COS 423 Problem Set 5 Due Wednesday December 12, 2012
Fall 2012
Tarjan

Collaboration allowed on problems 2 and 4, not on 1, 3, and 5.

1. (10 points) Show that *TQBF* (true quantified Boolean formulas) restricted to formulas in which the part following the quantifiers is in 3-CNF (conjunctive normal form with at most three literals per clause) is PSPACE-complete. You may use the fact (proved in class) that unrestricted *TQBF* is PSPACE-complete.

2. (a) (5 points) Let *A* be the language of properly nested parentheses. For example, (( )(( ))) is in *A* but ( ))(( ) is not. Show that *A* is in L.

(b) (5 points) Let *B* be the language of properly nested parentheses and brackets. For example, ([( )( )]( )[ ]) is in *B* but ([ )] is not. Show that *B* is in L. Hint: use the result of part (a) in an algorithm that checks each "(" for a matching ")" and each "[" for a matching "]".

3. (10 points) A directed graph is *strongly connected* if there is a path from any vertex to any other. Let $SG = \{\langle G \rangle \mid G \text{ is a strongly connected graph}\}$. Prove that *SG* is NL-complete.

4. (10 points) Prove that if EXPTIME ≠ NEXPTIME, then P ≠ NP. Hint: you may find helpful the function *pad*(*s*, *k*), which, given a string *s* of length at most *k*, returns *s* with $k - |s|$ copies of # appended.

5. (10 points) A 3DFA is a three-headed, two-way deterministic finite automaton: It has three heads and a read-only input tape, which contains an input word with a single blank on each side of the input word. The heads start at the first symbol of the input and can move independently in either direction, but cannot move past either blank. (The blanks allow the machine to know when it has reached the beginning or the end of the input word.) Prove that there is a language in P that is not accepted by a 3DFA. Hint: show that there is some fixed *k* such that every language accepted by a 3DFA can be accepted by an $O(n^k)$-time-bounded Turing machine. Then use the time hierarchy theorem.