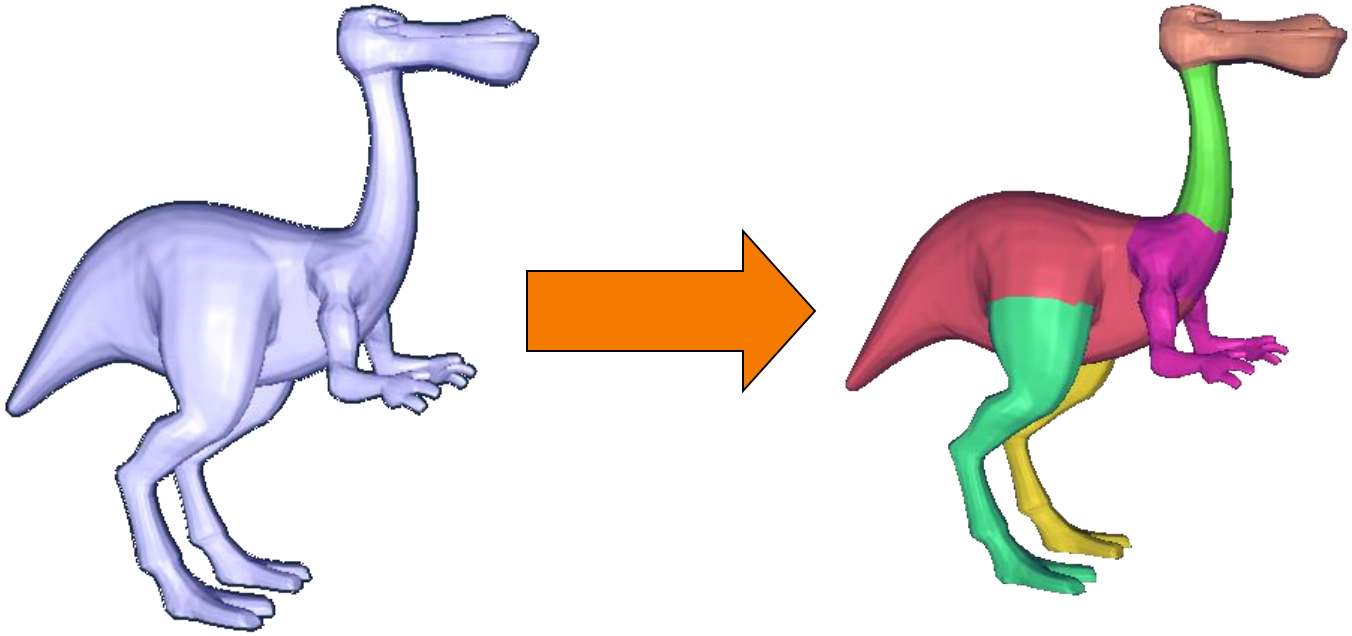
Simplification

Beautification

Texture synthesis

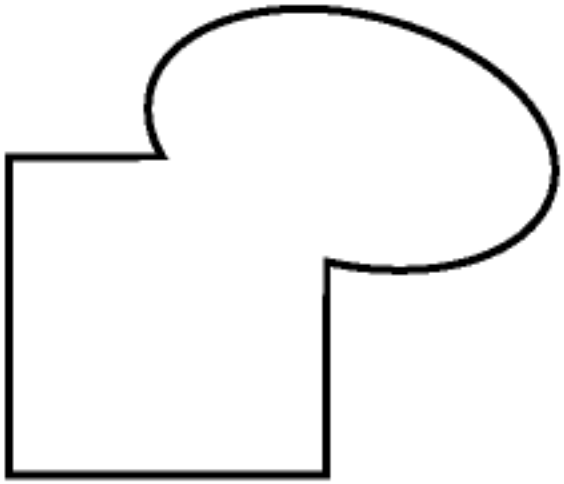
Application: Segmentation

Motivation: animation, modeling by parts, etc.



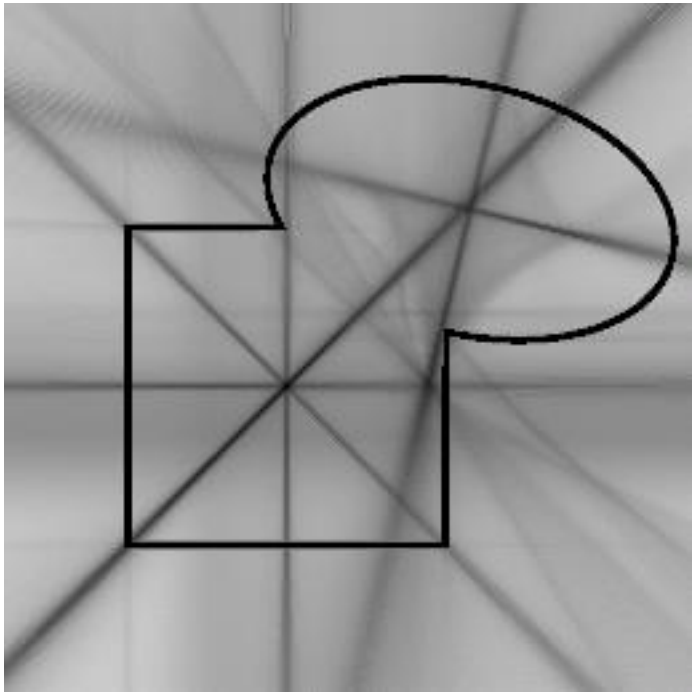
Application: Segmentation

Observation: distinct parts have strong local symmetries not shared by other parts



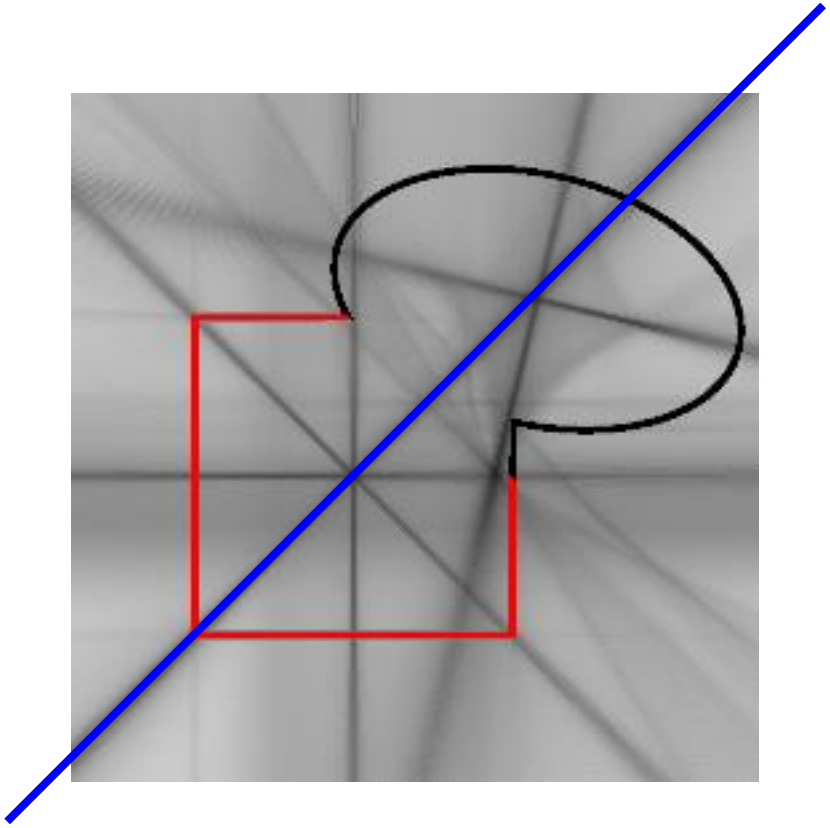
Application: Segmentation

Observation: distinct parts have strong local symmetries not shared by other parts



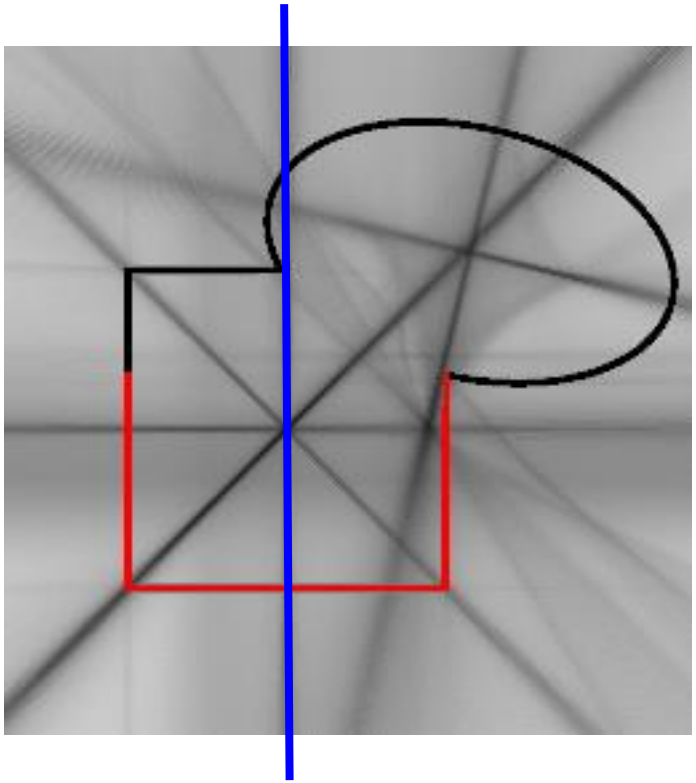
Application: Segmentation

Observation: distinct parts have strong local symmetries not shared by other parts



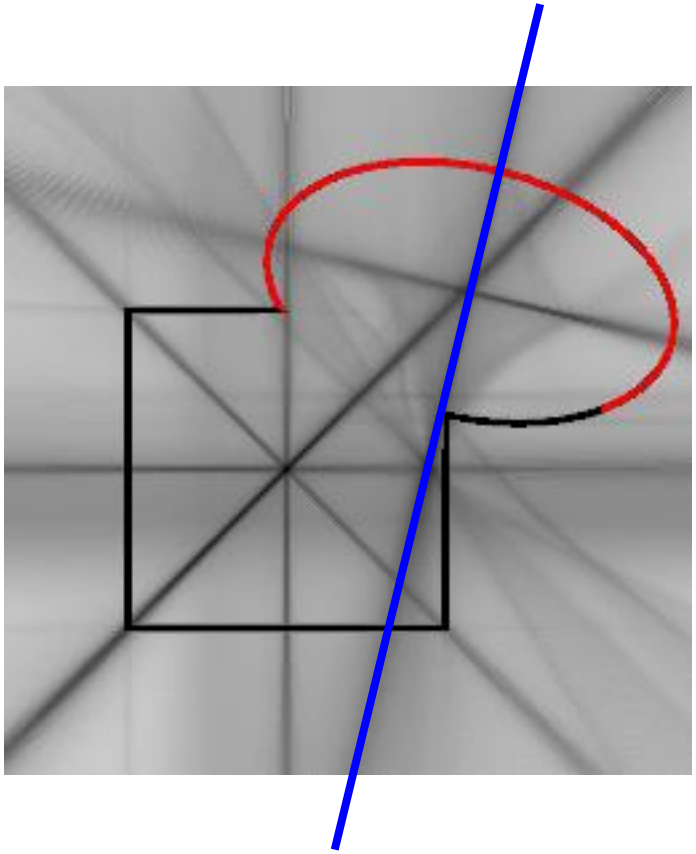
Application: Segmentation

Observation: distinct parts have strong local symmetries not shared by other parts



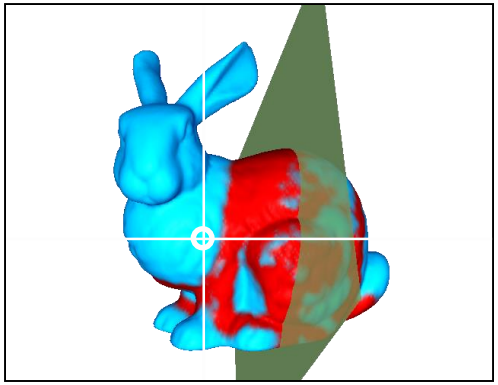
Application: Segmentation

Observation: distinct parts have strong local symmetries not shared by other parts

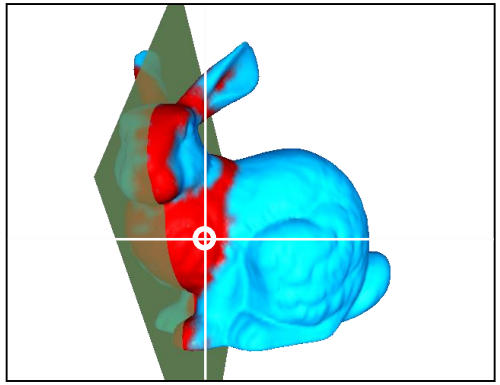


Application: Segmentation

Approach: cluster points on the surface by how much they support different symmetries

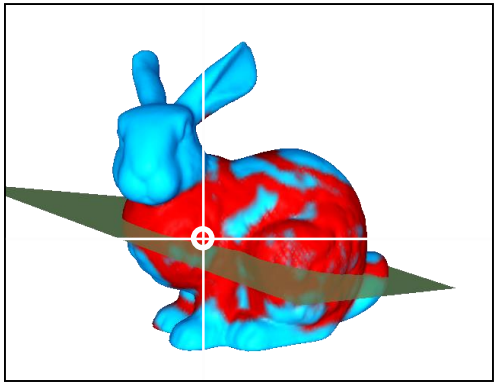


Support = 0.1



Support = 0.5

.....

