## Outline

- Differential surface representation
- Ideas and applications
- Compact shape representation
- Mesh editing and manipulation
- Membrane and flattening
- Generalizing Fourier basis for surfaces


## Motivation

- Meshes are great, but:
- Geometry is represented in a global coordinate system
- Single Cartesian coordinate of a vertex doesn't say much



## Motivation

- Meshes are difficult to edit



## Laplacian Mesh Editing

- Meshes are difficult to edit



## Motivation

- Meshes are difficult to edit



## Differential coordinates

- Represent a point relative to it's neighbors.
- Represent local detail at each surface point - better describe the shape
- Linear transition from global to differential
- Useful for operations on surfaces where surface details are important



## Connection to the smooth case

- The direction of $\delta_{i}$ approximates the normal
- The size approximates the mean curvature


$$
\mathbf{\delta}_{\mathbf{i}}=\frac{1}{d_{i}} \sum_{v \in N(i)}\left(\mathbf{v}_{\mathbf{i}}-\mathbf{v}\right) \quad \frac{1}{\operatorname{len}(\gamma)} \int_{\mathbf{v} \in \gamma}\left(\mathbf{v}_{\mathbf{i}}-\mathbf{v}\right) d s
$$

$$
\lim _{\operatorname{len}(\gamma) \rightarrow 0} \frac{1}{\operatorname{len}(\gamma)} \int_{\mathbf{v} \in \gamma}\left(\mathbf{v}_{\mathbf{i}}-\mathbf{v}\right) d s=H\left(\mathbf{v}_{\mathbf{i}}\right) \mathbf{n}_{\mathbf{i}}
$$

## Weighting schemes

$$
\delta_{i}=\frac{\sum_{j \in N(i)} w_{i j}\left(\mathbf{v}_{i}-\mathbf{v}_{j}\right)}{\sum_{j \in N(i)} w_{i j}}
$$

- Ignore geometry
$\delta_{\text {umbrella }}: w_{i j}=1$
- Integrate over circle around vertex
$\delta_{\text {mean value }}: w_{\mathrm{ij}}=\tan \phi_{\mathrm{ij}} / 2+\tan \phi_{\mathrm{ij}+1} / 2$
- Integrate over Voronoi region of vertex

$$
\delta_{\text {cotangent }}: w_{\mathrm{ij}}=\cot \alpha_{\mathrm{ij}}+\cot \beta_{\mathrm{ij}}
$$

$\delta_{\text {cotangent }}: w_{\mathrm{ij}}=\cot \alpha_{\mathrm{ij}}+\cot \beta_{\mathrm{ij}}$


## Differential coordinates

"Local control for mesh morphing", Alexa 01

- Detail = surface - smooth(surface)
- Smoothing = averaging


$$
\begin{aligned}
\boldsymbol{\delta}_{i} & =\mathbf{v}_{i}-\frac{1}{d_{i}} \sum_{j \in N(i)} \mathbf{v}_{j} \\
\boldsymbol{\delta}_{i} & =\sum_{j \in N(i)} \frac{1}{d}\left(\mathbf{v}_{i}-\mathbf{v}_{j}\right)
\end{aligned}
$$



The mesh


The symmetric Laplacian $L_{s}$

## Laplacian mesh

- Vertex positions are represented by Laplacian coordinates ( $\delta_{x} \delta_{y} \delta_{z}$ )

$\delta_{i}=\sum_{j \in N(i)} w_{i j}\left(\mathbf{v}_{i}-\mathbf{v}_{j}\right)$



## Basic properties

- $\operatorname{rank}(L)=\mathrm{n}-\mathrm{c} \quad(\mathrm{n}-1$ for connected meshes)
- We can reconstruct the xyz geometry from $\delta$ up to translation



## Reconstruction



## Cool underlying idea

- Mesh vertex positions are defined by minimizer of an objective function


$$
\tilde{\mathbf{x}}=\underset{\mathbf{x}}{\arg \min }\left(\left\|L \mathbf{x}-\boldsymbol{\delta}_{x}\right\|^{2}+\sum_{s=1}^{k}\left|x_{k}-c_{k}\right|^{2}\right)
$$

## Reconstruction



## Reconstruction



A $\mathbf{x}=\mathbf{b}$
Normal Equations:

$$
\begin{aligned}
\mathbf{A}^{\mathrm{T}} \mathbf{A} \mathbf{x} & =\mathbf{A}^{\mathrm{T}} \mathbf{b} \\
\mathbf{x} & =\underbrace{\left(\mathbf{A}^{\mathrm{T}} \mathbf{A}\right)^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{b}}_{\begin{array}{c}
\text { compute } \\
\text { once }
\end{array}}
\end{aligned}
$$

## What we have so far

- Laplacian coordinates $\delta$
- Local representation
- Translation-invariant
- Linear transition from $\delta$ to $x y z$
- can constrain more that 1 vertex
- least-squares solution

Editing using differential coordinates

- The editing process from the user's point of view:

1) First, a ROI , anchors and a handle vertex should be set.
2) Then the edit is Performed By moving this vertex.


## Mesh Editing Example



## Mesh Editing Example



Editing using differential coordinates

- The user moves the handle and interactively the surface changes.
- The stationary anchors are responsible for smooth transition of the edited part to the rest of the mesh.
- This is done using increasing weight with geodesic distance in the soft spatial equations.



## Mesh Editing Example



## Mesh Editing Example



What else can we do with it?
$\qquad$

## Parameterization

- Use zero Laplacians.


In 2D:


Texture Mapping *


## Feature Preserving Smoothing

- Weighted positional and smoothing constraints



## Detail transfer

- "Peel" the coating of one surface and transfer to another


Detail transfer


## Mesh transplanting

- Geometrical stitching via Laplacian mixing



## Detail transfer


$\Longrightarrow$


- Taking weighted average of $\delta_{i}$ and $\delta_{i}{ }^{\text {a }}$


Mesh transplanting

- Details gradually change in the transition area



## Mesh transplanting

- Details gradually change in the transition area



## The End

