COS522: Computational Complexity Fall 2011

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Lecture 13 notes Oct 27. Toda’s theorem

1. Recall: counting class #P. Defn of #P-completeness; #SAT.
2. Toda’s theorem.
3. Parity-P and Parity-SAT. Step 1: PH reduces probabilistically to Parity SAT.
4. Unclear even for NP!
5. Valiant Vazirani Lemma: Proof via inclusion exclusion. If we map a set of size between 2^k/4 and 2^k/2 to k-bit vectors by a pairwise indep. Hash function, then prob.the inverse of the all-zeros string is unique is at least 1/8.
6. Pr[has an inverse] is about ¼, and pr. Of collision is about 1/8.
7. Boosting in Parity-P. Write formulae for +, X of two formulae. Now can express the OR of two parity-SAT formulae and the AND. So just do the obvious thing of repeating VV reduction poly(n) times, and taking the OR.
8. Relativistic version of VV.
9. Making the reduction deterministic. (The counting trick, using polynomial 4x^3 + 3x^4.)