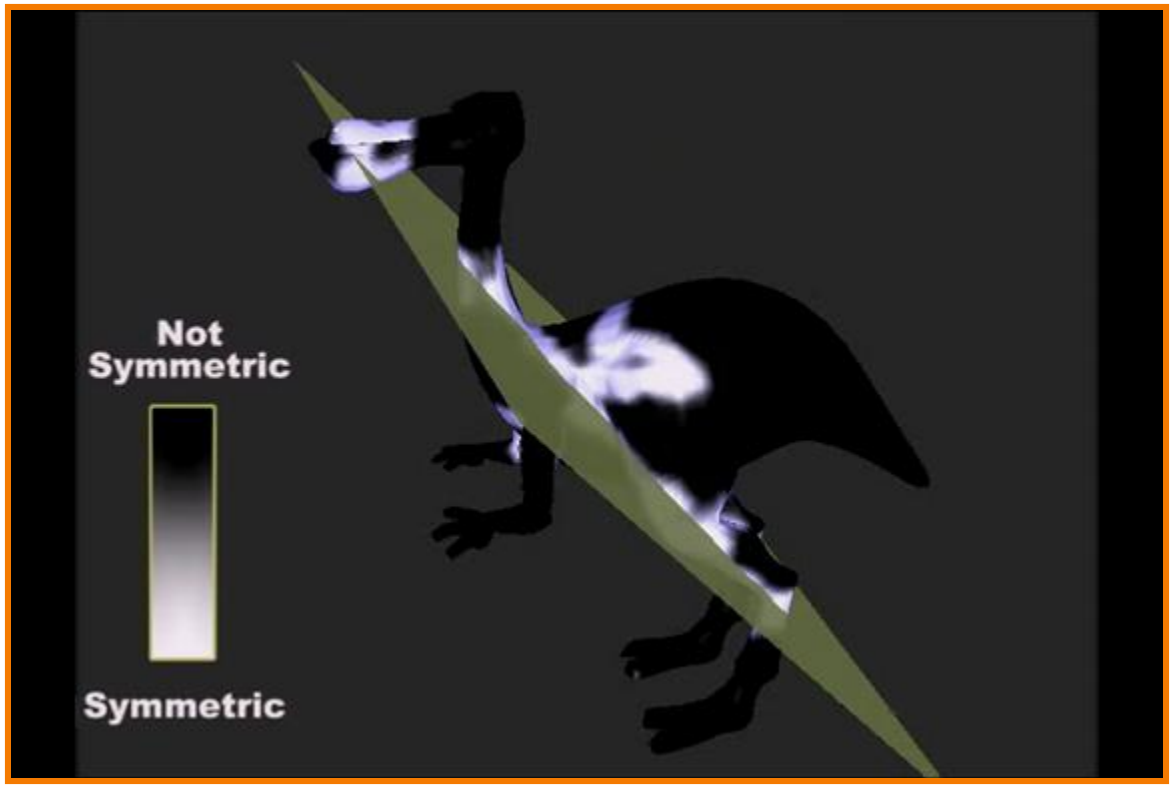


Support = 0.9

Symmetry Vector = { 0.1 , 0.5 , …. , 0.9 }

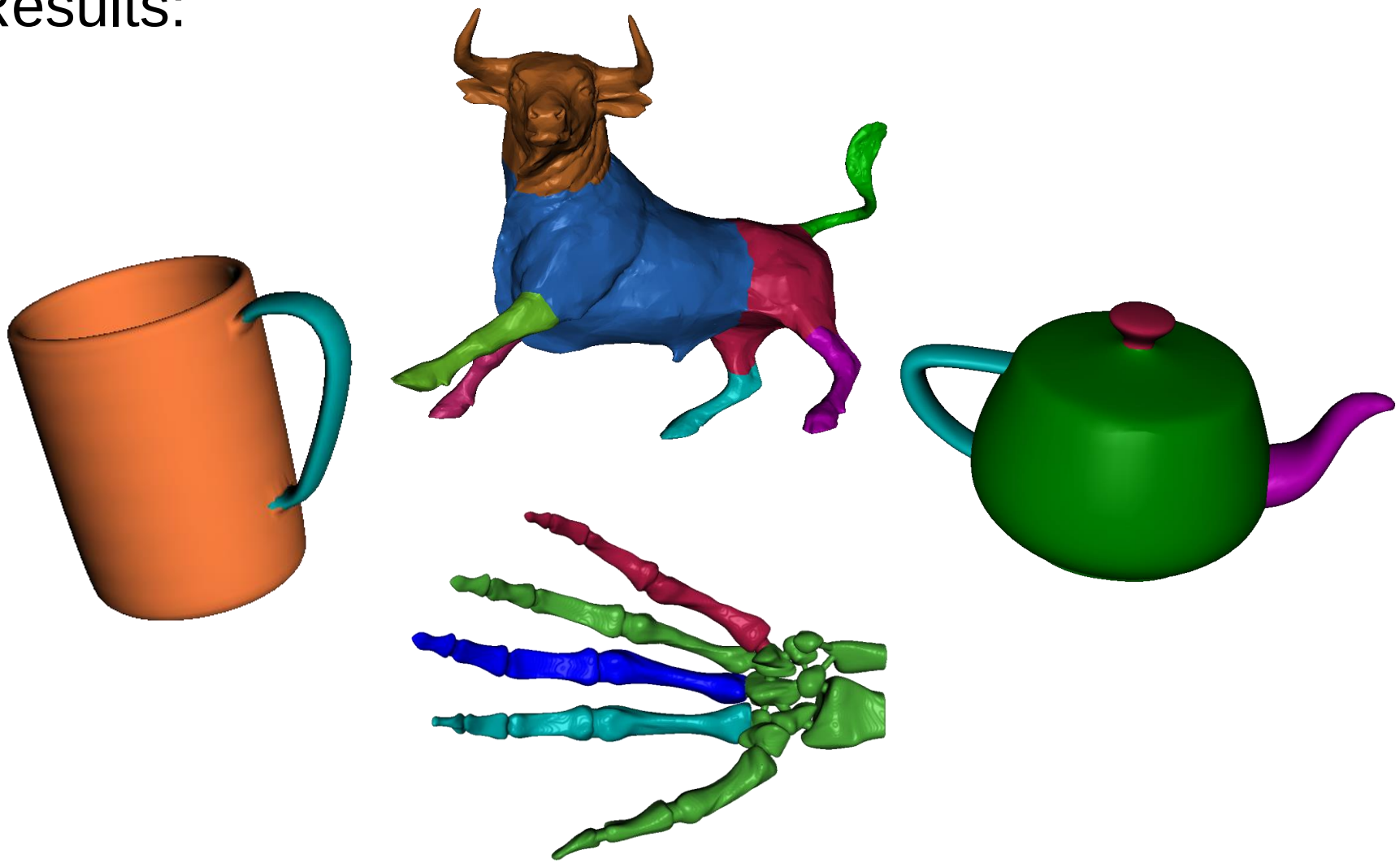
Application: Segmentation

Approach: cluster points on the surface by how much they support different symmetries



Application: Segmentation

Results:



Applications

Alignment

Matching

Segmentation

Viewpoint selection ←

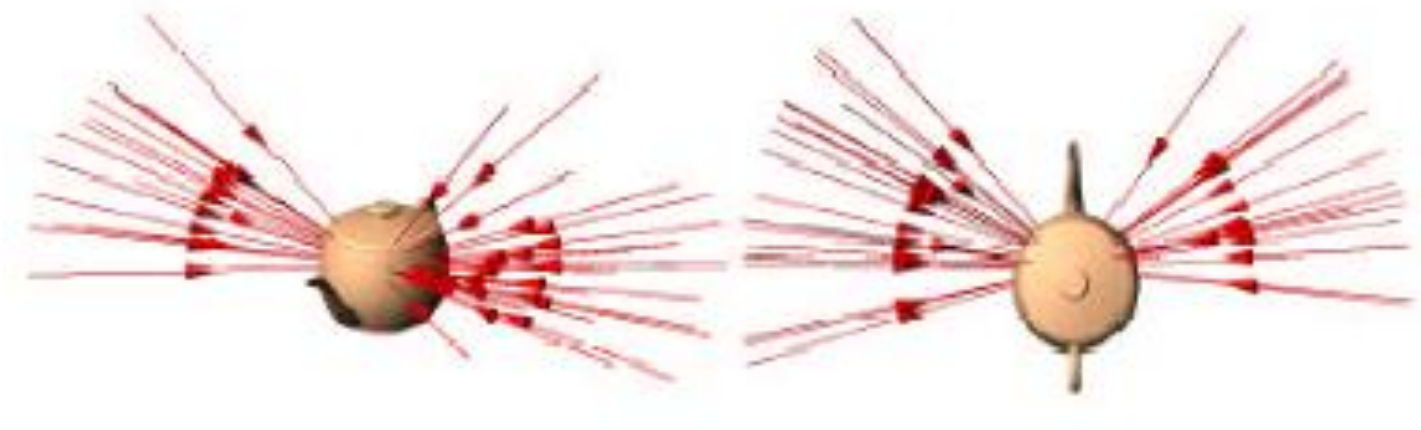
Simplification

Beautification

Texture synthesis

Application: Viewpoint Selection

Motivation: visualization, icon generation, etc.



Picture from Blanz et al. ‘99

Application: Viewpoint Selection

Observation: symmetry represents redundancy

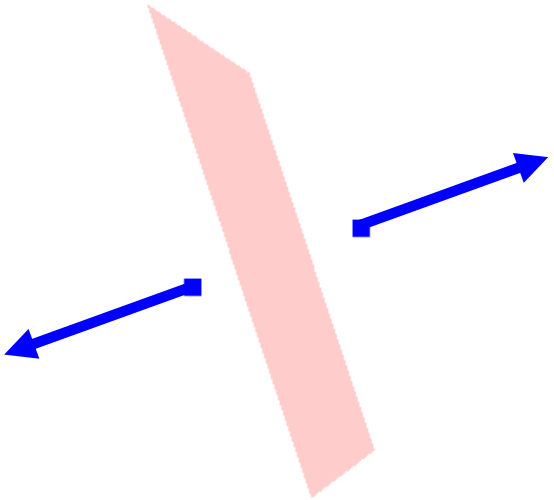
Bad Viewpoint

Application: Viewpoint Selection

Approach: Minimize visible symmetry

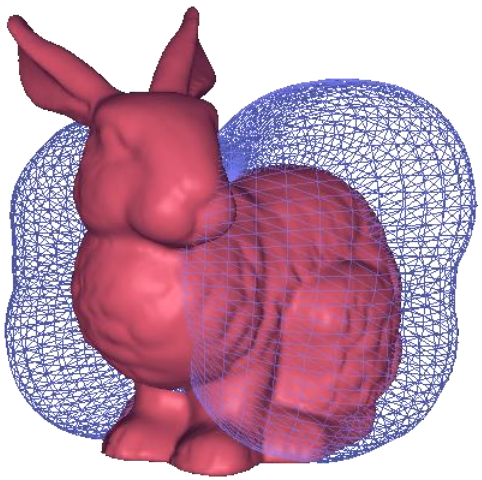
- Every plane of symmetry votes for a viewing direction perpendicular to it

