COS 226	Algorithms and Data Structures	Fall 2011

Midterm

This test has 9 questions worth a total of 60 points. You have 80 minutes. The exam is closed book, except that you are allowed to use a one page cheatsheet. No calculators or other electronic devices are permitted. Give your answers and show your work in the space provided. **Write out and sign the Honor Code pledge before turning in the test.**

"I pledge my honor that I have not violated the Honor Code during this examination."

Problem	Score
0	
1	
2	
3	
4	
Sub 1	

Problem	Score
5	
6	
7	
8	
Sub 2	

Total	

Name:

Login ID:

Precept:

P01	11	Maia Ginsburg
P01A	11	Aman Dhesi
P02	12:30	Sasha Koruga
P02A	12:30	Joey Dodds
P03	1:30	Maia Ginsburg
P03A	1:30	Joey Dodds

0. **Miscellaneous. (1 point)**

   In the space provided on the front of the exam, write your name and Princeton NetID; circle your precept number; and write and sign the honor code.

1. **Union find. (6 points)**

   Circle the letters corresponding to arrays that *cannot* possibly occur during the execution of weighted quick union.

```
          i:  0  1  2  3  4  5  6  7  8  9
              ----------------------------
A.   a[i]:  1  2  3  0  1  1  1  4  4  5

B.   a[i]:  9  0  0  0  0  0  9  9  9  9

C.   a[i]:  1  2  3  4  5  6  7  8  9  9

D.   a[i]:  0  0  0  0  0  1  1  1  6  2

E.   a[i]:  0  0  0  0  0  1  1  1  6  8

F.   a[i]:  0  0  0  1  1  3  3  7  7  7
```

2. **Analysis of algorithms. (6 points)**

   Suppose that you collect the following timing data for a program as a function of the input size N.

N	time
125	0.03 sec
1,000	1.00 sec
8,000	32.00 sec
64,000	1,024.00 sec
512,000	32,768.00 sec

   Estimate the running time of the program (in seconds) as a function of N and use tilde notation to simplify your answer.

   Hint: recall that $\log_b a = \lg a / \lg b$.

3. **Data structures. (9 points)**

   Suppose that the Java library `java.util.LinkedList` is implemented using a doubly-linked list, maintaining a reference to the first and last node in the list, along with its size.

```
public class LinkedList<Item> {
   private Node first;          // the first node in the linked list
   private Node last;           // the last node in the linked list
   private int N;               // number of items in the linked list

   private class Node {
      private Item item;        // the item
      private Node next, prev;  // the next and previous nodes
   }
   ...
}
```

   (a) Using the 64-bit memory cost model from the textbook, how much memory (in bytes) does a `Node` object use and how much does a `LinkedList` object use to store N items? Do *not* include the memory for the items themselves but do include the memory for the references to them.

   - Memory of a `Node`:

   - Memory of a `LinkedList` with N items:

   (b) What is the order of growth of the *worst-case running time* of each of operation below? Write down the best answer in the space provided, using one of the following possibilities.

   $1 \qquad \log N \qquad \sqrt{N} \qquad N \qquad N \log N \qquad N^2$

`addFirst(item)`	*prepend the item to the beginning of the list*	
`get(i)`	*return the item at position i in the list*	
`set(i, item)`	*replace position i in the list with the item*	
`removeLast()`	*delete and return the item at the end of the list*	
`contains(item)`	*is the item in the list?*	

4. **8 sorting and shuffling algorithms. (8 points)**

The column on the left is the original input of strings to be sorted or shuffled; the column on the right are the string in sorted order; the other columns are the contents at some intermediate step during one of the 8 algorithms listed below. Match up each algorithm by writing its number under the corresponding column. Use each number exactly once.

```
navy  coal  corn  blue  blue  blue  wine  bark  mist  bark
plum  jade  mist  gray  coal  coal  teal  blue  coal  blue
coal  navy  coal  rose  gray  corn  silk  cafe  jade  cafe
jade  plum  jade  mint  jade  gray  plum  coal  blue  coal
blue  blue  blue  lime  lime  jade  sage  corn  cafe  corn
pink  gray  cafe  navy  mint  lime  pink  dusk  herb  dusk
rose  pink  herb  jade  navy  mint  rose  gray  gray  gray
gray  rose  gray  teal  pink  navy  jade  herb  leaf  herb
teal  lime  leaf  coal  plum  pink  navy  jade  dusk  jade
ruby  mint  dusk  ruby  rose  plum  ruby  leaf  mint  leaf
mint  ruby  mint  plum  ruby  rose  pine  lime  lime  lime
lime  teal  lime  pink  teal  ruby  palm  mint  bark  mint
silk  bark  bark  silk  bark  silk  coal  silk  corn  mist
corn  corn  navy  corn  corn  teal  corn  plum  navy  navy
bark  silk  silk  bark  dusk  bark  bark  navy  wine  palm
wine  wine  wine  wine  leaf  wine  gray  wine  silk  pine
dusk  dusk  ruby  dusk  silk  dusk  dusk  pink  ruby  pink
leaf  herb  teal  leaf  wine  leaf  leaf  ruby  teal  plum
herb  leaf  rose  herb  cafe  herb  herb  rose  sage  rose
sage  sage  sage  sage  herb  sage  blue  sage  rose  ruby
cafe  cafe  pink  cafe  mist  cafe  cafe  teal  pink  sage
mist  mist  plum  mist  palm  mist  mist  mist  pine  silk
pine  palm  pine  pine  pine  pine  mint  pine  palm  teal
palm  pine  palm  palm  sage  palm  lime  palm  plum  wine
----  ----  ----  ----  ----  ----  ----  ----  ----  ----
 0                                                     1
```

