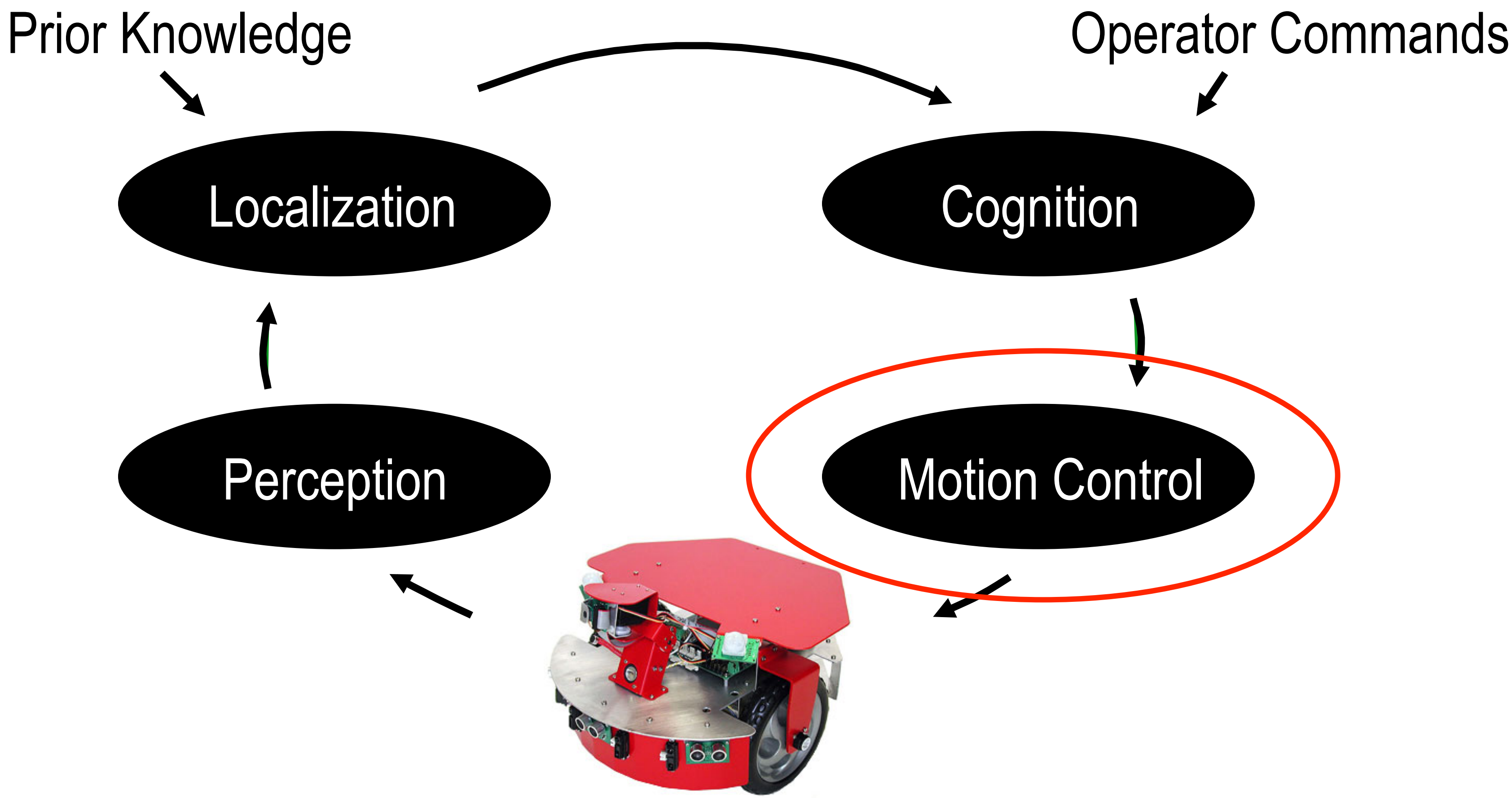
COS 495 - Lecture 5
Autonomous Robot Navigation

