COS 226
Final Exam Review
Fall 2014

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Logistics

- The final exam **time and location**
  - The final exam is from 9am to 12noon on Tuesday, January 20 in **McDonnell Hall A02.**
  - The exam will start and end promptly, so please do arrive on time.
- **Exam Format**
  - Closed book, closed note.
  - You may bring one 8.5-by-11 sheet (both sides) with notes in your own handwriting to the exam.
  - No electronic devices (e.g., calculators, laptops, and cell phones).

What to focus on

- focus on understanding basic issues, not memorizing details
- *For each algorithm*
  - understand how it works on typical input
  - Why do we care about this algorithm?
  - How is it different from other algorithms for the same problem?
  - When is it effective?

Material covered

- The exam will *stress* material covered since the midterm, including the following components.
  - Lectures 13–23.
  - *Algorithms in Java, 4th edition*, Chapters 4–6.
  - Exercises 12–22.
  - Programming assignments 6–8
    - Wordnet, seamcarving, burrows-wheeler

Topics covered

Depth-first search

Kruskal's algorithm

Key-indexed counting

Knuth-Morris-Pratt substring search

RE to NFA

Run-length coding

Topics covered

Breadth-first search

Dijkstra's algorithm

LSD radix sort

Boyer-Moore substring search

R-way tries

Huffman coding

Topics covered

Topological sort

Bellman-Ford algorithm

MSD radix sort

Rabin-Karp substring search

Ternary search tries

LZW compression

Topics covered

Prim's algorithm

Ford-Fulkerson algorithm

3-way radix quicksort

Reductions

Burrows-Wheeler

Algorithm Analysis

1. Order of growth

```
public static int f3(int N) {
   if (N == 0) return 1;
   int x = 0;
   for (int i = 0; i < N; i++)
      x += f3(N-1);
   return x;
}
```

```
public static int f2(int N) {
   int x = 0;
   for (int i = 0; i < N; i++)
      for (int j = 0; j < i; j++)
         x++;
   return x;
}
```

2. Order of growth

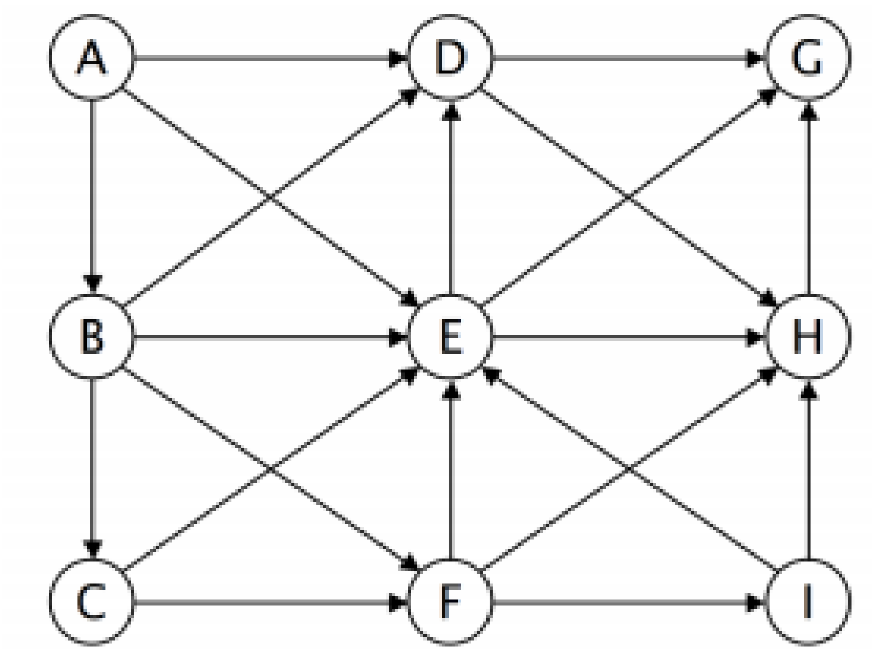
Suppose that you collect the following memory usage data for a program as a function of the input size N.

N	memory
1,000	10,000 bytes
8,000	320,000 bytes
64,000	10,240,000 bytes
512,000	327,680,000 bytes

Estimate the memory usage of the program (in bytes) as a function of N and use tilde notation to simplify your answer.

