Laplacian Meshes

COS 526 – Fall 2012

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Outline

Differential surface representation

- Ideas and applications
 - Compact shape representation
 - Mesh editing and manipulation
 - Membrane and flattening
 - Generalizing Fourier basis for surfaces



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Motivation

Meshes are great, but:

- Geometry is represented in a global coordinate system
 - Single Cartesian coordinate of a vertex doesn't say much

Laplacian Mesh Editing

· Meshes are difficult to edit



Motivation

Meshes are difficult to edit



Motivation

- Meshes are difficult to edit



Differential coordinates

- Represent a point *relative* to it's neighbors.Represent *local detail* at each surface point
- better describe the shape
- Linear transition from global to differential
- Useful for operations on surfaces where surface details are important



Differential coordinates

"Local control for mesh morphing", Alexa 01

- Detail = surface smooth(surface)
- Smoothing = averaging



Connection to the smooth case

- The direction of δ_i approximates the normal
- The size approximates the mean curvature



$$\lim_{len(\gamma)\to 0} \frac{1}{len(\gamma)} \int_{\mathbf{v}\in\gamma} (\mathbf{v}_i - \mathbf{v}) ds = H(\mathbf{v}_i) \mathbf{n}_i$$

Laplacian matrix





The mesh

The symmetric Laplacian Ls



Laplacian mesh

Vertex positions are represented by Laplacian coordinates (δ, δ, δ,)





Basic properties

rank(L) = n - c (n - 1 for connected meshes)

• We can reconstruct the xyz geometry from δ up to translation



Reconstruction



Reconstruction



Reconstruction

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Cool underlying idea

 Mesh vertex positions are defined by minimizer of an objective function



What we have so far

- _____
- Laplacian coordinates $\boldsymbol{\delta}$
 - Local representation
 - Translation-invariant
- Linear transition from δ to xyz
 - can constrain more that 1 vertex
 - least-squares solution

Editing using differential coordinates

- The editing process from the user's point of view:
- 1) First, a ROI , <u>anchors</u> and a <u>handle vertex</u> should be set.
- Then the edit is Performed By moving this vertex.



Editing using differential coordinates

- The user moves the handle and interactively the surface changes.
- The stationary anchors are responsible for smooth transition of the edited part to the rest of the mesh.
- This is done using increasing weight with geodesic distance in the soft spatial equations.



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Mesh Editing Example



Mesh Editing Example



Mesh Editing Example



Mesh Editing Example



What else can we do with it?

Parameterization





Feature Preserving Smoothing

. . . · Weighted positional and smoothing constraints



Feature Preserving Smoothing

•• · Weighted positional and smoothing constraints





Smoothed

Detail transfer

"Peel" the coating of one surface and transfer to another



Detail transfer



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Detail transfer



Mixing Laplacians

• Taking weighted average of δ_i and δ_i'



Mesh transplanting

Geometrical stitching via Laplacian mixing



Mesh transplanting

Details gradually change in the transition area



Mesh transplanting

Details gradually change in the transition area



The End

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