Best Viewing Directions

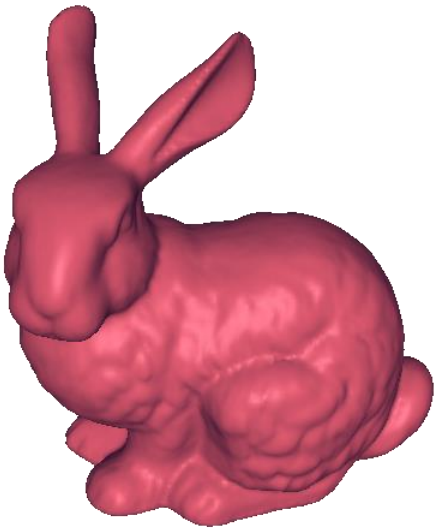


Application: Viewpoint Selection

Results:



Viewpoint Function

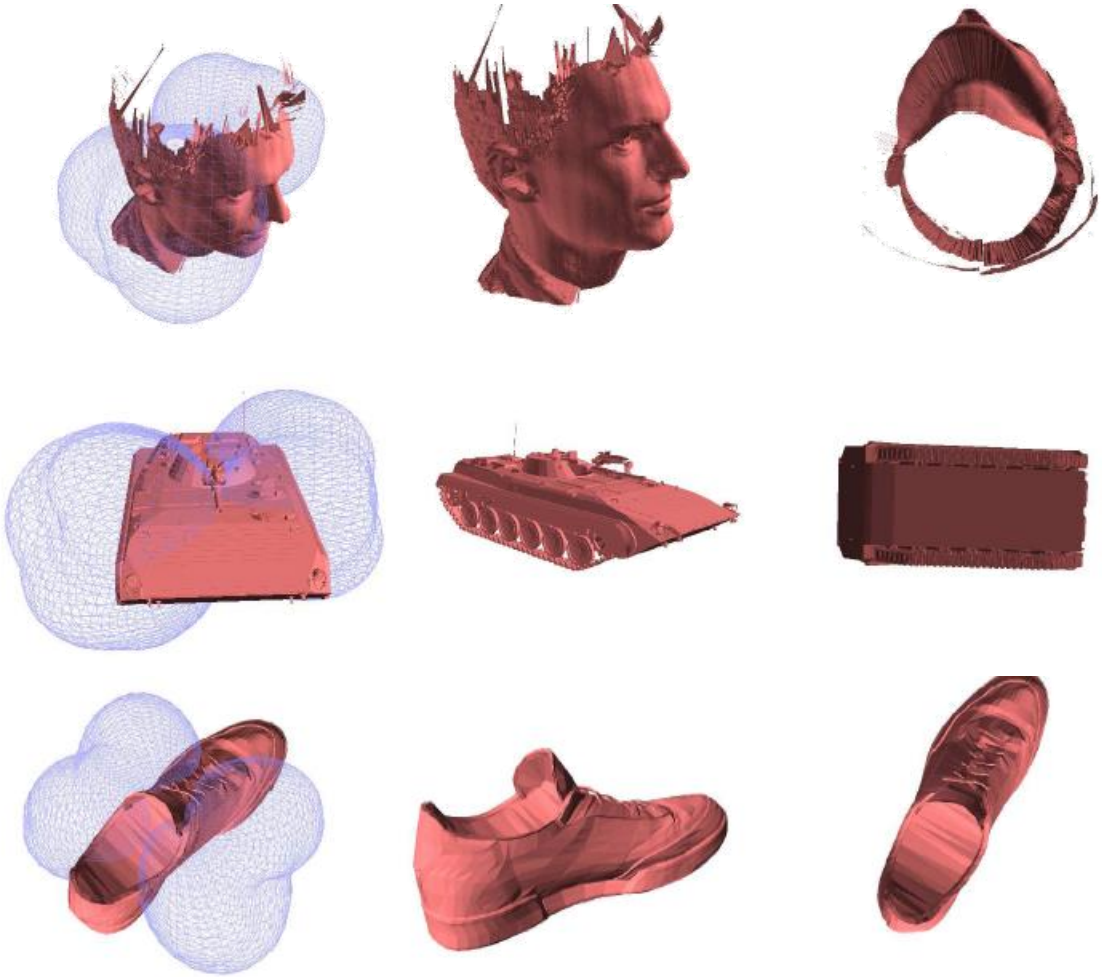


Best Viewpoint

Worst Viewpoint

Application: Viewpoint Selection

Results:



Viewpoint Function | Best Viewpoint | Worst Viewpoint

Applications

Alignment

Matching

Segmentation

Viewpoint selection

Simplification ←

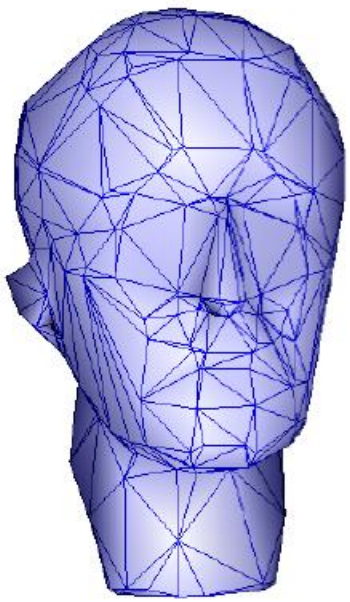
Beautification

Texture synthesis

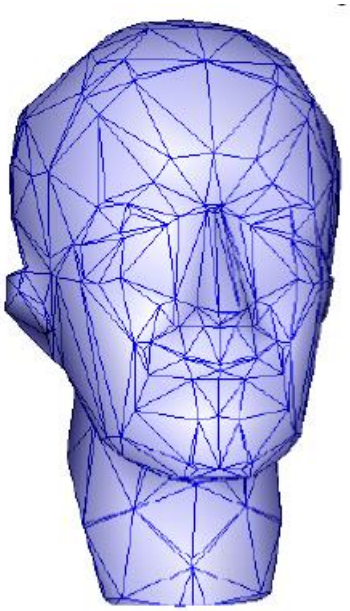
Application: Simplification

Motivation: preserve symmetry when simplify mesh

Input
Mesh



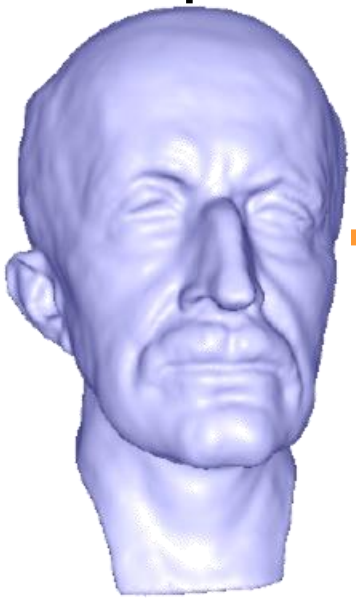
Standard
Simplification



Symmetric
Simplification

Application: Simplification

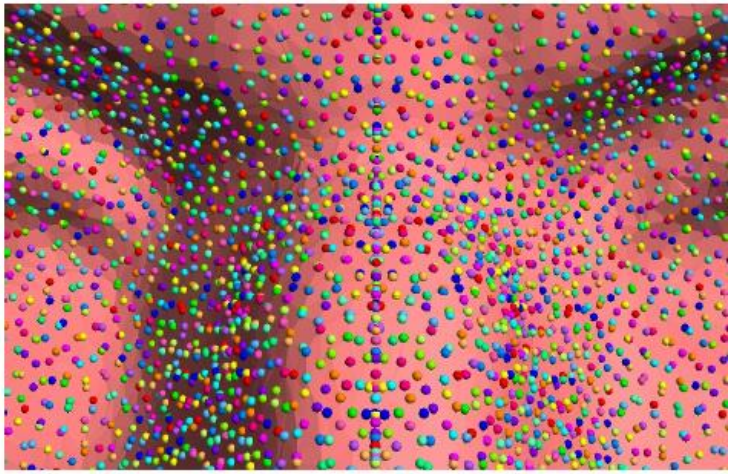
Approach 1: detect (approximate) symmetries and then preserve them as decimate



