COS 495 – Lecture 21
Autonomous Robot Navigation

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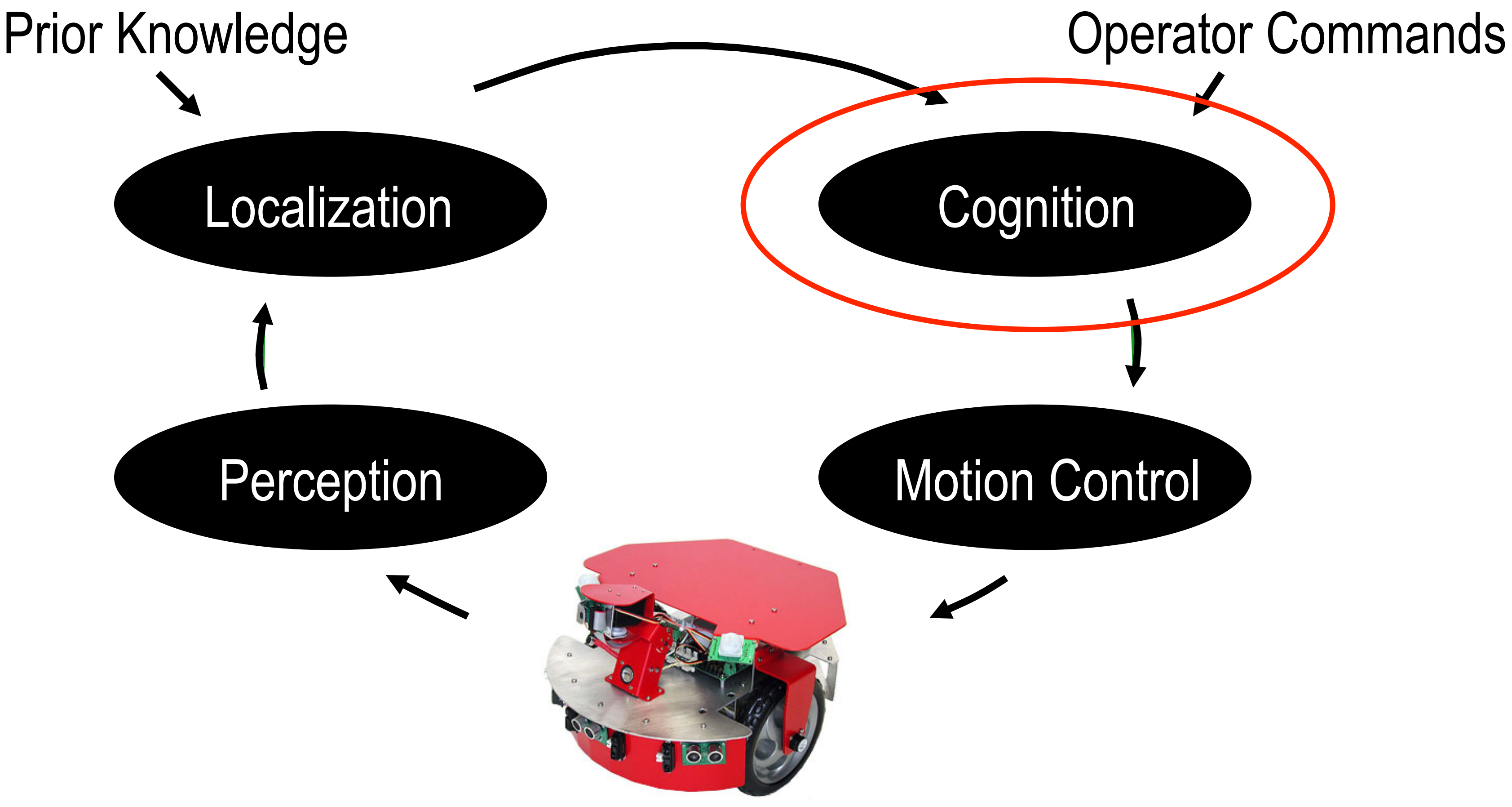
Semester: Fall 2011

