COS 226 – Data Structures and Algorithms
Fall 2014 – Flipped Lecture Section
Group worksheet
Week 8 – 11.06.14
Topics covered: digraphs, MST

Instructions: This worksheet covers directed graphs (digraphs) and Minimum spanning trees (MST). Answer questions as a group (3-4 students)

1. Minimum spanning tree. (fin-s08)

Suppose you know the MST of a weighted graph G. Now, a new edge v-w of weight c is inserted into G to form a weighted graph G'. Design an O(V) time algorithm to determine if the MST in G is also an MST in G'. You may assume all edge weights are distinct. Your answer will be graded for correctness, clarity, and conciseness.

(a) State the algorithm.

(b) Explain briefly why it takes O(V) time.

2. Problem identification [fin-s14]

You are applying for a job in a software company. Determine if each of the following tasks is possible, impossible or unknown.

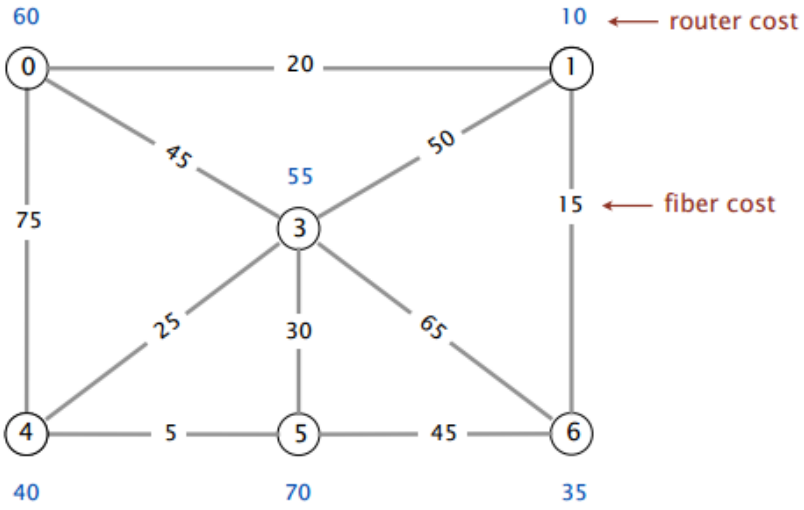
_____ Given an undirected graph, determine if there exists a path of length $V - 1$ with no repeated vertices in time proportional to EV in the worst case.

_____ Given a digraph, determine if there exists a directed path between every pair of vertices in time proportional to $E + V$ in the worst case.

_____ Given a digraph, design an algorithm to determine whether it is a *rooted DAG* (i.e., a DAG in which there is a path from every vertex to some root r) in time proportional to $E + V$ in the worst case.

3. Algorithm Design [fin-s14]

There are N dorm rooms, each of which needs a secure internet connection. It costs $w_i > 0$ dollars to install a secure router in dorm room i and it costs $c_{ij} > 0$ dollars to build a secure fiber connection between rooms i and j. A dorm room receives a secure internet connection if either there is a router installed there or there is some path of fiber connections between the dorm room and a dorm room with an installed router. The goal is to determine in which dorm rooms to install the secure routers and which pairs of dorm rooms to connect with fiber so as to minimize the total cost.



Formulate the problem as a minimum spanning tree problem. To demonstrate your formulation, modify the figure above to show the MST problem that you would solve to find the minimum cost set of routers and fiber connections