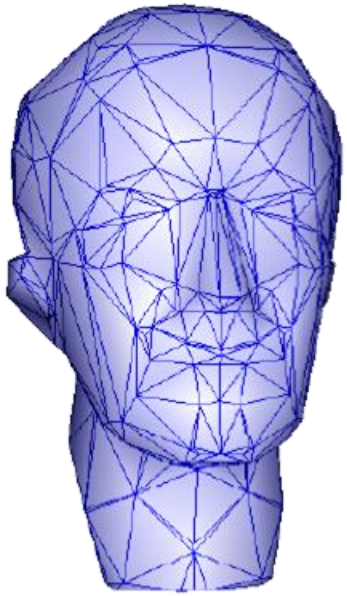
Input Mesh

Symmetry Analysis

Symmetry Aware Decimation



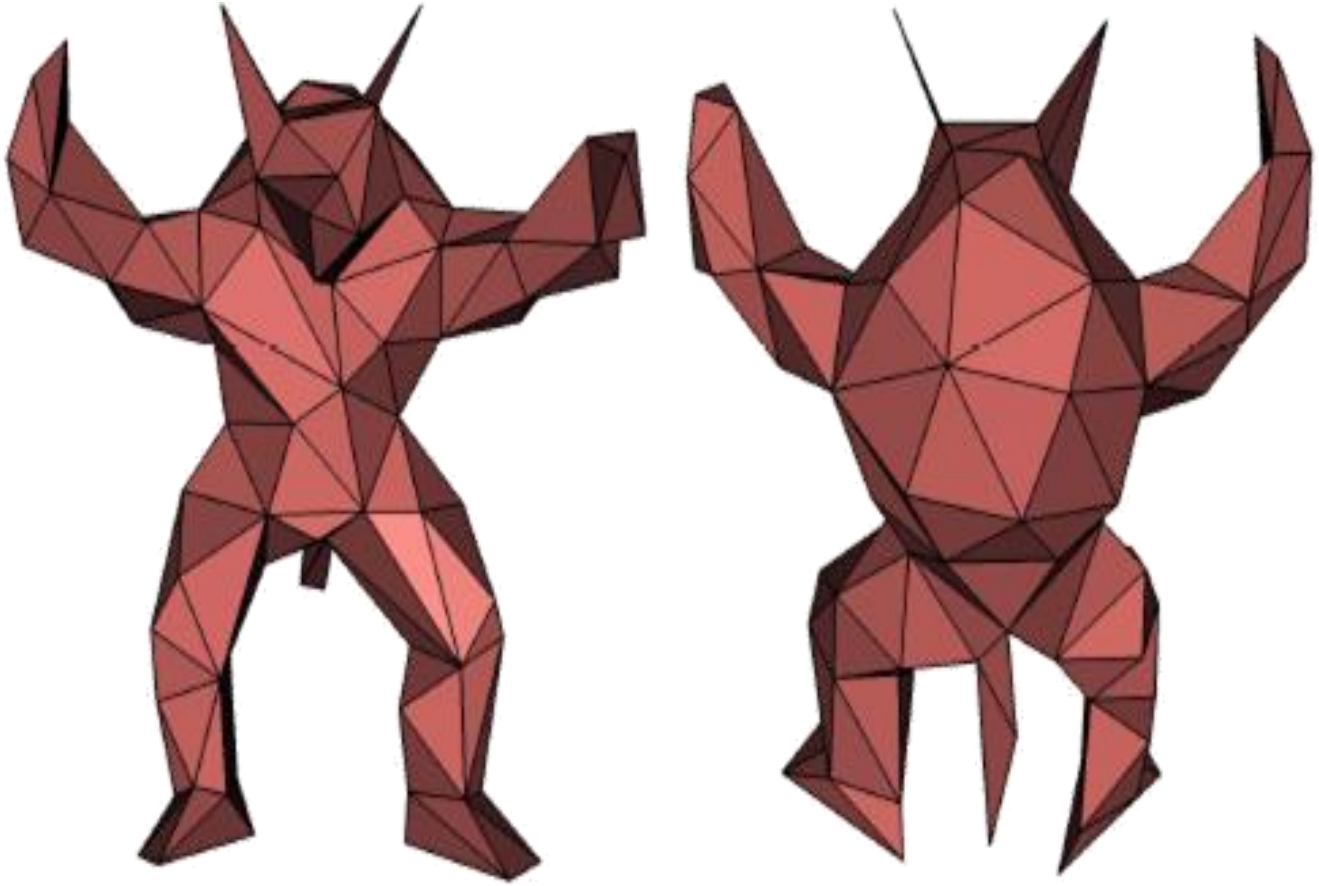
Symmetric Correspondences



Symmetric Simplification

Application: Simplification

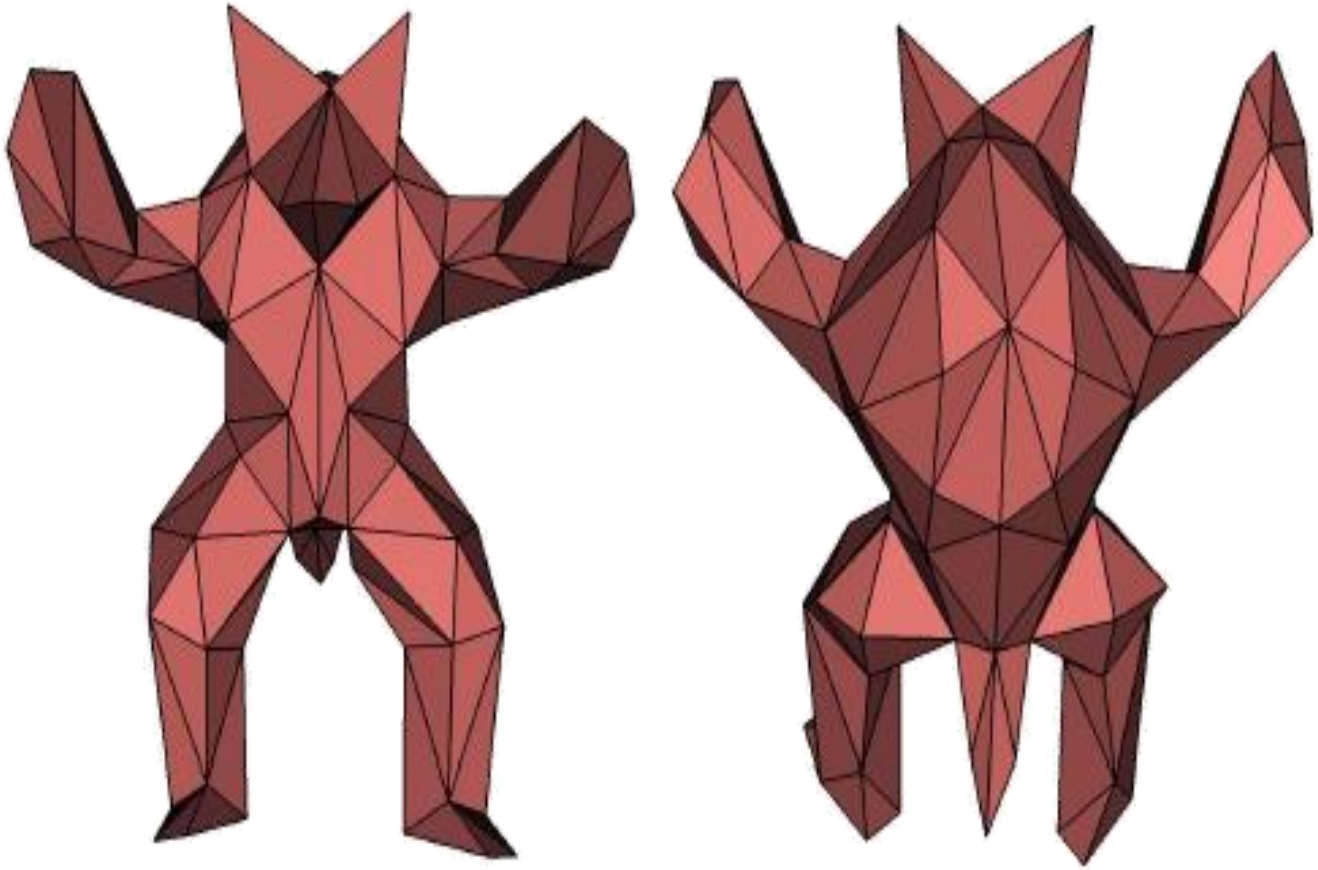
Results:



Original Qslim

Application: Simplification

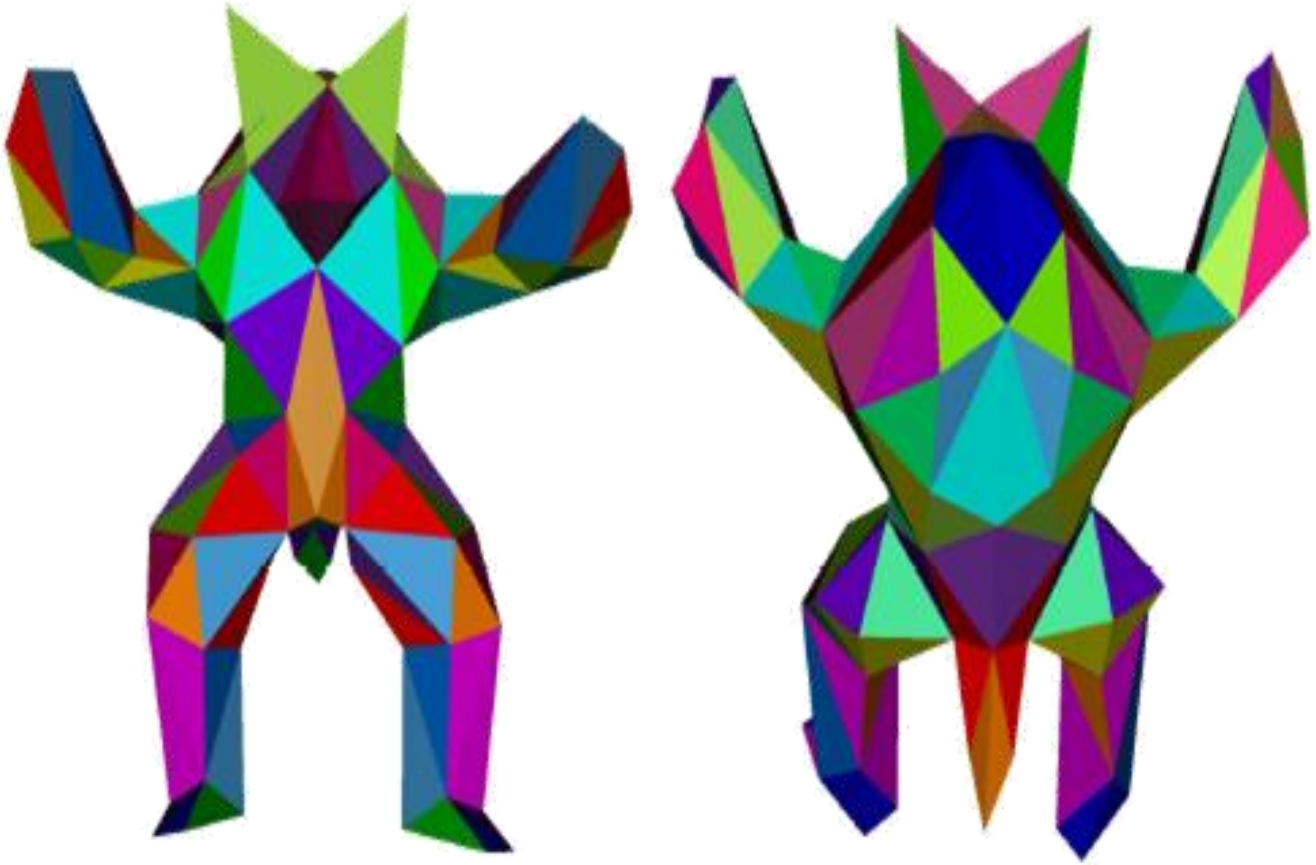
Results:



Symmetry-Preserving Qslim

Application: Simplification

Results:

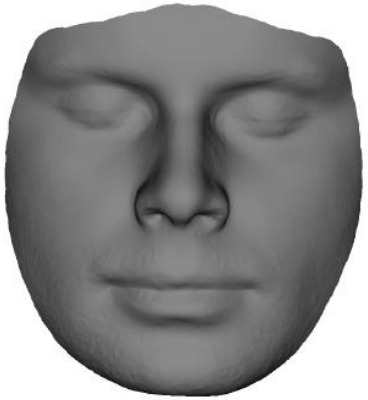


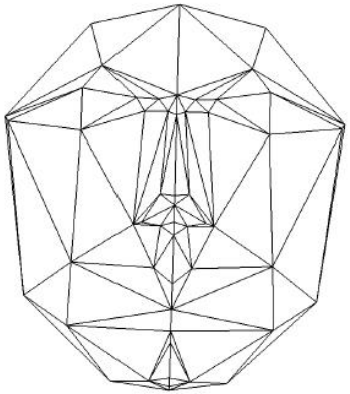
Symmetry-Preserving Qslim

Application: Parameterization

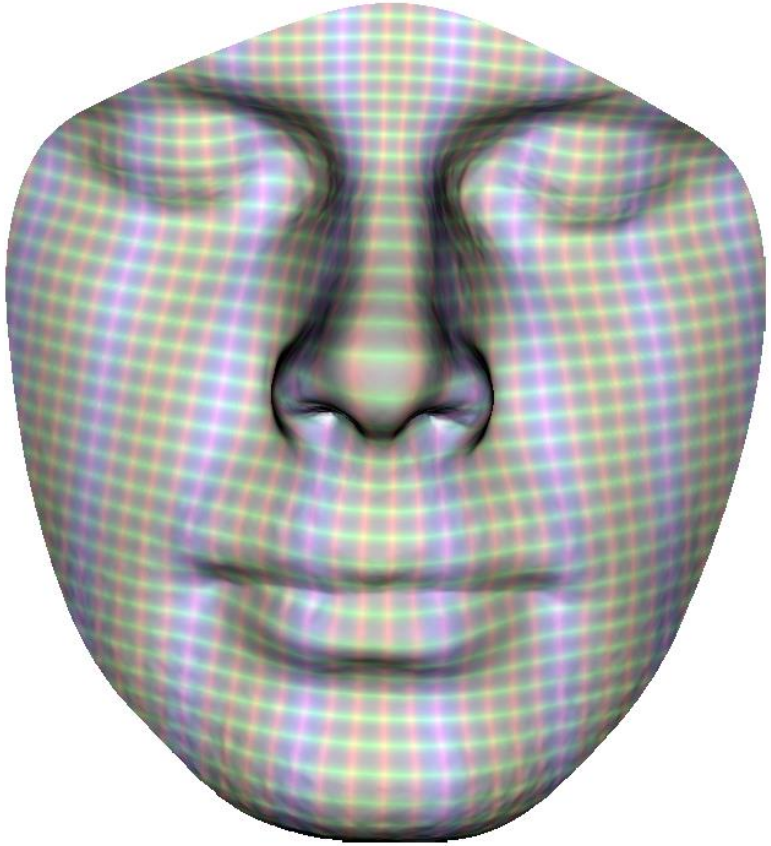
Preserve symmetries in base domain

Input Mesh





Base Domain



Parameterization

Applications

Alignment

Matching

Segmentation

Viewpoint selection

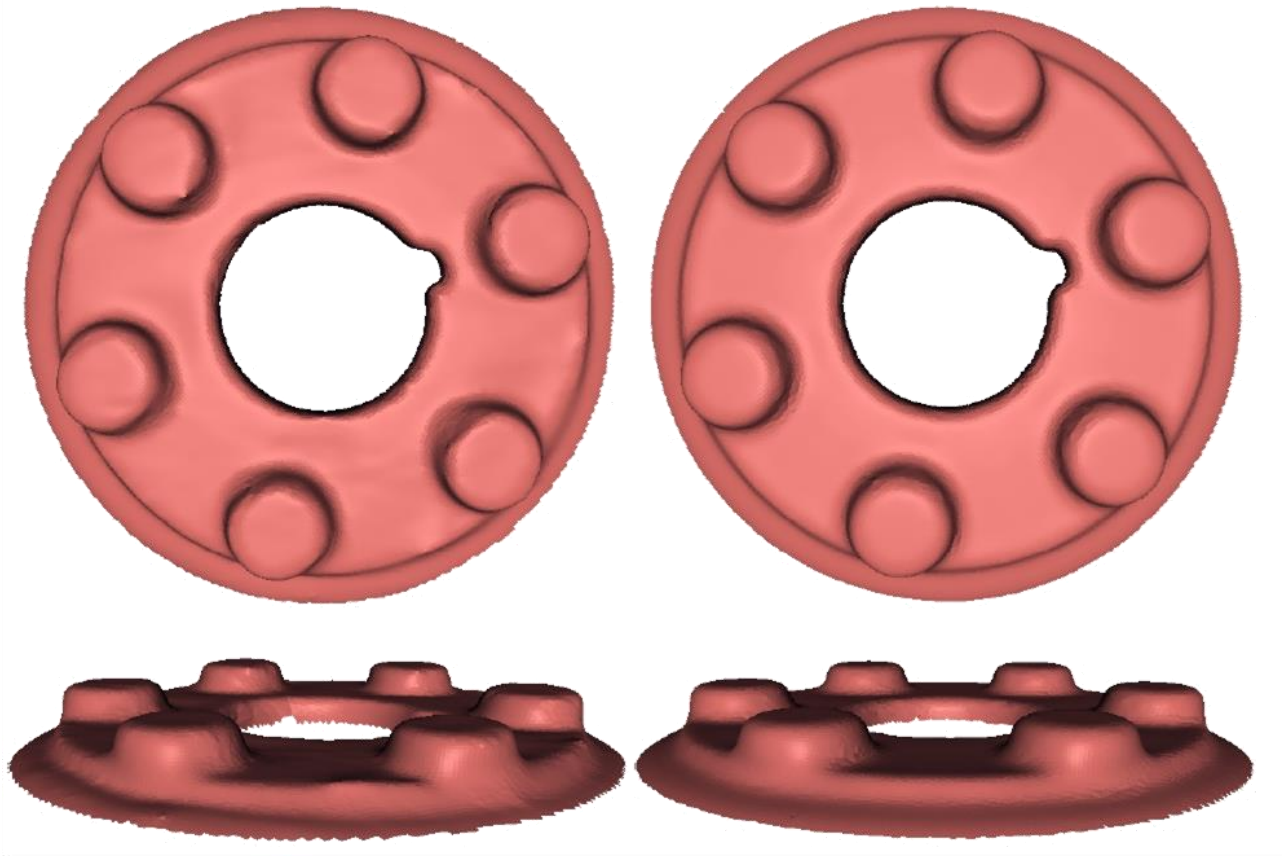
Simplification

Beautification ←

Texture synthesis

Application: Beautification

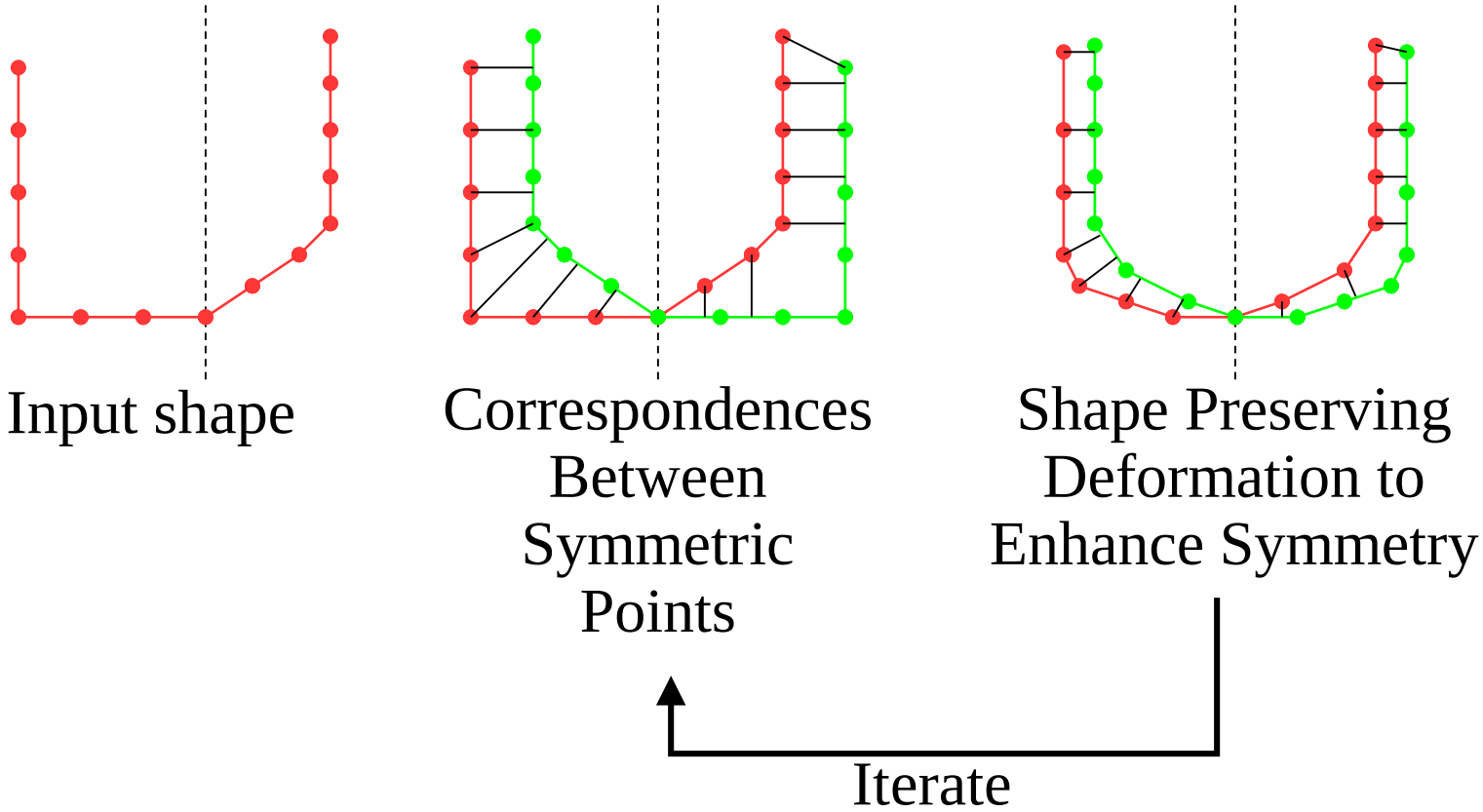
Goal: make meshes of symmetric objects perfectly symmetric



Input Mesh Symmetric Mesh

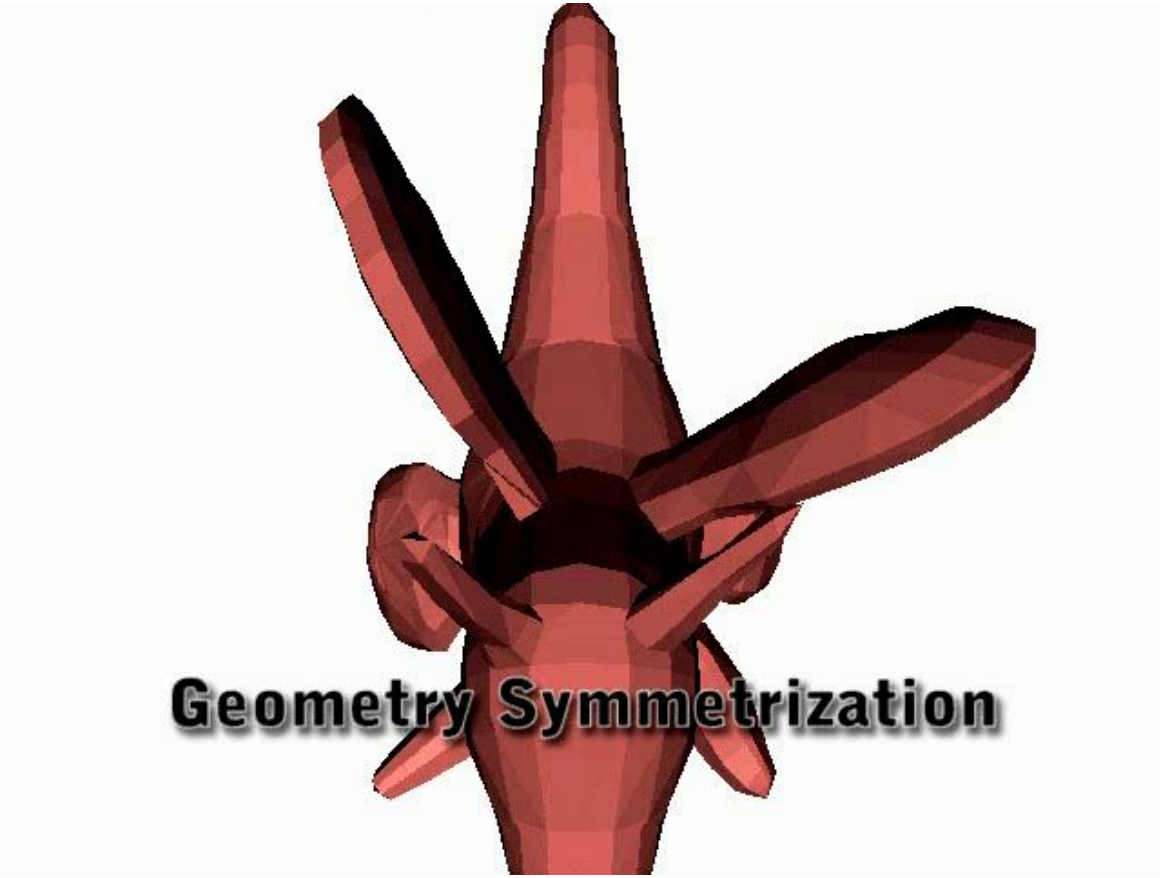
Application: Beautification

Approach: iterative non-rigid deformation to align symmetric points (symmetrization)



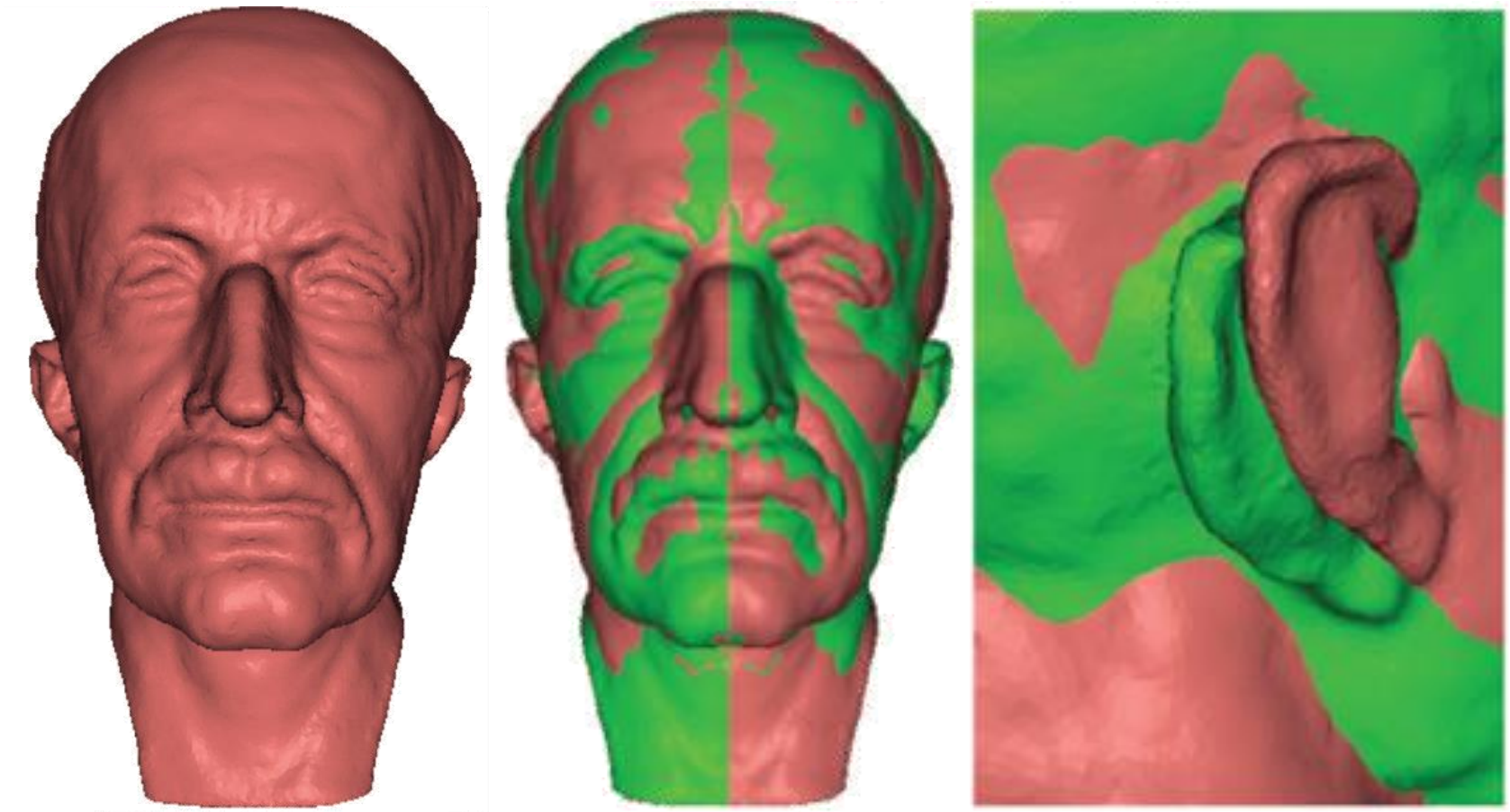
Application: Beautification

Approach: iterative non-rigid deformation to align symmetric points (symmetrization)