Figures courtesy of Siegwart & Nourbakhsh

Control Structure



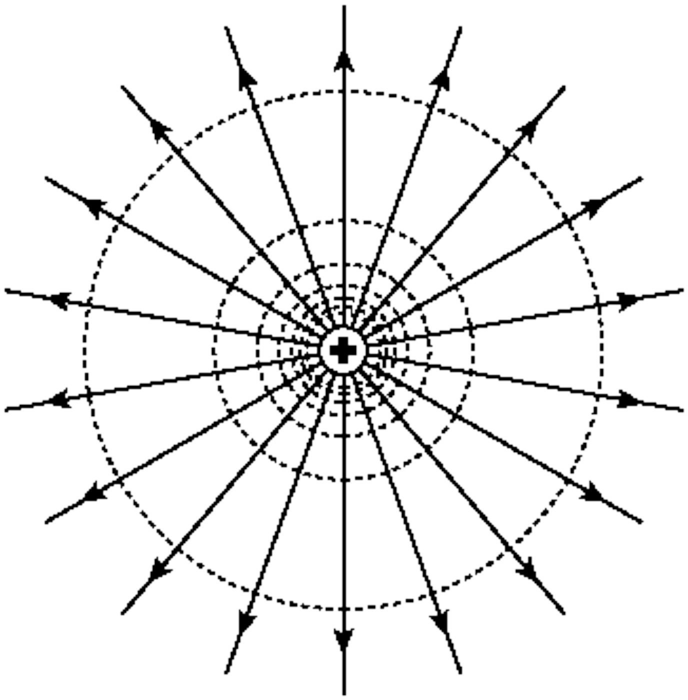
Cell Decomposition & Potential Fields: Outline

- Discretizations
  1. Single-Query Probabilistic Road Maps
  2. Artificial Potential Fields
  3. Cell Decompositions

Artificial Potential Fields

- Electric Potentials
  - The electric potential V_E (J C^{-1}) created by a point charge Q, at a distance r from the charge (relative to the potential at infinity), can be shown to be

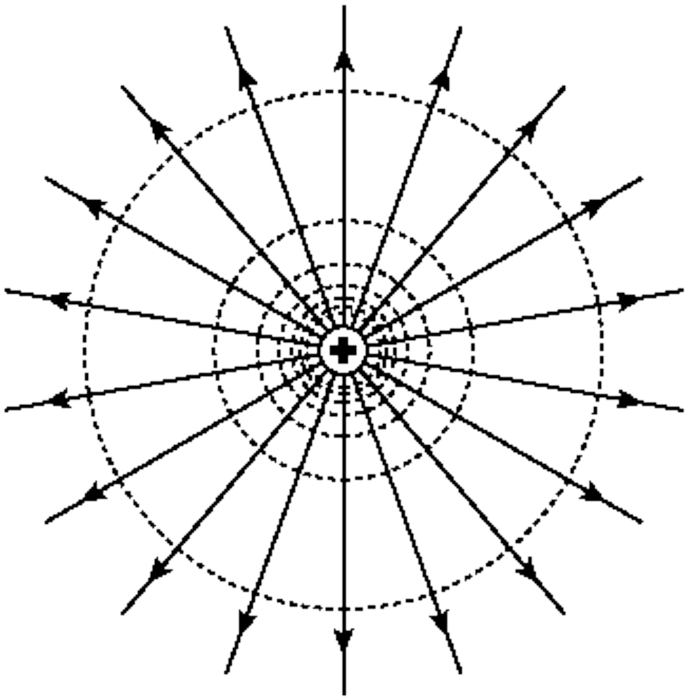
$$V_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



Artificial Potential Fields

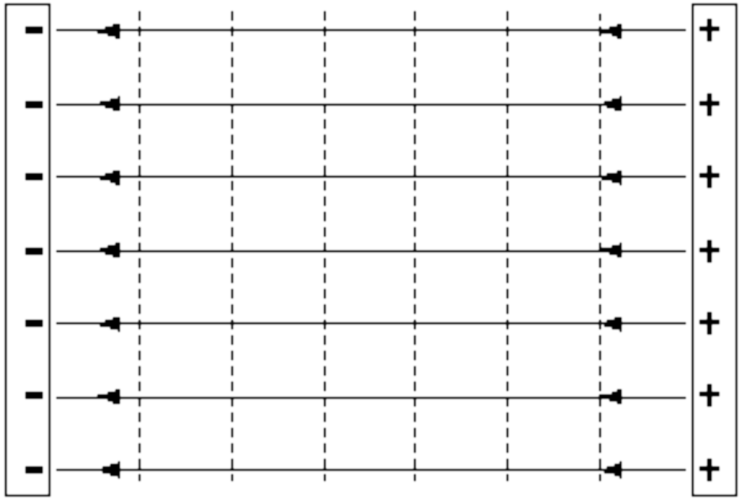
- Electric Fields
  - The electric field intensity E is defined as the force per unit positive charge that would be experienced by a point charge
  - It is obtained by taking the negative gradient of the electric potential

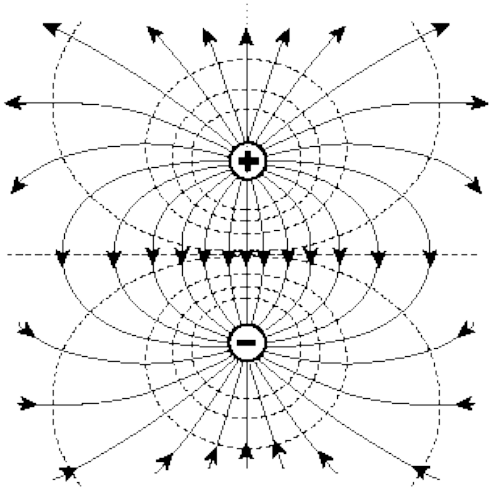
$$E = -\nabla V_E$$



Artificial Potential Fields

- Electric Potential Fields
  - Different arrangements of charges can lead to various fields

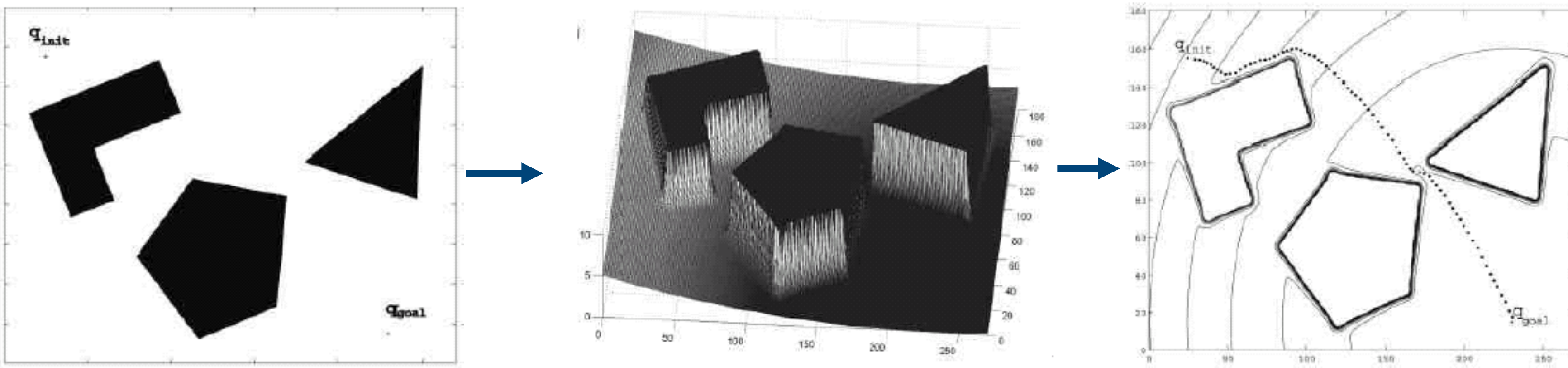




Artificial Potential Fields

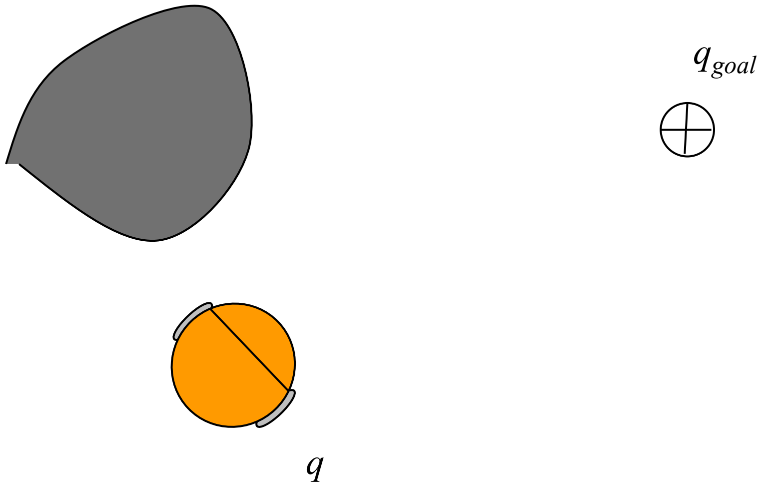
- In APFs, the robot is treated as a *point under the influence* of an artificial potential field.