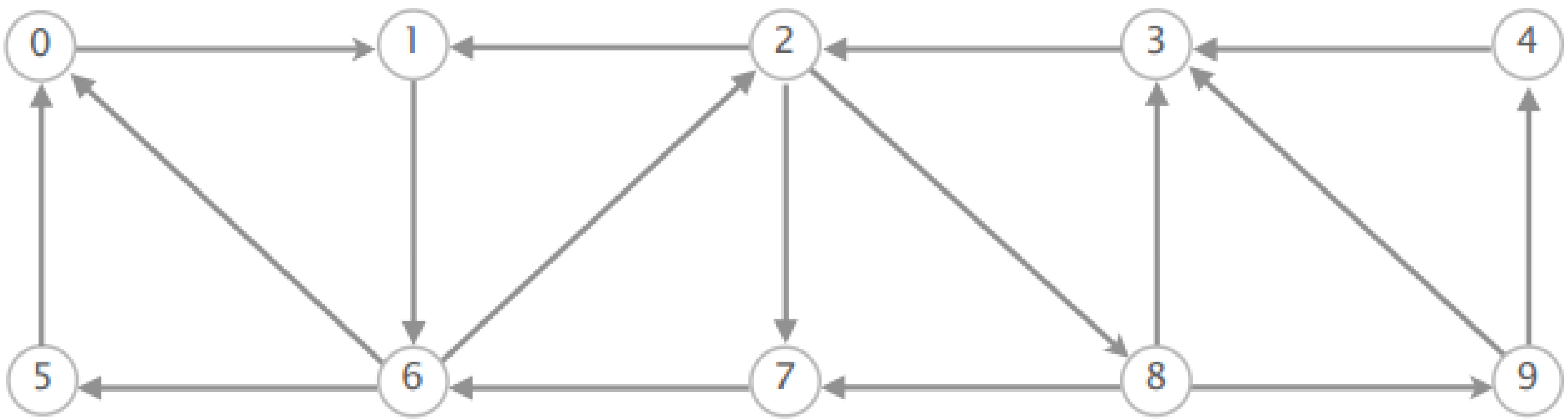
Graph Algorithms

3. Graph Search



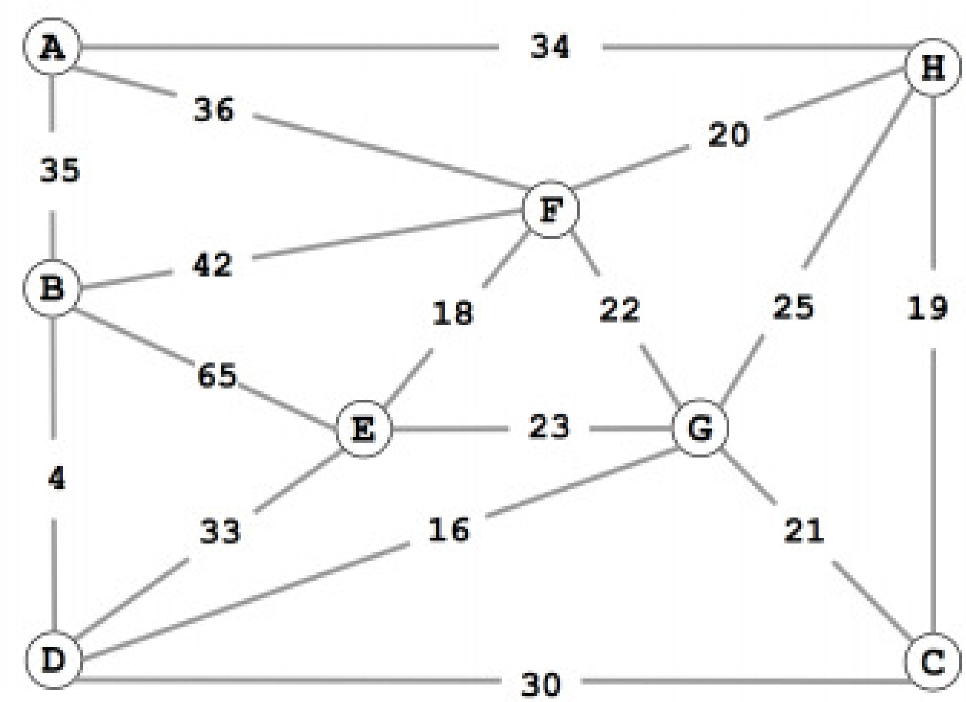
- Run BFS, DFS (starting from vertex A)
- Identify one situation where you would need to use BFS instead of DFS.
- Identify one situation where you would need to use DFS instead of BFS.

