COS 487 Problem Set 2 Due Wednesday October 10
Fall 2012
Tarjan

Collaboration is allowed on all problems.

1. (5 points per part) Given a language L (set of strings over some finite alphabet) and a positive integer k, we define $Drop(L, k)$ to be the set of strings formed from the strings in L by taking each string in L whose length n is evenly divisible by k and dropping every n/k^{th} symbol. For example,

$Drop(L, 1) = \{w \mid wa \in L$ for some symbol $a\}$, and

$Drop(L, 2) = \{vw \mid |v| = |w|$ and $vawb \in L$ for some symbols a and $b\}$.

Suppose L is regular.

(a) Is $Drop(L, 1)$ necessarily regular? Context-free?
(b) Is $Drop(L,2)$ necessarily regular? Context-free?
(c) Is $Drop(L, 3)$ necessarily regular? Context-free?

In each case prove your answer.

2. (5 points per part) For a fixed positive integer k, let $L(k)$ be the set of strings over {0, 1} such that the k^{th} symbol from the right end is a 0. (Thus every string in $L(k)$ contains at least k symbols.) Prove that (a) there is an NFA with $k + 1$ states that accepts $L(k)$, but that (b) every DFA that accepts $L(k)$ has at least 2^k states. Thus an exponential blowup in the number of states is needed to convert an NFA to a DFA in the worst case.

3. (15 points) Prove the following stronger version of the pumping lemma for CFL's: If L is a CFL, there is an integer p such that if $s \in L$ and $|s| \geq p$, then s may be divided into five pieces, $s = uvxyz$, such that $uv^ixy^iz \in L$ for all $i \geq 0$, $|vxy| \leq p$, $|v| \geq 1$, and $|y| \geq 1$. Hint: Consider parse trees such that the longest path must contain some non-terminal at least three times (as compared to two, in the proof of the standard pumping lemma).

4. (10 points) If A and B are languages, define $A \lozenge B = \{vw \mid v \in A$ and $w \in B$ and $|v| = |w|\}$. Prove that if A and B are regular, then $A \lozenge B$ is context-free.