  - Electrical analogy: The generated robot movement is similar to an electric charge under the force of an electric field
  - Mechanical analogy: The generated robot movement is similar to a ball rolling down the hill
  - Goal generates attractive force
  - Obstacles generate repulsive forces

Artificial Potential Fields



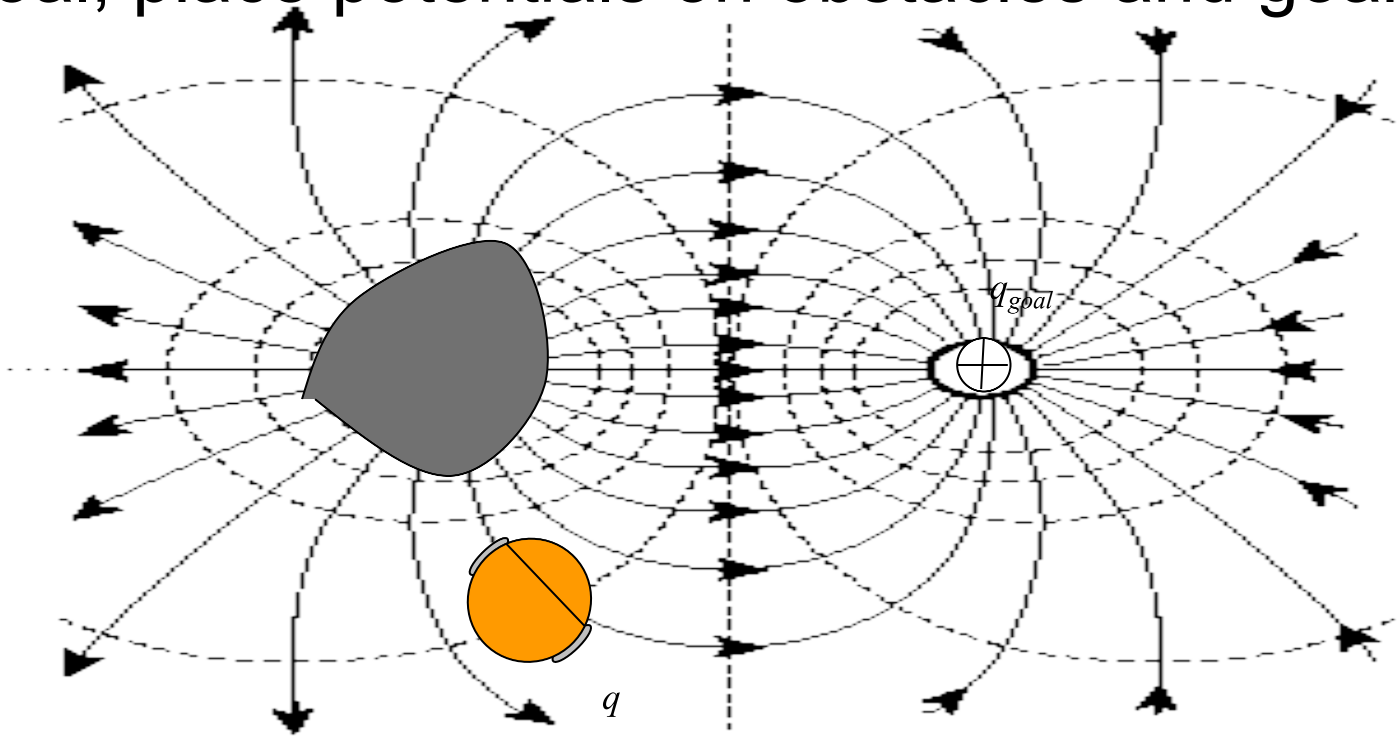
Artificial Potential Fields

- For a given configuration space and desired goal, place potentials on obstacles and goals