4. Order traversals



List the vertices in preorder, post order and reverse post order (starting with node 0)

5. MST



- Run Kruskals

- Prims

6. MST Algorithm Design

Suppose you know the MST of a weighted graph G. Now, a new edge v-w of weight c is inserted into G to form a weighted graph G'. Design an $O(V)$ time algorithm to determine if the MST in G is also an MST in G'. You may assume all edge weights are distinct.

Your answer will be graded for correctness, clarity, and *conciseness*.

1. State the algorithm

2. Explain why your algorithm takes O(V) time

7. Match Algorithms

___ T9 texting in a cell phone

___ 1D range search

___ 2D range search

___ Document similarity

___ Traveling salesperson problem

___ Web crawler

___ Google maps

___ PERT/CPM (Program Evaluation and Review Technique / Critical Path Method).

A. Trie

B. Hashing

C. 3-way radix quicksort

D. Binary search tree

E. Kd tree

F. Depth-first search

G. Breadth-first search

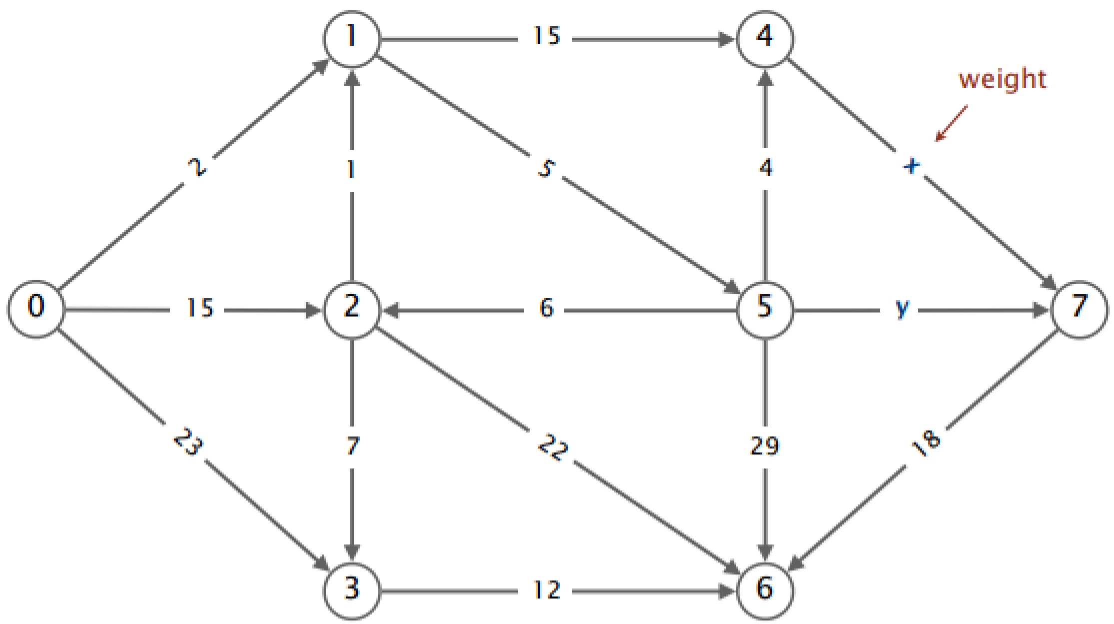
H. Dijkstra's algorithm

I. Topological sort

J. Bellman-Ford

K. Enumerate permutations

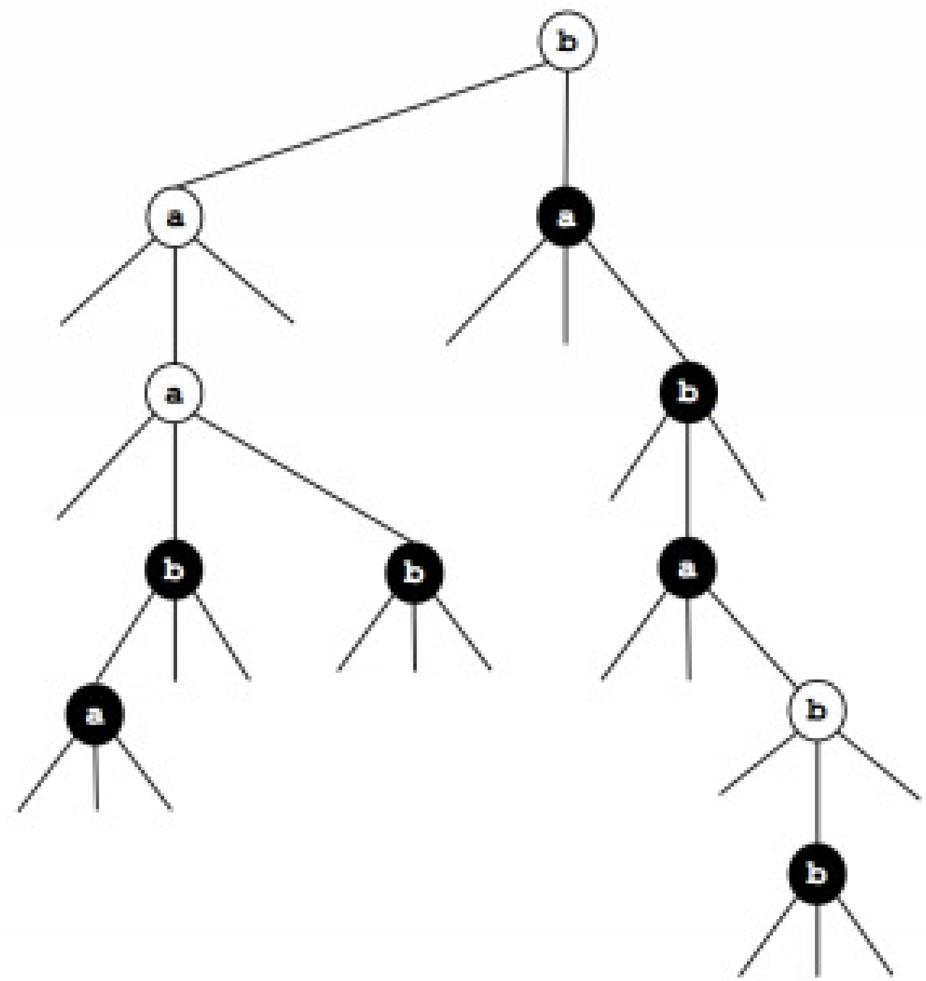
8. Dijkstra's algorithm



v	distTo[]	edgeTo[]
0		
1		
2		
3		
4		
5		
6		
7		

Strings

9. TST



1. List the words in alphabetical order (black nodes denote the end of a word)
2. Insert aaca to TST

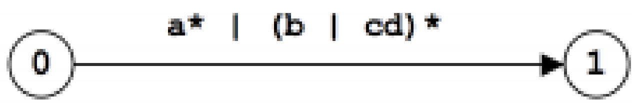
10. String Sorting

Put an X in each box if the string sorting algorithm (the standard version considered in class) has the corresponding property.

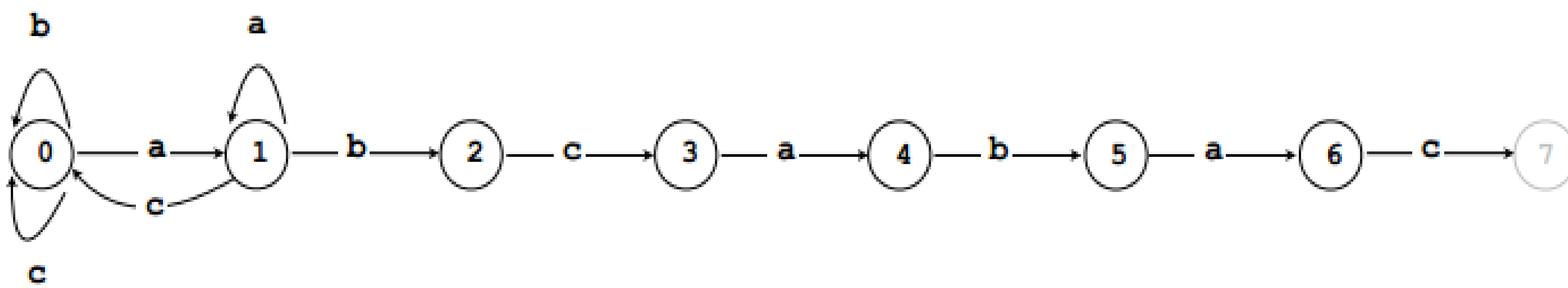
	mergesort	LSD radix sort	MSD radix sort	3-way radix quicksort
stable				
in-place				
sublinear time (in best case)				
fixed-length strings only				

12. Regular Expression to NFA

Convert the RE `a* | (b | c d)*` into an equivalent NFA using the algorithm described in lecture, showing the result after applying each transformation.



