COS 487 Problem Set 3 Due Wednesday October 24, 2012, 1:30 PM
Fall 2012
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No collaboration. Please work on all these problems by yourself. Ten points per problem (five each on parts (a) and (b) of Problem 4)

1. Let $C = \{\langle G, x\rangle \mid G$ is a CFG and x is a substring of some $y \in L(G)\}$. Show that C is decidable.

2. Let $PAL_{DFA} = \{\langle M\rangle \mid M$ is a DFA that accepts some palindrome$\}$. Prove that PAL_{DFA} is decidable.

3. A *useful* state in a deterministic Turing machine is one that is entered during the machine's computation on at least one accepted input word. Consider $L = \{\langle M, q\rangle \mid$ State q is a useful state of deterministic Turning machine $M\}$. Prove that L is Turing-recognizable but not decidable.

4. (a) Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left when its head is on the first tape cell of the input. Formulate this problem as a language and prove that it is Turing-recognizable but not decidable.

(b) Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left at any point during its computation on w. Formulate this problem as a language problem and prove that it is decidable.

5. Prove that the language $OVERLAP_{CFG} = \{\langle G, H\rangle \mid G$ and H are CFG's such that $L(G) \cap L(H) \neq \varphi\}$ is undecidable.