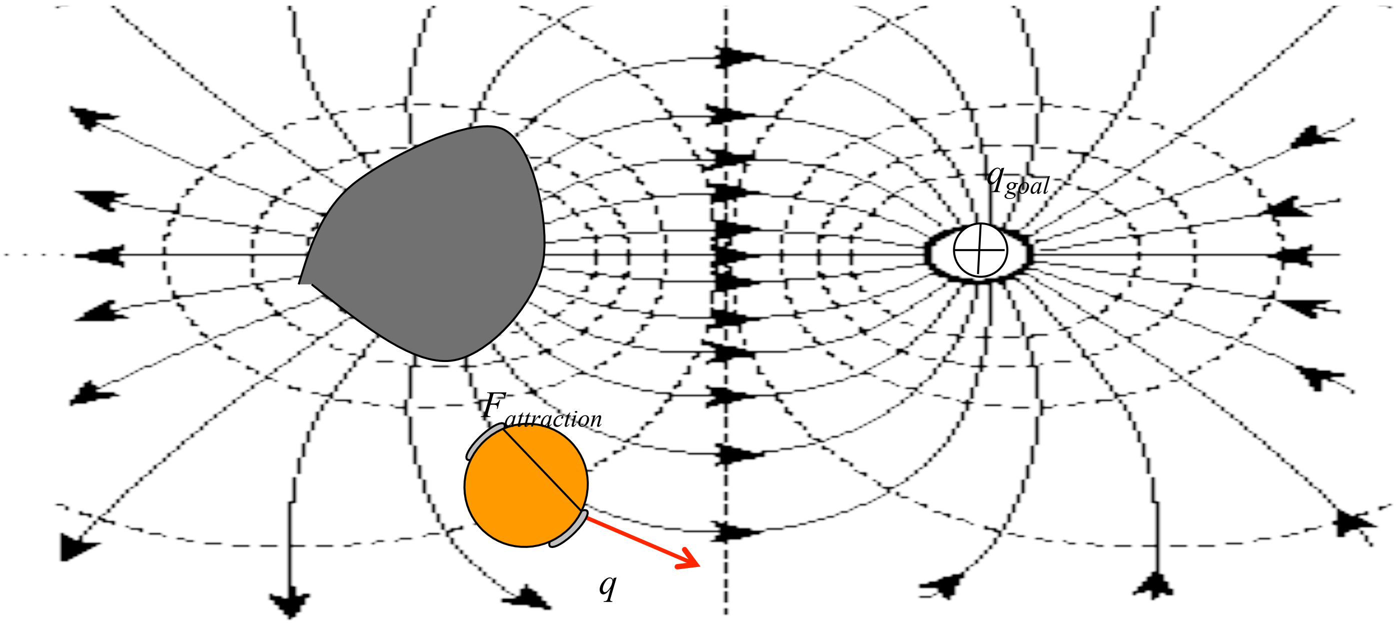
Artificial Potential Fields

- For a given configuration space and desired goal, place potentials on obstacles and goals

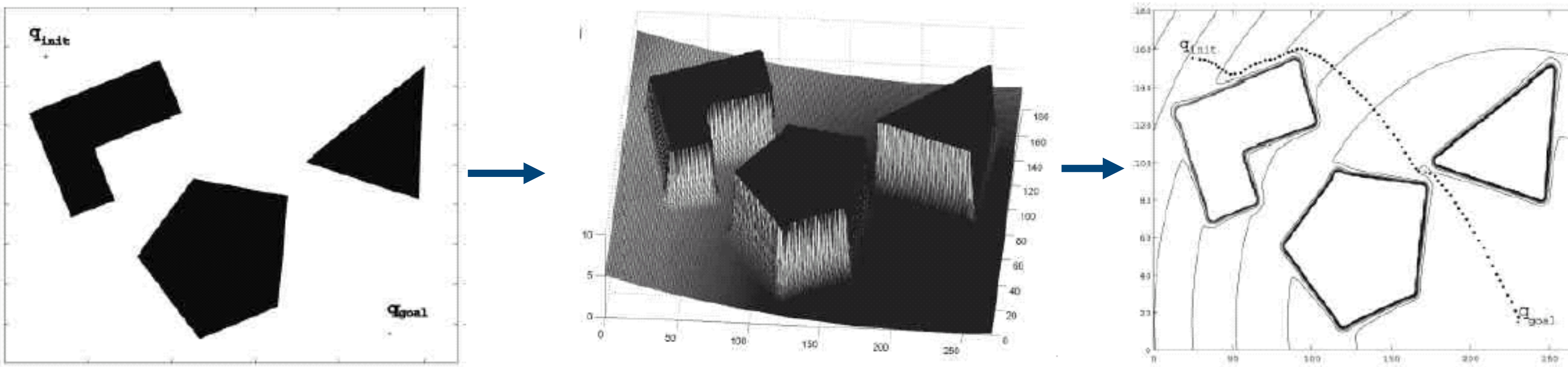


Artificial Potential Fields

- For any robot configuration q, the forces felt by the robot can be calculated to steer the robot towards the goal.



Artificial Potential Fields



Potential Field Generation

- Given potential functions U, Generate artificial force field $F(q)$
  - Sum all potentials (repulsive and attractive).
  - Differentiate to determine forces
  - Note: functions must be differentiable

$$
\begin{aligned}
F(q) &= -\nabla U(q) \\
&= -\nabla U_{att}(q) - \nabla U_{rep}(q) \\
&= \begin{bmatrix} \delta U / \delta x \\ \delta U / \delta y \end{bmatrix}
\end{aligned}
$$

Attractive Potential Fields

- Parabolic function representing the Euclidean distance $\rho_{goal}(q) = || q - q_{goal} ||$ to the goal.

$$U_{att}(q) = \frac{1}{2} k_{att} \rho^2_{goal}(q)$$

- Attracting force converges linearly towards 0 (goal)

$$F_{att}(q) = - \nabla U_{att}(q)$$
$$= - k_{att} (q - q_{goal})$$

Repulsive Potential Fields

- Should generate a barrier around all the obstacle
  - Strong if close to the obstacle
  - Does not influence robot if far from the obstacle

$$U_{rep}(q) = \begin{cases} \frac{1}{2} k_{rep} \left[\frac{1}{\rho(q)} - \frac{1}{\rho_0} \right]^2 & if \quad \rho(q) \leq \rho_0 \\ 0 & if \quad \rho(q) > \rho_0 \end{cases}$$

- Where $\rho(q)$ is the minimum distance to the object

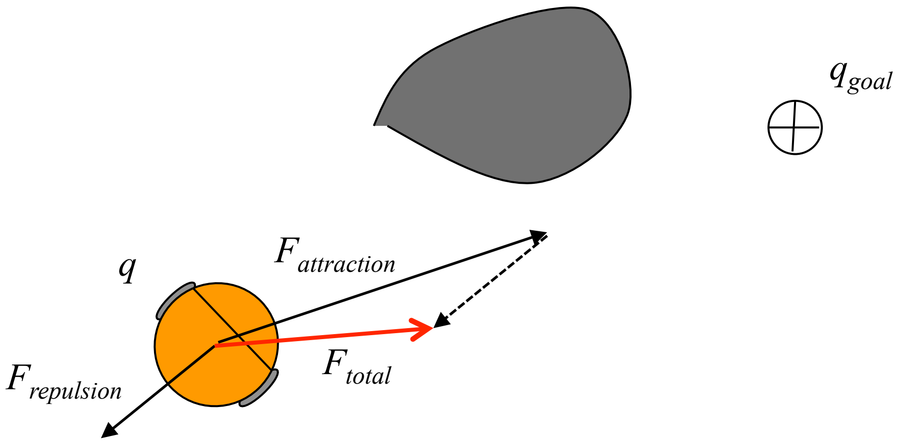
Repulsive Potential Fields

- Field is positive or zero and tends to infinity as q gets closer to the object

$$
\begin{aligned}
F_{rep}(q) &= -\nabla U_{rep}(q) \\
&= \begin{cases} k_{rep}\left(\dfrac{1}{\rho(q)} - \dfrac{1}{\rho_0}\right)\dfrac{q - q_{obj}}{\rho^3(q)} & \text{if } \rho(q) \le \rho_0 \\ 0 & \text{if } \rho(q) > \rho_0 \end{cases}
\end{aligned}
$$