Instructor: Chris Clark

Semester: Fall 2011

Figures courtesy of Siegwart & Nourbakhsh

Control Structure



Motion Uncertainty

1. Odometry & Dead Reckoning
2. Modeling motion
3. Odometry on the X80

Odometry & Dead Reckoning

- Odometry
  - Use wheel sensors to update position
- Dead Reckoning
  - Use wheel sensors and heading sensor to update position
- Straight forward to implement
- Errors are integrated, unbounded

Odometry & Dead Reckoning

- Odometry Error Sources
  - Limited resolution during integration (time increments, measurement resolution).
  - Unequal wheel diameter (deterministic)
  - Variation in the contact point of the wheel (deterministic)
  - Unequal floor contact and variable friction can lead to slipping (non deterministic)

Odometry & Dead Reckoning

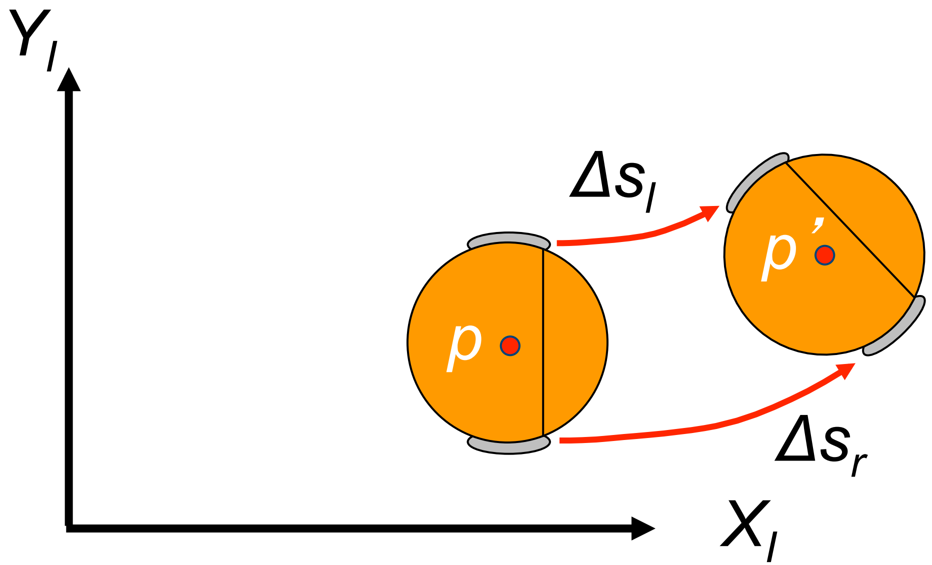
- Odometry Errors
  - Deterministic errors can be eliminated through proper calibration
  - Non-deterministic errors have to be described by error models and will always lead to uncertain position estimate.

Motion Uncertainty

1. Odometry & Dead Reckoning
2. Modeling motion
3. Odometry on the X80

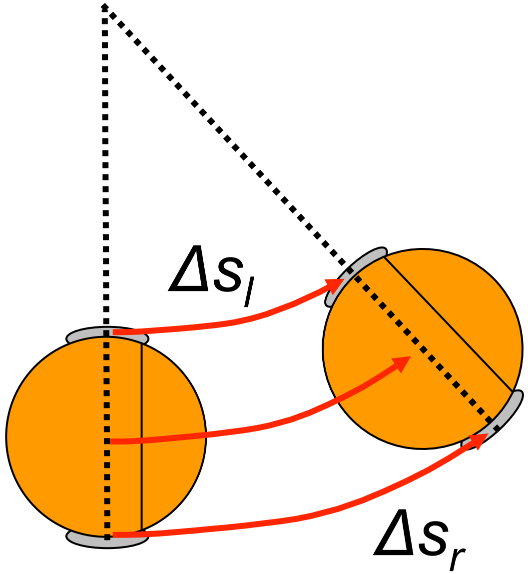
Modeling Motion

- If a robot starts from a position *p*, and the right and left wheels move respective distances Δs_r and Δs_l, what is the resulting new position *p'* ?