(0) Original input

(1) Sorted

(2) Selection sort

(3) Insertion sort

(4) Mergesort (*top-down*)

(5) Mergesort (*bottom-up*)

(6) Quicksort (*standard, no shuffle*)

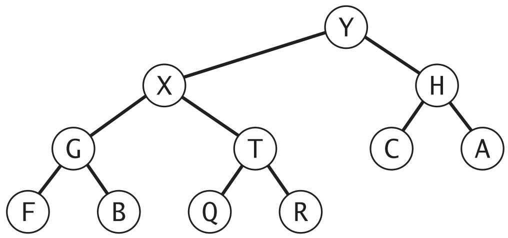
(7) Quicksort (*3-way, no shuffle*)

(8) Heapsort

(9) Knuth shuffle

5. **Binary heaps. (6 points)**

(a) Consider the following binary tree representation of a max-heap.



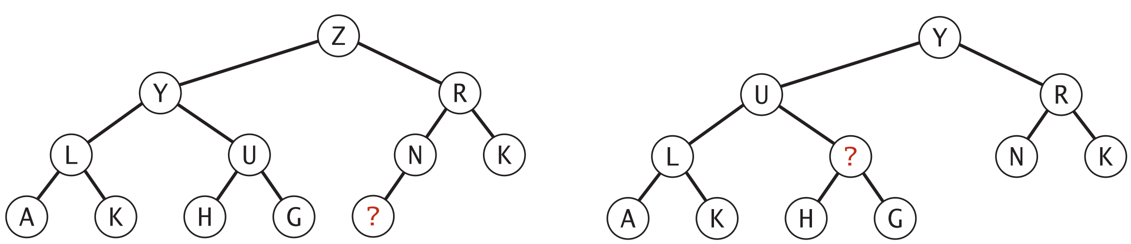
Give the array representation of the heap.

0	1	2	3	4	5	6	7	8	9	10	11	12
-												-

(b) *Insert* the key `P` into the binary heap above, *circling* any entries that changed.

0	1	2	3	4	5	6	7	8	9	10	11	12
-												

(c) A *delete-the-max* operation in the binary heap at left results in the binary heap at right.

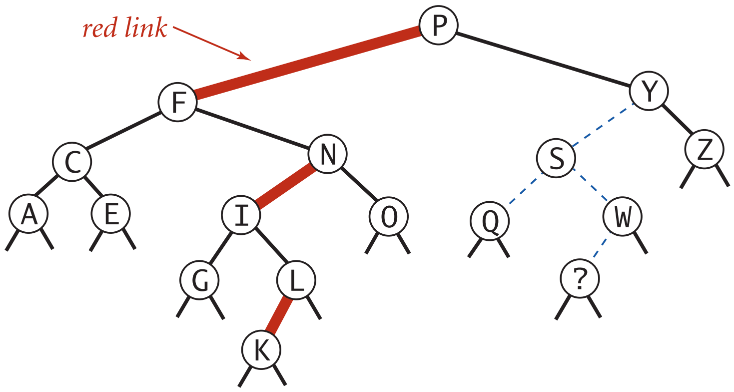


Which of the keys below could be the one labeled with a question mark?
Circle all possibilities.

`A B C D E F G H I J K L M N O P Q R S T U V W X Y Z`

6. **Red-black BSTs. (8 points)**

Consider the following left-leaning red-black BST. Some of the colors and key values are suppressed.



(a) Which of the keys below could be the one labeled with a question mark?
Circle all possibilities.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

(b) For each link from the left-hand column, select its possible color(s) from the right-hand column.

____ link between W and S	A. red
____ link between ? and W	B. black
____ link between S and Y	C. either red or black
____ link between Q and S	

(c) How many *left rotation*, *right rotation*, and *color flip* operations would be used to insert each key below into the original red-black BST above?

	H	D	B	J
rotateLeft()	1			
rotateRight()	0			
flipColors()	0			

7. **Comparing two arrays of points. (8 points)**

Given two arrays `a[]` and `b[]`, each containing N distinct points in the plane, design two algorithms (with the performance requirements specified below) to determine whether the two arrays contains precisely the same set of points (but possibly in a different order).

For each algorithm, give a crisp and concise English description of your algorithm. Your answer will be graded on correctness, efficiency, and clarity.

(a) Design an algorithm for the problem whose running time is linearithmic in the *worst case* and uses at most constant extra space.

(b) Design an algorithm for the problem whose running time is linear under reasonable assumptions and uses at most linear extra space. Be sure to state any assumptions that you make.

8. **Stabbing count queries. (8 points)**

Given a collection of x-intervals and a real value x, a *stabbing count query* is the number of intervals that contain x. Design a data structure that supports interval insertions intermixed with stabbing count queries by implementing the following API:

public class IntervalStab

	`IntervalStab()`	*create an empty data structure*
`void`	`insert(double xmin, double xmax)`	*insert the interval (xmin, xmax) into the data structure*
`int`	`count(double x)`	*number of intervals that contain x*

For example, after inserting the five intervals $(3, 10)$, $(4, 5)$, $(6, 12)$, $(8, 15)$, and $(19, 30)$ into the data structure, `count(9.1)` is 3 and `count(17.2)` is 0.

If there are N intervals in the data structure, you should support *insert* and *count* in time proportional to $\log N$ in the worst case (even if `count()` returns N).

For simplicity, assume that no two intervals contain a left or right endpoint in common and that the argument to the stabbing count query is not equal to a left or right endpoint.

Give a crisp and concise English description of your data structure.
Your answer will be graded on correctness, efficiency, and clarity.

- `IntervalStab()`:

- `insert(xmin, xmax)`:

- `count(x)`: