COS 487 Problem Set 1 Due Wednesday September 26, 1:30 PM
Fall 2012
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Collaboration Policy: On those problems marked "Collaboration allowed," you may discuss the problems with your fellow students and consult source material other than the textbook. Your write-ups of the solutions must be done entirely by you, however, and you must cite any collaborators or sources other than the textbook. On problems not marked "Collaboration allowed," solve the problem entirely on your own, using the only the textbook. On all problems you are welcome to discuss your questions and ideas with the assistant and/or the instructor. Problem sets will be due at the beginning of class on Wednesdays.

1. ("Strong" passwords). You want to establish an account with an on-line service, for which you need to choose a password. They require that your password contain at least eight symbols, including at least two different letters, at least two digits and at least one special symbol. Devise a DFA that accepts exactly the strings that satisfy these requirements, which you can use to test your password choice for validity. For simplicity, assume that the set of possible symbols is {a, b, c, d, 0, 1, #, $}, of which a, b, c, and d are letters, 0 and 1 are digits, and # and $ are special symbols.

2. (Collaboration allowed) Given a regular language L, the language $L(1/3)$ consists of every word w such that there is a word x twice as long as w for which wx is in L. Prove that is L is regular, so is $L(1/3)$. (See Problem 1.57 in the textbook.)

3. (a) Build a DFA that accepts exactly the strings over the alphabet {0, 1} that contain exactly the strings having 01011 as a substring.
(b) Build a DFA that accepts exactly the strings over the alphabet {0, 1} that contain the symbols 0, 1, 0, 1, 1 in this order, with any number (including zero) of symbols interspersed). (This problem is like (a) except that the symbols in the pattern need not occur consecutively.)

4. Let L be an infinite regular language. Prove that L can be split into two infinite regular subsets.

5. (a) Prove that the set $\{1^k y \mid y \in \{0, 1\}^*$ and y contains at least k 1's, for $k \geq 1\}$ is regular.
(b) Prove that the set $\{1^k y \mid y \in \{0, 1\}^*$ and y contains at most k 1's, for $k \geq 1\}$ is not regular.