Modeling Motion

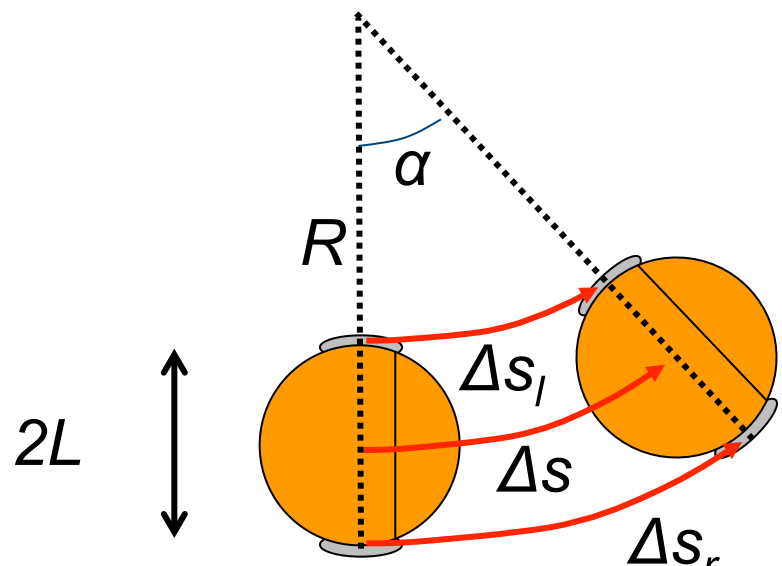
- To start, let's model the change in angle $\Delta\theta$ and distance travelled Δs by the robot.
  - Assume the robot is travelling on a circular arc of constant radius.



Modeling Motion

- Begin by noting the following holds for circular arcs:

$\Delta s_l = R\alpha$ $\Delta s_r = (R+2L)\alpha$ $\Delta s = (R+L)\alpha$



Modeling Motion

- Now manipulate first two equations:

$$\Delta s_l = R\alpha \qquad \Delta s_r = (R+2L)\alpha$$

To:

$$R\alpha = \Delta s_l$$

$$L\alpha = (\Delta s_r - R\alpha)/2$$

$$= \Delta s_r/2 - \Delta s_l/2$$

Modeling Motion

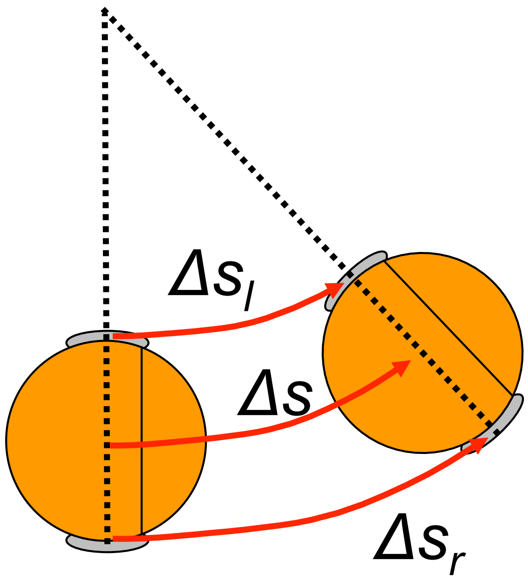
- Substitute this into last equation for *Δs:*

$$\Delta s = (R+L)\alpha$$
$$= R\,\alpha + L\alpha$$
$$= \Delta s_l + \Delta s_r /2 \;-\; \Delta s_l/2$$
$$= \Delta s_l /2 + \Delta s_r /2$$
$$= \frac{\Delta s_l + \Delta s_r}{2}$$

Modeling Motion

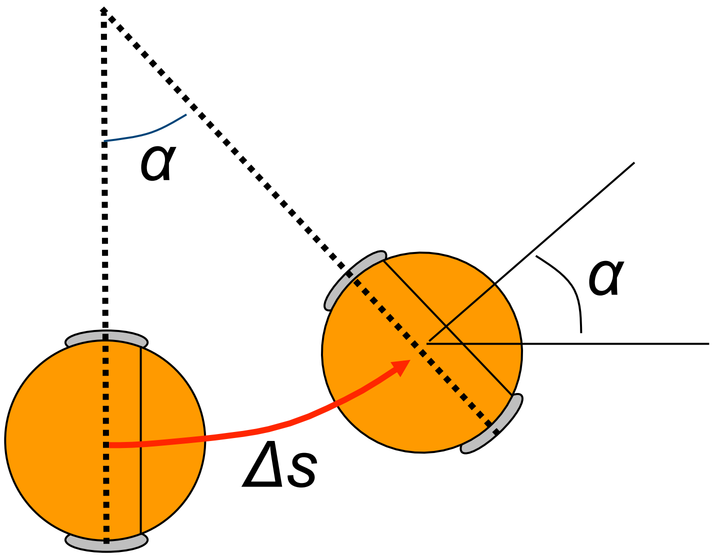
- Or, note the distance the center travelled is simply the average distance of each wheel:

$$\Delta s = \frac{\Delta s_r + \Delta s_l}{2}$$



Modeling Motion

- To calculate the change in angle $\Delta\theta$, observe that it equals the rotation about the circular arc's center point



$$\Delta\theta = \alpha$$

Modeling Motion

- So we solve for α by equating α from the first two equations:

$$\Delta s_l = R\alpha \qquad \Delta s_r = (R+2L)\alpha$$

This results in:

$$\Delta s_l / R = \Delta s_r / (R+2L)$$

$$(R+2L)\,\Delta s_l = R\,\Delta s_r$$

$$2L\,\Delta s_l = R\,(\Delta s_r - \Delta s_l)$$

$$\frac{2L\,\Delta s_l}{(\Delta s_r - \Delta s_l)} = R$$

Modeling Motion

- Substitute R into

$$\alpha = \Delta s_l / R$$

$$= \Delta s_l \,(\Delta s_r - \Delta s_l) / (2L\, \Delta s_l)$$

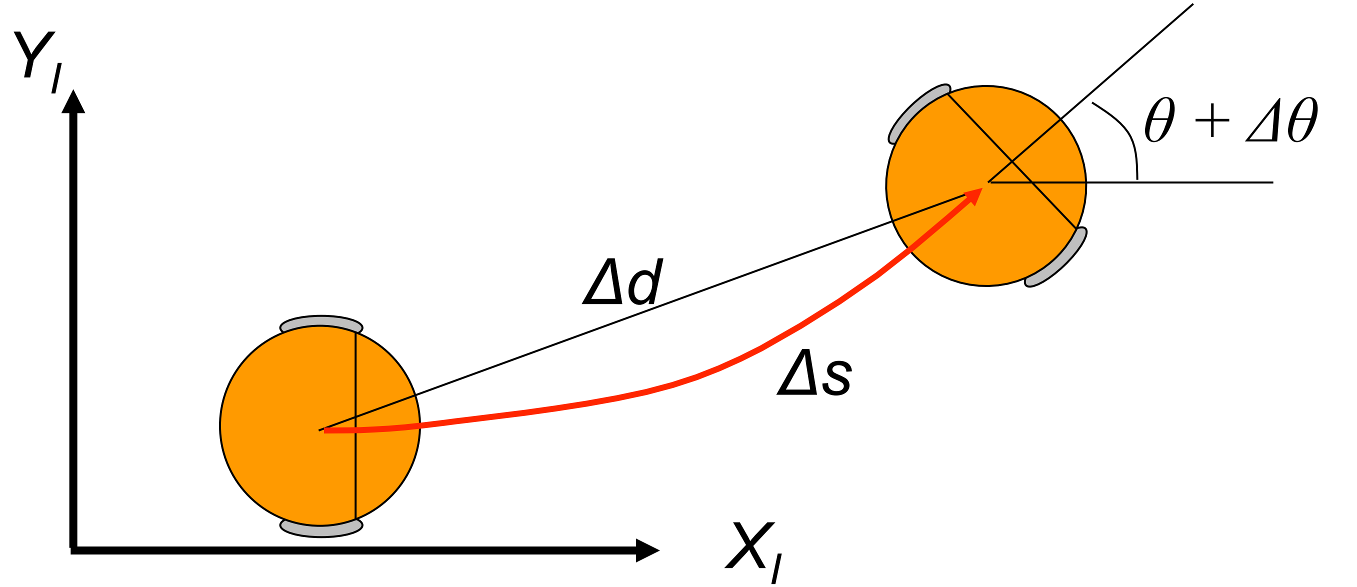
$$= \frac{(\Delta s_r - \Delta s_l)}{2L}$$

So…

$$\Delta\theta = \frac{(\Delta s_r - \Delta s_l)}{2L}$$

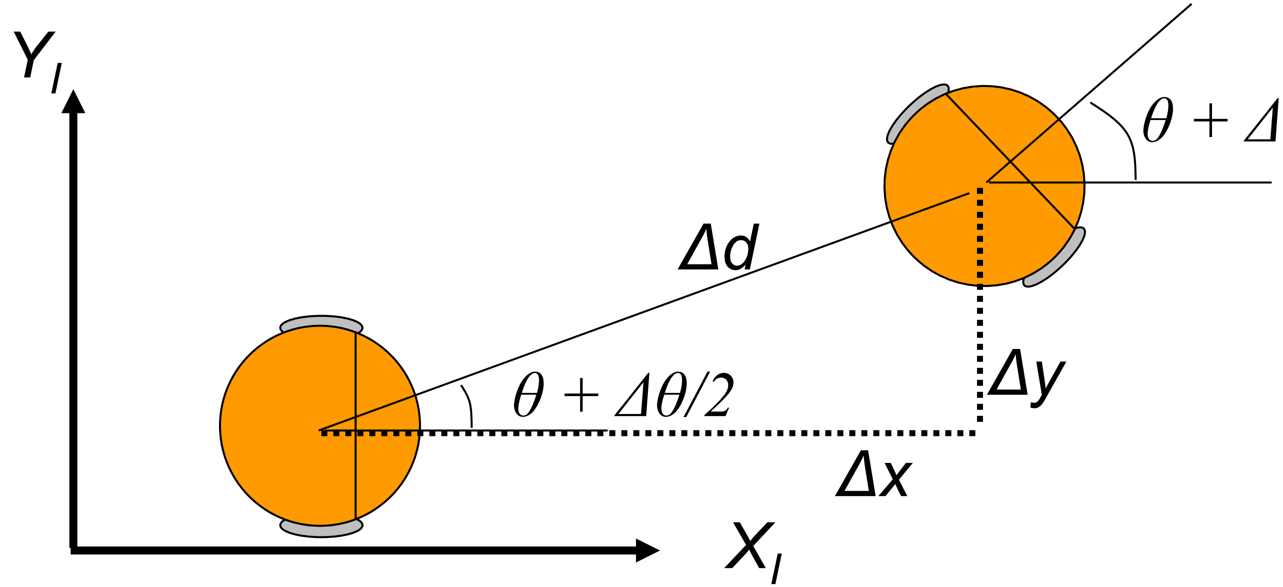
Modeling Motion

- Now that we have $\Delta\theta$ and Δs, we can calculate the position change in global coordinates.
  - We use a new segment of length Δd.



