COS522: Computational Complexity Fall 2011

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1. Recall: P, NP. Leftover defns: coNP. Complete problem for it.
2. Often unexpected connections between these different phenomena. Example: NEXPTIME and EXPTIME vs P /NP. Padding argument.
3. Space complexity: the model (separate work tape when space is sublinear).
4. Space(S(n)) \subseteq TIME(2^{O(S(n)}).
5. NSpace(S(n)). Also in TIME(2^{O(S(n)}).
6. Sublinear space. L, NL, coNL.
7. Motivating problem: PATH (is there a path from s to t in the given directed graph?)
8. Show PATH is in SPACE(log^2 n). (“space can be reused.”)
9. NL completeness. Main problem: where to store things. Logspace transducer.
10. PATH is NL-complete.
11. Certificate definition of NL. X is in the language iff there is a certificate of poly(X) size which a deterministic logspace machine can verify using one-way access.
12. Complement of PATH is in NL (NL=coNL)!
13. Proof: S\_k = set of vertices at distance <= k from s. Suppose you know S\_n. How can somebody convince you that t is not connected to s?
14. How can somebody convince you that a vertex is in S\_k?
15. How can somebody convince you that a vertex is not in S\_k (assume you know |S\_k|)?
16. How can somebody convince you of the size of S\_{k+1} assuming you know |S\_k|?
17. PSPACE. Contains P, NP, coNP.
18. Essence of PSPACE: 2-person games of perfect information.
19. TQBF, a complete problem for PSPACE.
20. Proof of PSPACE-completeness (sketch)