13. KMP Table



Compression

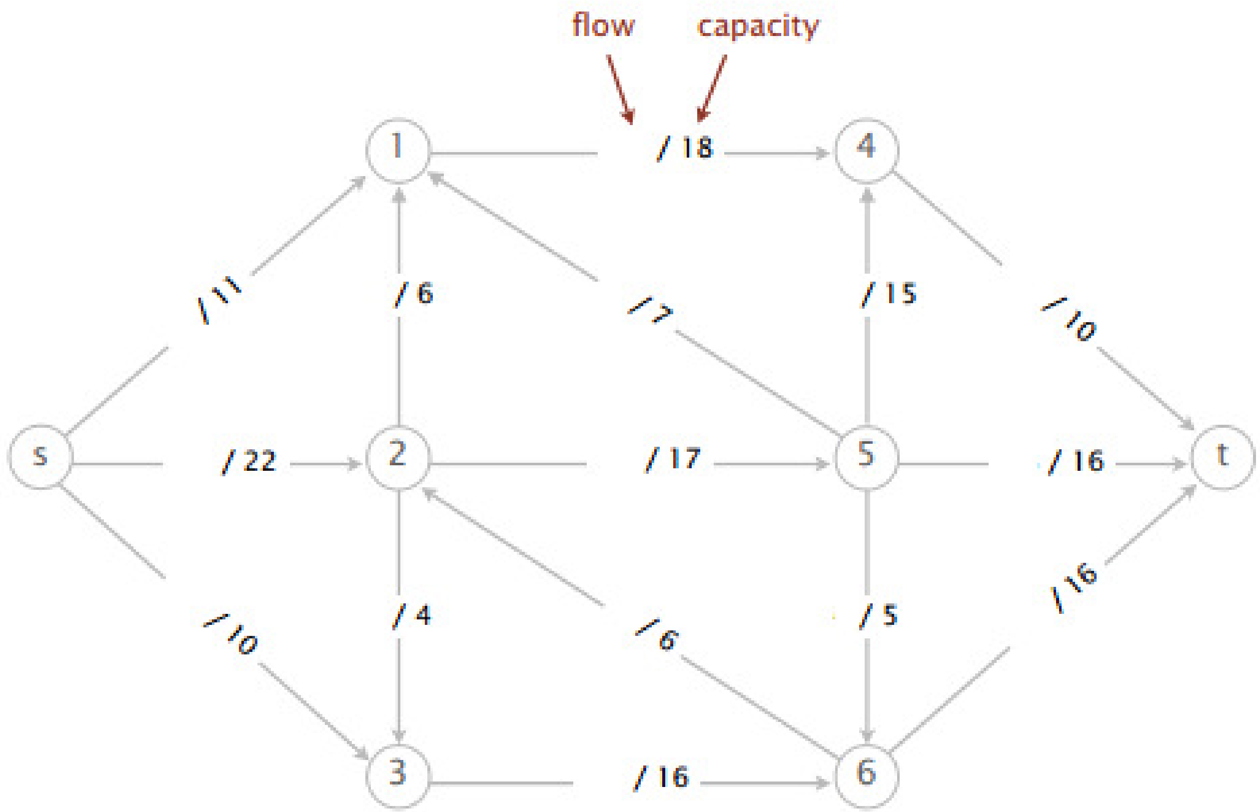
14. LZW compression

You receive the following message encoded using LZW compression.

97
98
128
129
131
132
130

Decode the message (a -97, b-98 … and next code starts - 128

15. MaxFlow-MinCut



17. Algorithm Design

In data compression, a set of binary code words is *prefix-free* if no code word is a prefix of another. For example, $\{01, 10, 0010, 1111\}$ is prefix free, but $\{01, 10, 0010, 10100\}$ is not because 10 is a prefix of 10100.

1. Design an efficient algorithm to determine if a set of binary code words is prefix-free

2. What is the order of growth of the worst-case running time of your algorithm as a function of N and W, where N is the number of binary code words and W is the total number of bits in the input?