Modeling Motion

- Now calculate the change in position as a function of *Δd.*

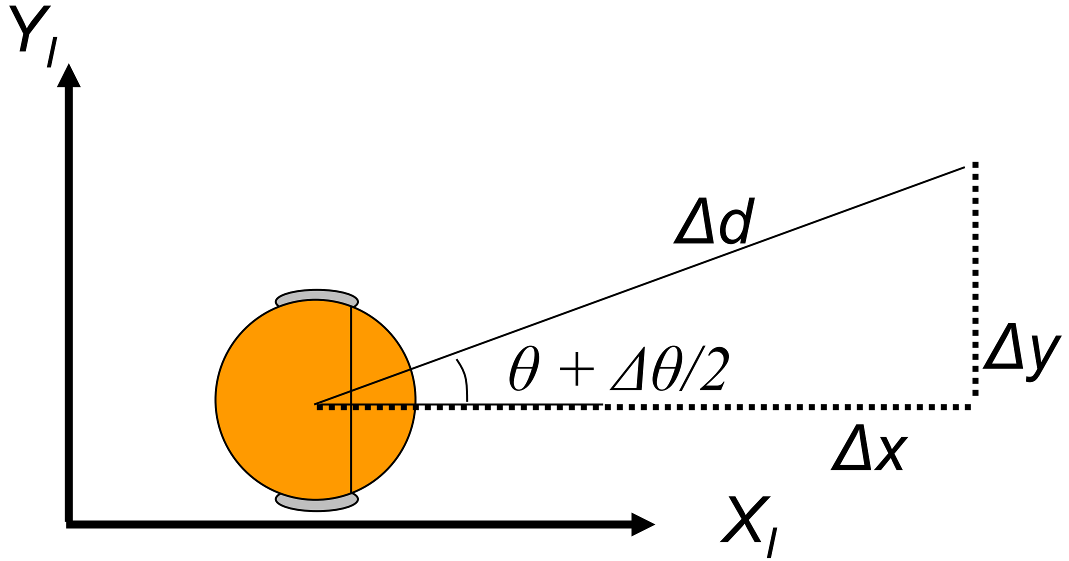


Modeling Motion

- Using Trig:

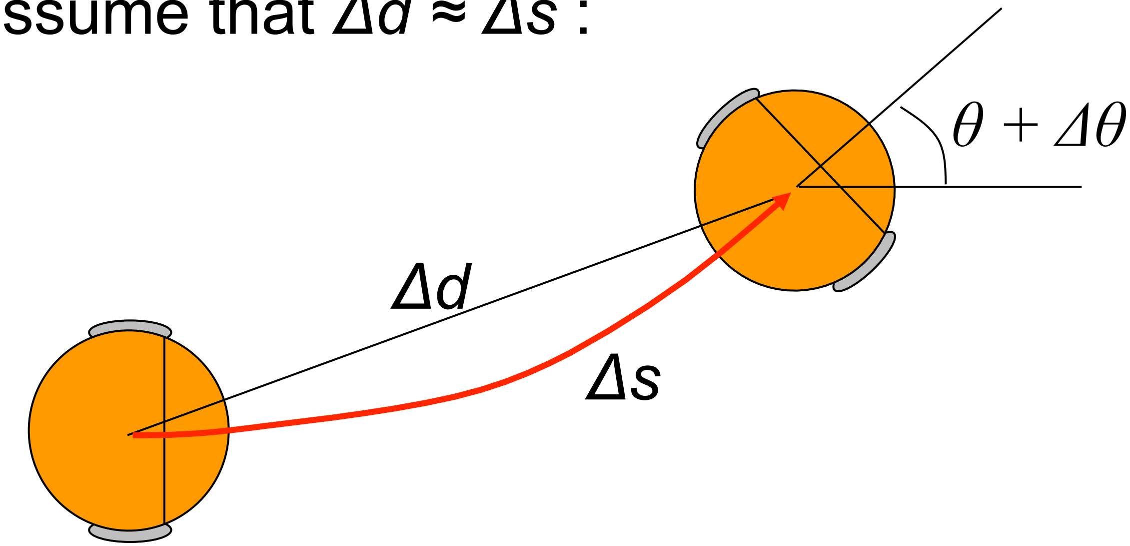
$$\Delta x = \Delta d \cos(\theta + \Delta\theta/2)$$

$$\Delta y = \Delta d \sin(\theta + \Delta\theta/2)$$



Modeling Motion

- Now if we assume that the motion is small, then we can assume that $\Delta d \approx \Delta s$:



- So…

$$\Delta x = \Delta s \cos(\theta + \Delta\theta/2)$$

$$\Delta y = \Delta s \sin(\theta + \Delta\theta/2)$$

Modeling Motion

- Summary:

$$\Delta x = \Delta s \cos(\theta + \Delta\theta / 2)$$

$$\Delta y = \Delta s \sin(\theta + \Delta\theta / 2)$$

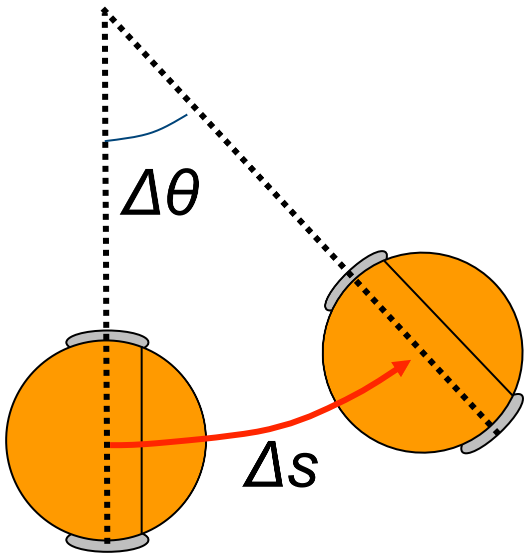
$$\Delta\theta = \frac{\Delta s_r - \Delta s_l}{b}$$

$$\Delta s = \frac{\Delta s_r + \Delta s_l}{2}$$

$$p' = f(x, y, \theta, \Delta s_r, \Delta s_l) = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta s_r + \Delta s_l}{2} \cos\left(\theta + \frac{\Delta s_r - \Delta s_l}{2b}\right) \\ \frac{\Delta s_r + \Delta s_l}{2} \sin\left(\theta + \frac{\Delta s_r - \Delta s_l}{2b}\right) \\ \frac{\Delta s_r - \Delta s_l}{b} \end{bmatrix}$$

Modeling Uncertainty in Motion

- Let's look at delta terms as errors in wheel motion, and see how they propagate into positioning errors.
  - Example: the robot is trying to move forward 1 m on the x axis.



If:

$$\Delta s = 1 + e_s$$

$$\Delta\theta = 0 + e_\theta$$

where e_s and e_θ are error terms

Modeling Uncertainty in Motion

- According to the following equations, the error e_s = 0.001m produces errors in the direction of motion.

$$\Delta x = \Delta s \cos(\theta + \Delta\theta/2)$$

$$\Delta y = \Delta s \sin(\theta + \Delta\theta/2)$$

- However, the $\Delta\theta$ term affects each direction differently. If e_θ = 2 deg and e_s = 0 meters, then:

$$\cos(\theta + \Delta\theta/2) = 0.9998$$

$$\sin(\theta + \Delta\theta/2) = 0.0175$$

Modeling Uncertainty in Motion

- So

$$\Delta x = 0.9998$$

$$\Delta y = 0.0175$$

- But the robot is supposed to go to *x=1*,*y=0*, so the errors in each direction are

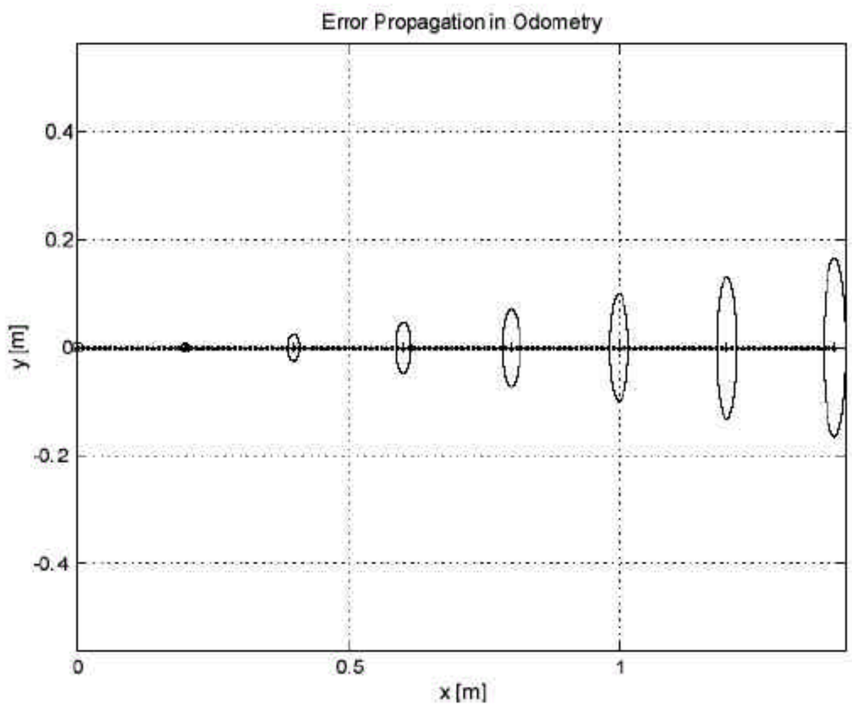
$$\Delta x = +0.0002$$

$$\Delta y = -0.0175$$

- *THE ERROR IS BIGGER IN THE "Y" DIRECTION!*

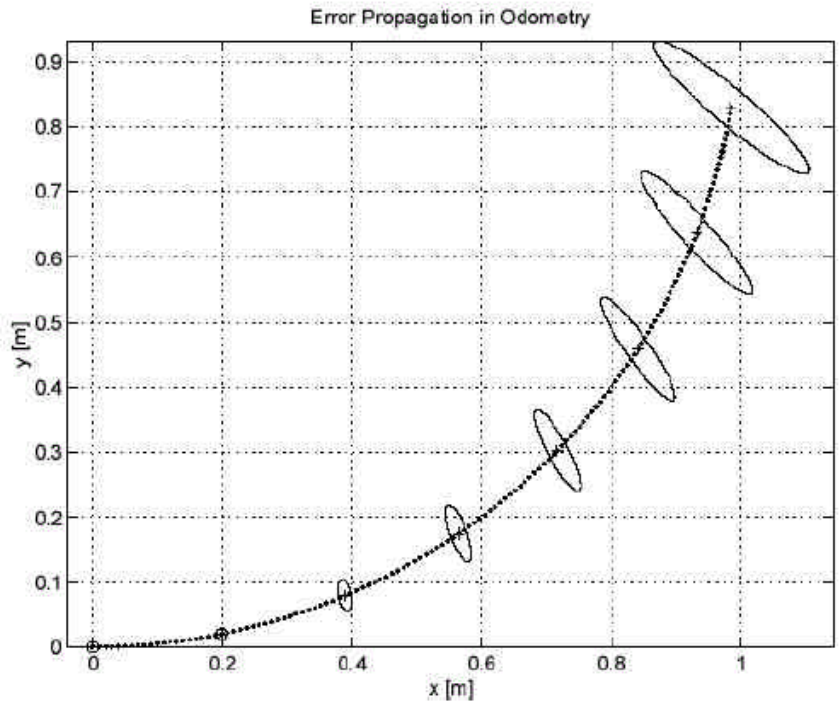
Modeling Uncertainty in Motion

- Errors perpendicular to the direction grow much larger.



Modeling Uncertainty in Motion

- Error ellipse does not remain perpendicular to direction.



Motion Uncertainty

1. Odometry & Dead Reckoning
2. Modeling Uncertainty in motion
3. Odometry on the X80

Odometry on the X80

- Goals:
  - Calculate the resulting robot position and orientation from wheel encoder measurements.
  - Display them on the GUI.
- Method:
  - Use the forward kinematics equation from last lecture.

Odometry on the X80

- Method cont':
  - Make use of the fact that your encoder has resolution of 1200 counts per revolution. Be able to convert this to a distance travelled by the wheel.

$$r\varphi_r = \Delta s_r$$

  - Given the distance travelled by each wheel, we can calculate the change in the robot's distance and orientation.

$$\Delta s = \frac{\Delta s_r + \Delta s_l}{2} \qquad \Delta\theta = \frac{(\Delta s_r - \Delta s_l)}{2L}$$

Odometry on the X80

- Method cont':
  - Now you should be able to update the position/orientation in global coordinates.

$$\Delta x = \Delta s \cos(\theta + \Delta\theta/2)$$

$$\Delta y = \Delta s \sin(\theta + \Delta\theta/2)$$