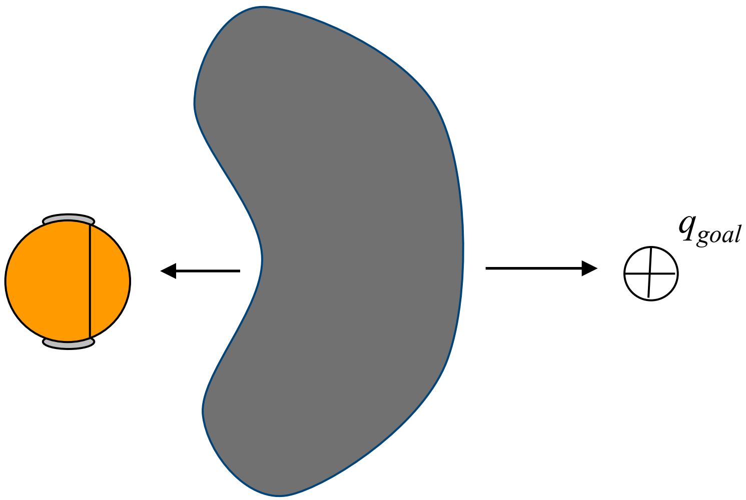
Artificial Potential Fields

- Given current configuration of the robot q
  1. **Sum** total force **vectors** $F(q)$ generated by the potential fields.
  2. Set desired robot velocity (v, w) proportional to the force $F(q)$



Artificial Potential Fields

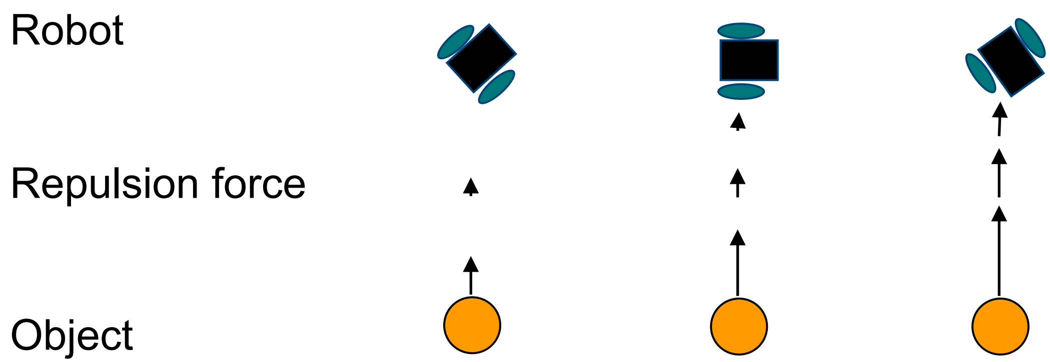
- Local minimums



  - If objects are not *convex* (i.e. concave), there exist situations where several minimal distances exist and can result in oscillations
  - Not complete

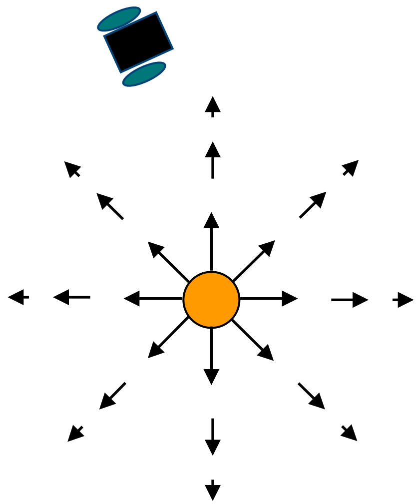
Artificial Potential Fields

- Extended Potential Fields
  - Many modifications to potential fields have been done in order to improve completeness, optimality.
  - Example: Orientation based potentials
    - Can increase potential depending on orientation of robot