Application: Beautification

Results:

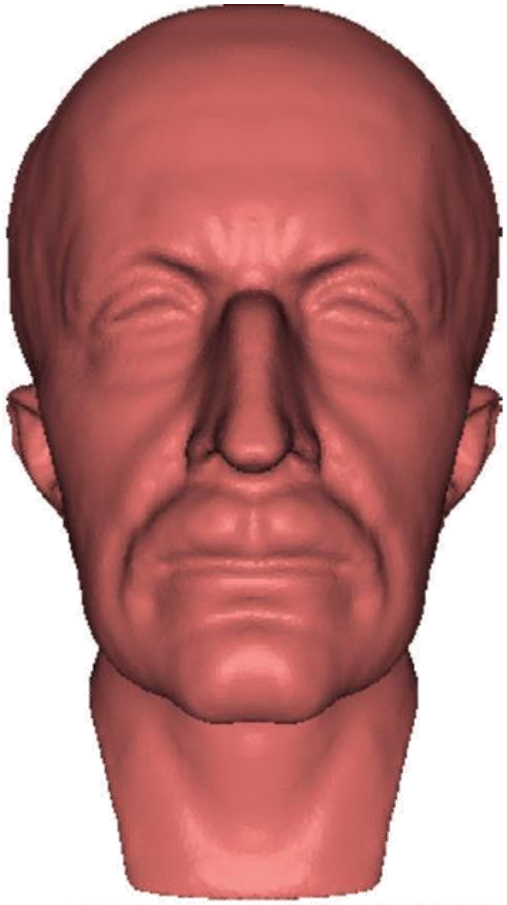


Input Mesh

Mesh overlaid with its reflection

Application: Beautification

Results:

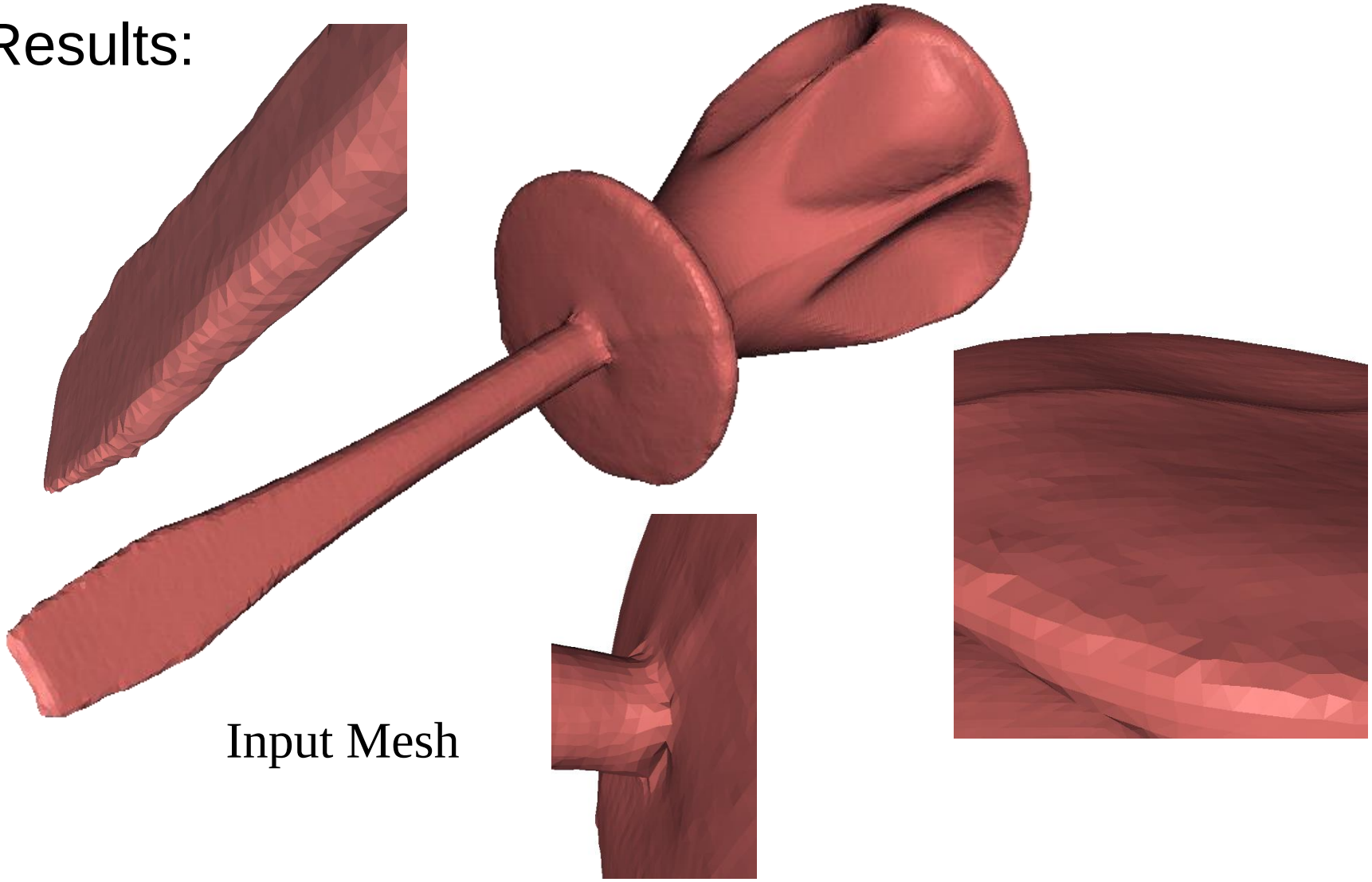


Symmetrized Mesh

Mesh overlaid with its reflection

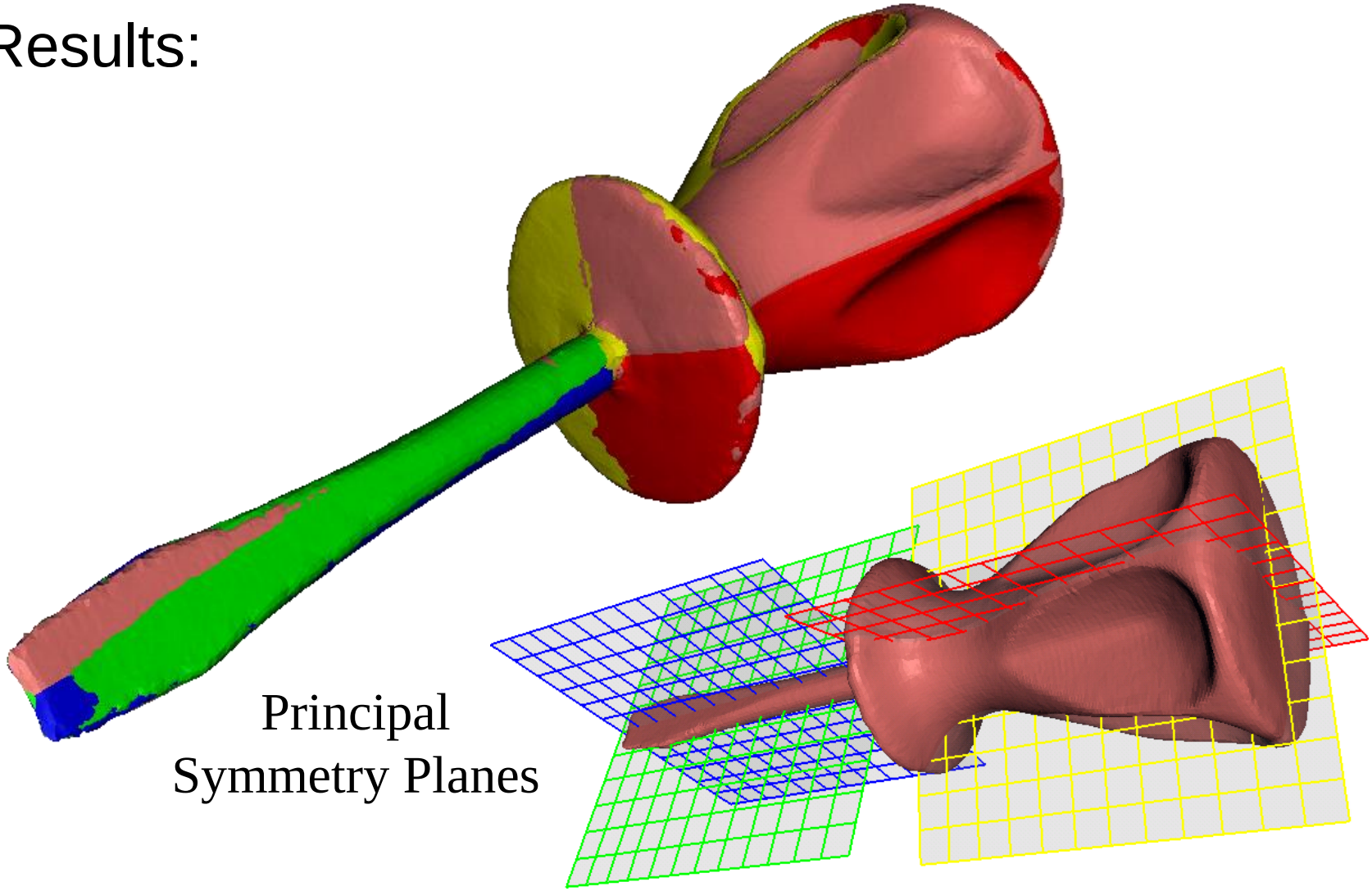
Application: Beautification

Results:



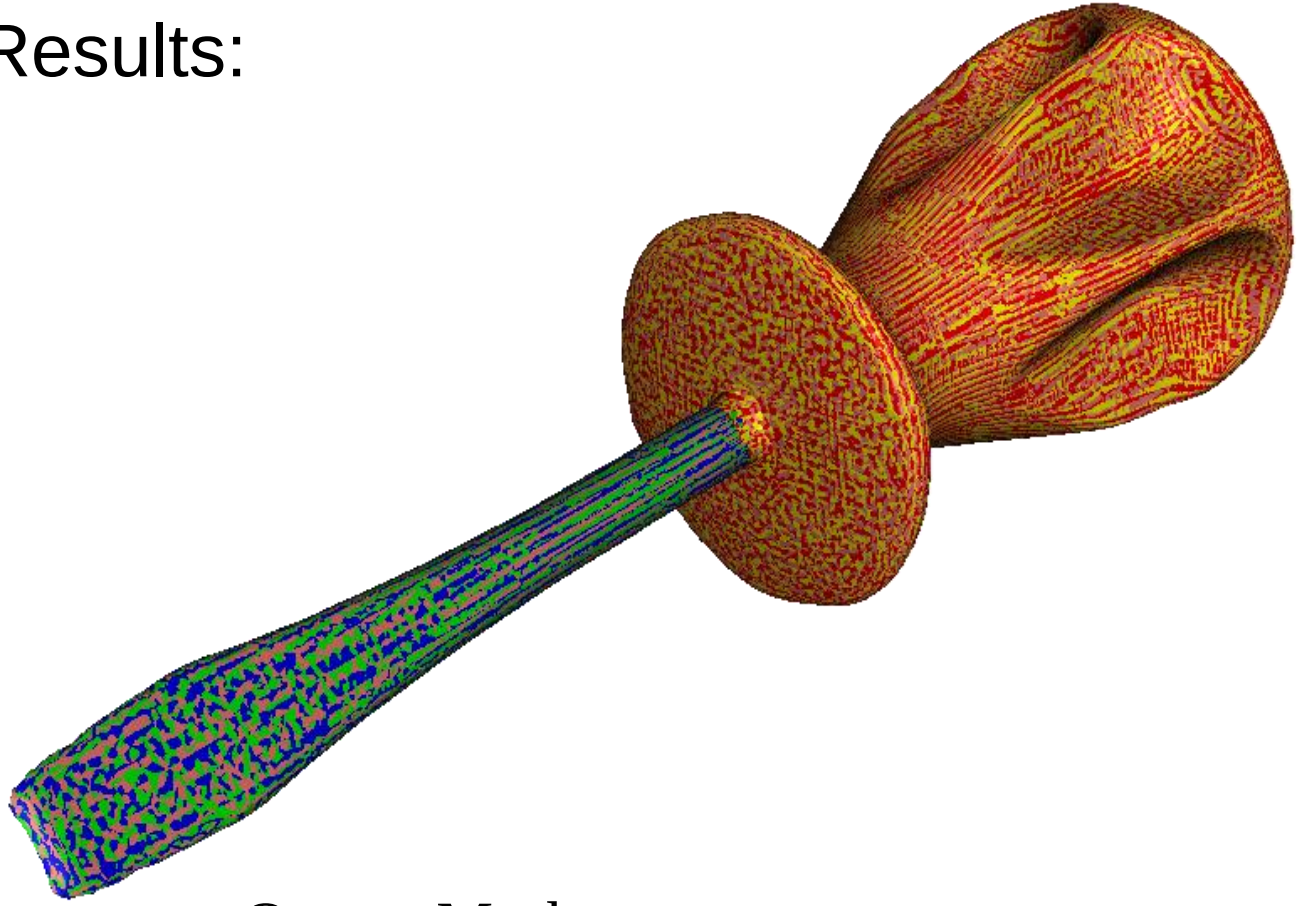
Application: Beautification

Results:



Application: Beautification

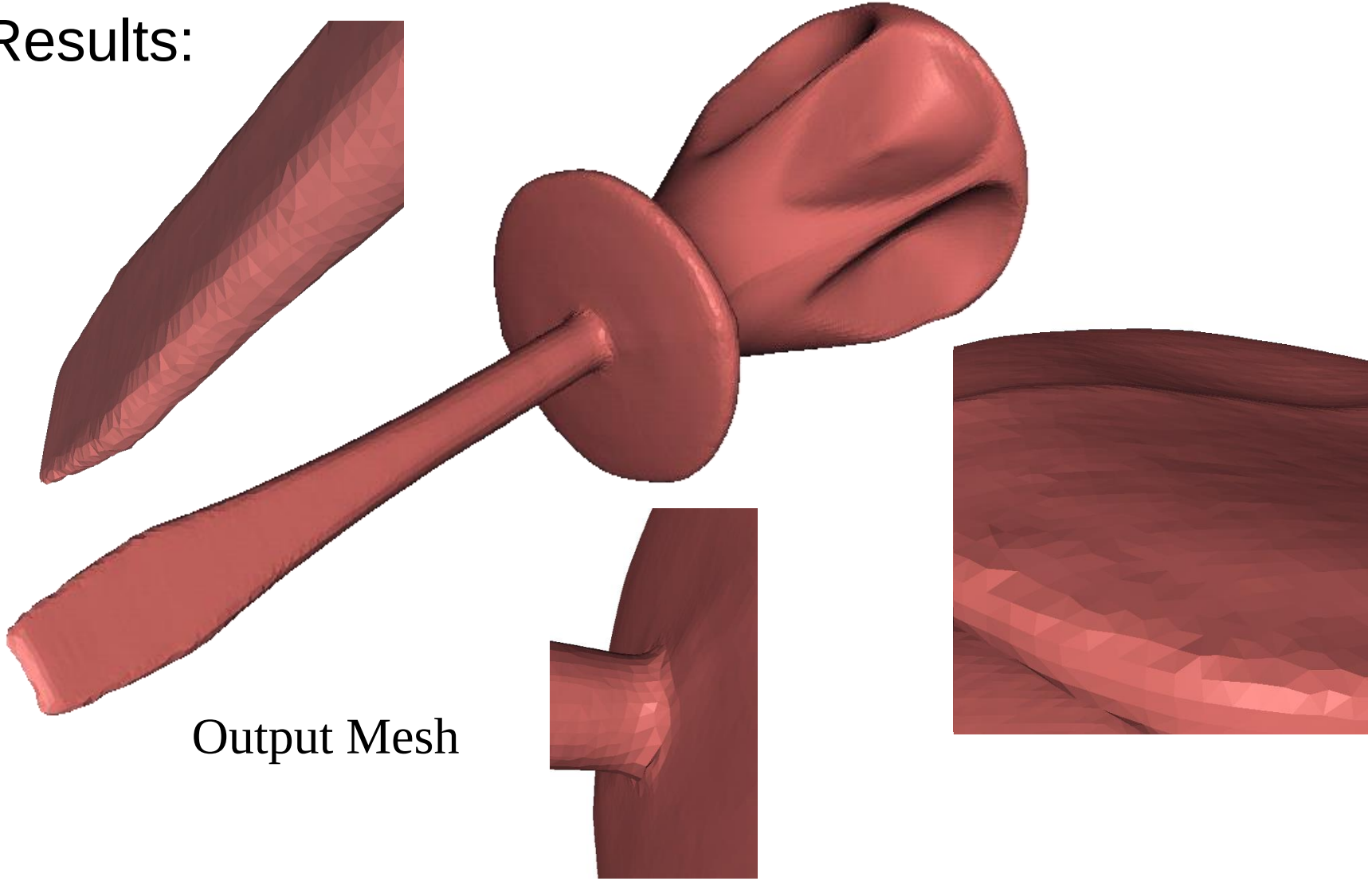
Results:



Output Mesh

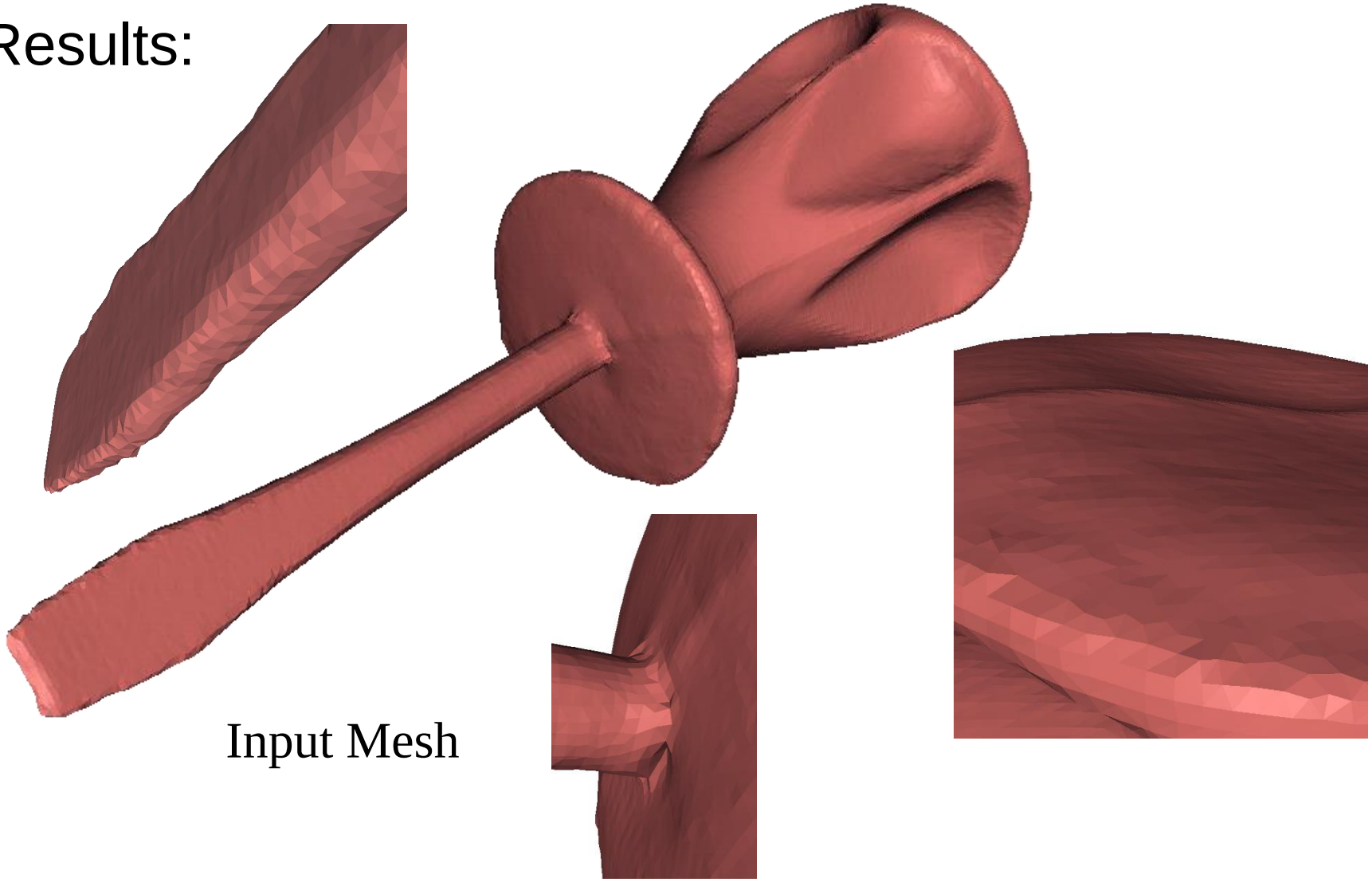
Application: Beautification

Results:



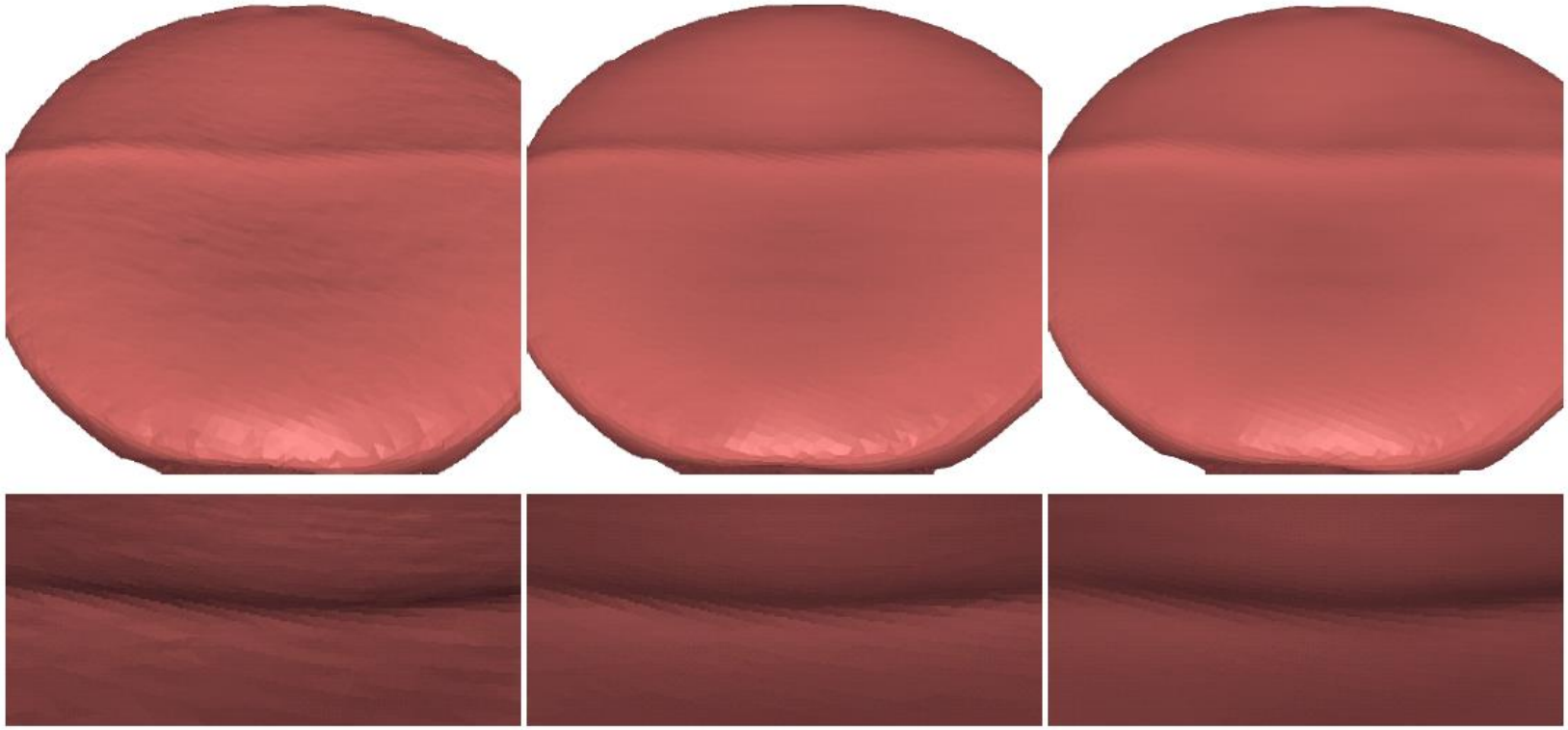
Application: Beautification

Results:



Application: Denoising

Results:



Input Mesh | Symmetrized Mesh | Bilateral Filtering

Applications

Alignment

Matching

Segmentation

Viewpoint selection

Simplification

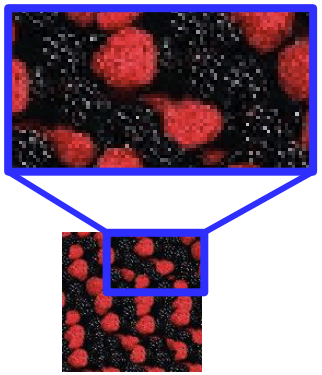
Beautification

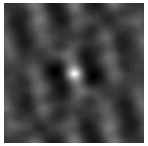
Texture synthesis ←

Application: Texture Synthesis

Maintain symmetries during texture synthesis

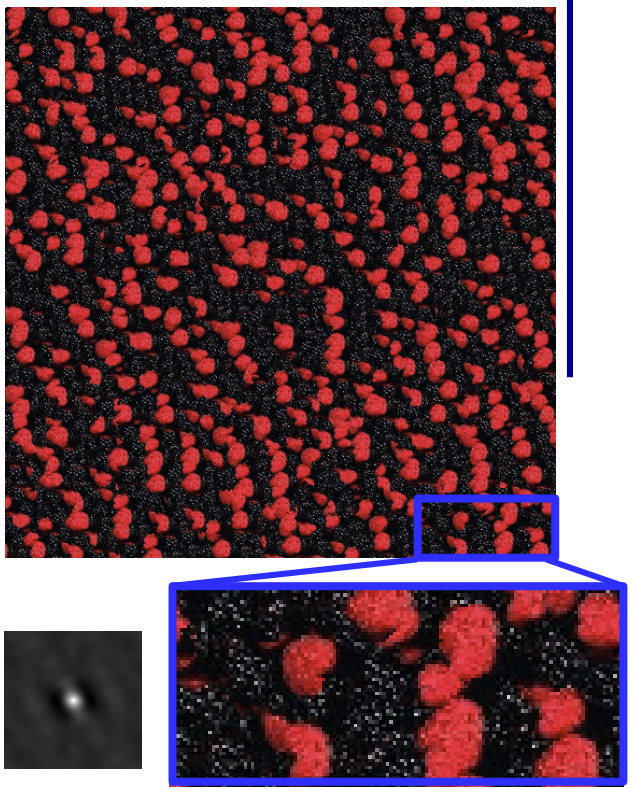
Input



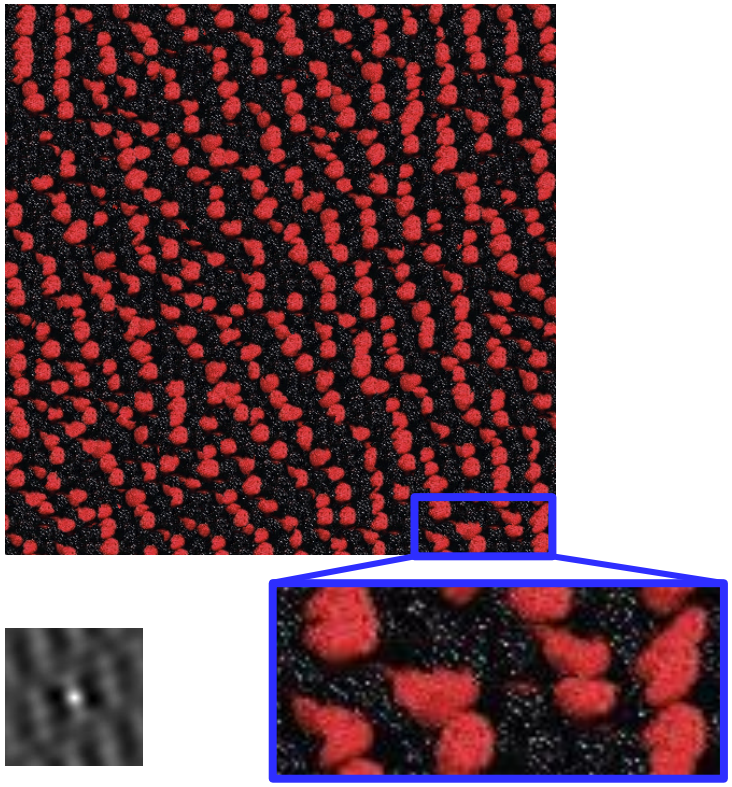


Symmetry Representation

Not Symmetry-guided

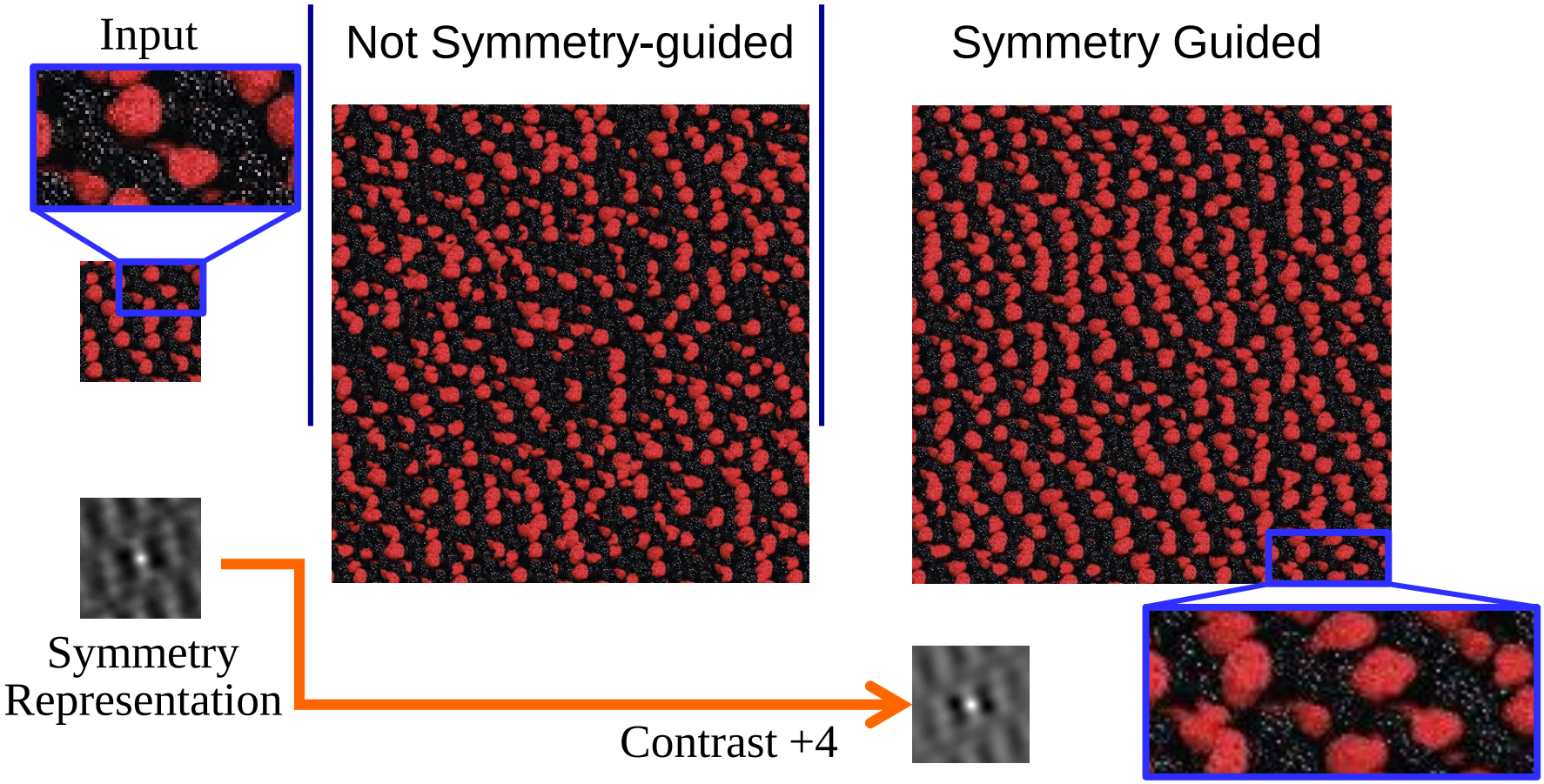


Symmetry Guided



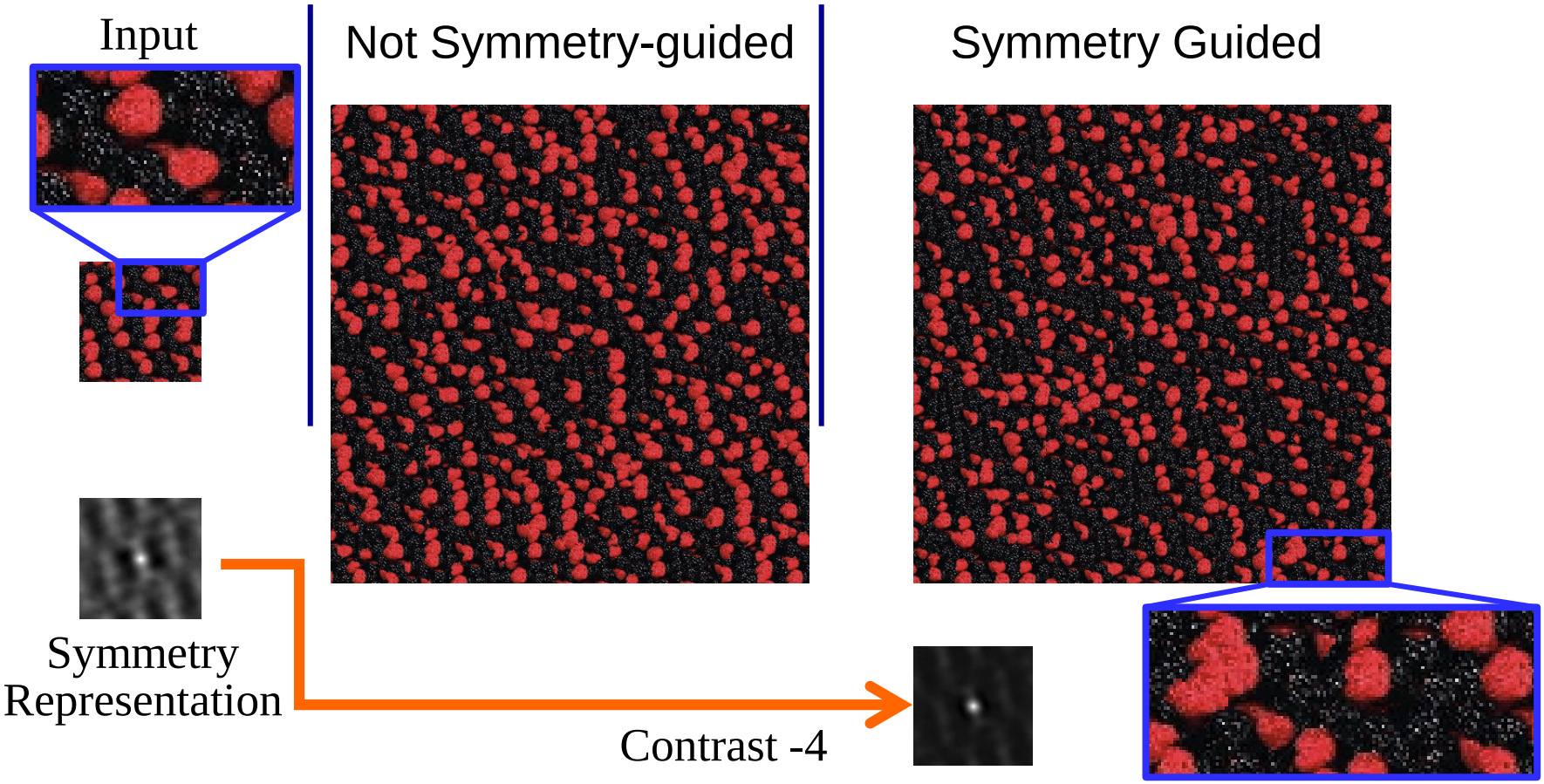
Application: Texture Manipulation

Edit symmetries during texture synthesis



Application: Texture Manipulation

Edit symmetries during texture synthesis



Summary

Representations

- Symmetry descriptor
- Symmetry transform
- Principal symmetries

Applications

- Alignment
- Matching
- Segmentation
- Viewpoint Selection
- Simplification
- Beautification
- Texture synthesis

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