3. What is the order of growth of the memory usage of your algorithm?

19. Burrows-Wheeler

What is the Burrows-Wheeler transform of

b a b a a b a c

What is the Burrows-Wheeler inverse transform of

7
b b b b a a a a a

21. counting memory

- standard data types
- object overhead – 16 bytes
- array overhead – 24 bytes
- references – 8 bytes
- Inner class reference – 8 bytes

```
public class TwoThreeTree<Key extends Comparable<Key>, Value> {
   private Node root;

   private class Node {
      private int count;                  // subtree count
      private Key key1, key2;             // the one or two keys
      private Value value1, value2;       // the one or two values
      private Node left, middle, right;   // the two or three subtrees
   }
   ...
}
```

- How much memory is needed for a 2-3 tree that holds N nodes?

22. String Sorting

KISS	ABBA	ENYA	ABBA	ENYA	ACDC	SOAD	SADE	ABBA
ENYA	ACDC	INXS	ACDC	ABBA	ABBA	WHAM	CAKE	ACDC
INXS	AQUA	DIDO	AQUA	AQUA	AQUA	ABBA	CARS	AQUA
STYX	BECK	CARS	BECK	ACDC	BUSH	MOBY	JAYZ	BECK
SOAD	BLUR	ACDC	BLUR	SOAD	BLUR	BECK	ABBA	BLUR
ACDC	BUSH	FUEL	BUSH	CAKE	BECK	ACDC	ACDC	BUSH
KORN	CAKE	BUSH	CAKE	MUSE	CAKE	SADE	BECK	CAKE
FUEL	CARS	ABBA	CARS	HOLE	CARS	DIDO	WHAM	CARS
BUSH	DIDO	AQUA	DIDO	SADE	DIDO	FUEL	DIDO	DIDO
ABBA	ENYA	CAKE	ENYA	BUSH	ENYA	CAKE	KISS	ENYA
WHAM	FUEL	BLUR	FUEL	RUSH	FUEL	HOLE	BLUR	FUEL
CAKE	HOLE	JAYZ	HOLE	BECK	HOLE	TSOL	INXS	HOLE
BLUR	INXS	BECK	INXS	FUEL	INXS	KORN	ENYA	INXS
MUSE	JAYZ	HOLE	JAYZ	TSOL	JAYZ	CARS	SOAD	JAYZ
BECK	KISS	KORN	KISS	WHAM	KISS	MUSE	MOBY	KISS
MOBY	KORN	KISS	KORN	KORN	KORN	BUSH	HOLE	KORN
HOLE	MUSE	TSOL	TSOL	DIDO	MUSE	RUSH	KORN	MOBY
TSOL	MOBY	MOBY	MOBY	BLUR	MOBY	KISS	AQUA	MUSE
JAYZ	RUSH	MUSE	MUSE	KISS	RUSH	AQUA	TSOL	RUSH
AQUA	STYX	SADE	SADE	INXS	STYX	BLUR	STYX	SADE
SADE	SOAD	WHAM	WHAM	CARS	SOAD	INXS	FUEL	SOAD
CARS	SADE	SOAD	SOAD	STYX	SADE	ENYA	MUSE	STYX
DIDO	TSOL	RUSH	RUSH	MOBY	TSOL	STYX	BUSH	TSOL
RUSH	WHAM	STYX	STYX	JAYZ	WHAM	JAYZ	RUSH	WHAM
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0								1

(0) Original input

(1) Sorted

(2) LSD radix sort

(3) MSD radix sort

(4) 3-way string quicksort (no shuffle)

23. Reductions

Consider the following two problems:

- 3SUM. Given N integers $x_1, x_2, \ldots, x_N$, are there three distinct indices i, j, and k such that $x_i + x_j + x_k = 0$?

- 3SUMPLUS. Given N integers $x_1, x_2, \ldots, x_N$ and an integer b, are there three distinct indices i, j, and k such that $x_i + x_j + x_k = b$?

(a) Show that 3SUM linear-time reduces to 3SUMPLUS. To demonstrate your reduction, give the 3SUMPLUS instance that you would construct to solve the following 3SUM instance: $x_1, x_2, \ldots, x_N$.

(b) Show that 3SUMPLUS linear-time reduces to 3SUM. To demonstrate your reduction, give the 3SUM instance that you would construct to solve the following 3SUMPLUS instance: $b, x_1, x_2, \ldots, x_N$.