COS 487 Problem Set 4 Due Wednesday Nov. 21
Fall 2012
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Collaboration allowed on Problems 1 and 2; cite your collaborators. No collaboration allowed on 3, 4, and 5.

1. (a) (5 points) Give a polynomial-time algorithm that determines whether two DFA's accept the same language.

(b) (5 points) Call a language L *star-closed* if $L^* = L$. Give a polynomial-time algorithm to determine whether a DFA accepts a star-closed language.

2. (10 points) Let *SET-SPLITTING* = {<*S*, *C*>| S is a finite set and $C = \{C_1, C_2, \ldots, C_k\}$ is a collection of $k \geq 1$ subsets of S such that the elements of S can be colored red or blue so that each subset C_i has at least one red element and at least one blue one}. Prove that *SET-SPLITTING* is NP-complete.

3. (10 points) Consider the following scheduling problem. You are given a list of final exams to be scheduled, and a list of students taking each exam. (A student can be taking one or more exams.) Each exam takes the same amount of time. You must schedule the exams into slots so that no student is required to take two or more exams in the same slot. The problem is to determine whether there is a schedule that fits into k slots. Formulate this problem as a language and show that it is NP-complete.

4. (10 points) Show that if P = NP, there is a polynomial-time algorithm to factor any integer represented in binary. Note: See the discussion in Sipser's Problem 7.38. Hint: How can you test whether a given interval contains a factor? Use this plus binary search.

5. (10 points) Given a CNF formula with m variables and c clauses, show how to construct in polynomial time an NFA with $O(cm)$ states that accepts all truth assignments that make the formula false. (Each assignment is encoded as a string of m bits.) Use this result to show that P ≠ NP implies that NFA's cannot be minimized in polynomial time. (That is, there is no polynomial algorithm that, given an NFA, will find an equivalent NFA with as few states as possible.)