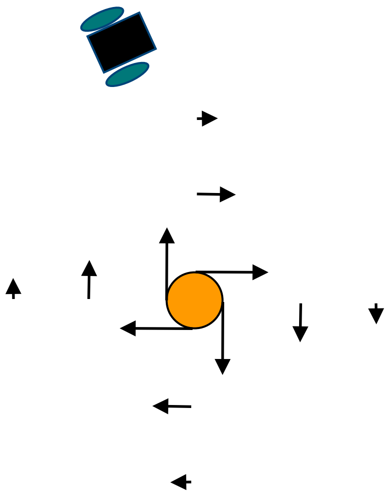


Artificial Potential Fields

- Extended Potential Fields
  - Also, can use rotational fields in one direction



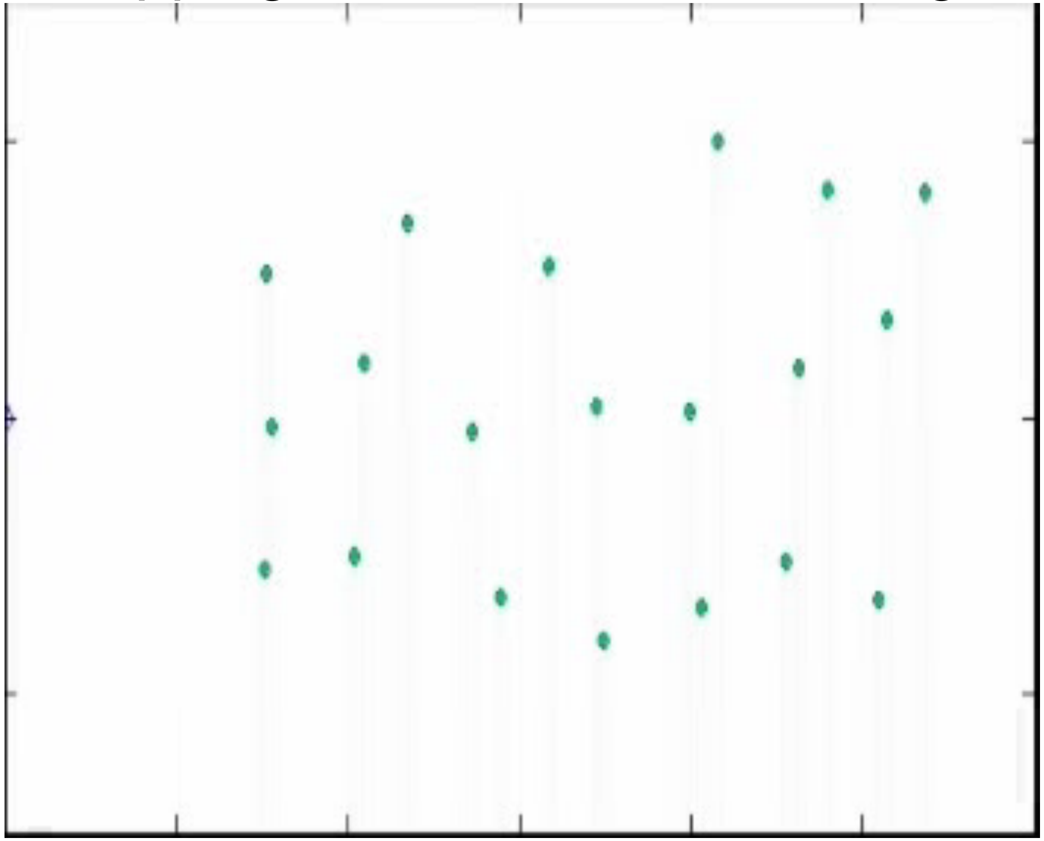
Linear source



Rotational source

Artificial Potential Fields

- Example: Mapping obstacles and avoiding them.



Motion Planning: Potential Fields

- Example: Mapping obstacles and avoiding them.

Artificial Potential Fields

- Final Notes on APFs
  - APFs are easy to understand and implement
  - Fast to compute
  - Can be difficult to tune parameters for optimal performance
  - Not always complete
  - Ideal for cluttered open spaces

Cell Decomposition & Potential Fields: Outline

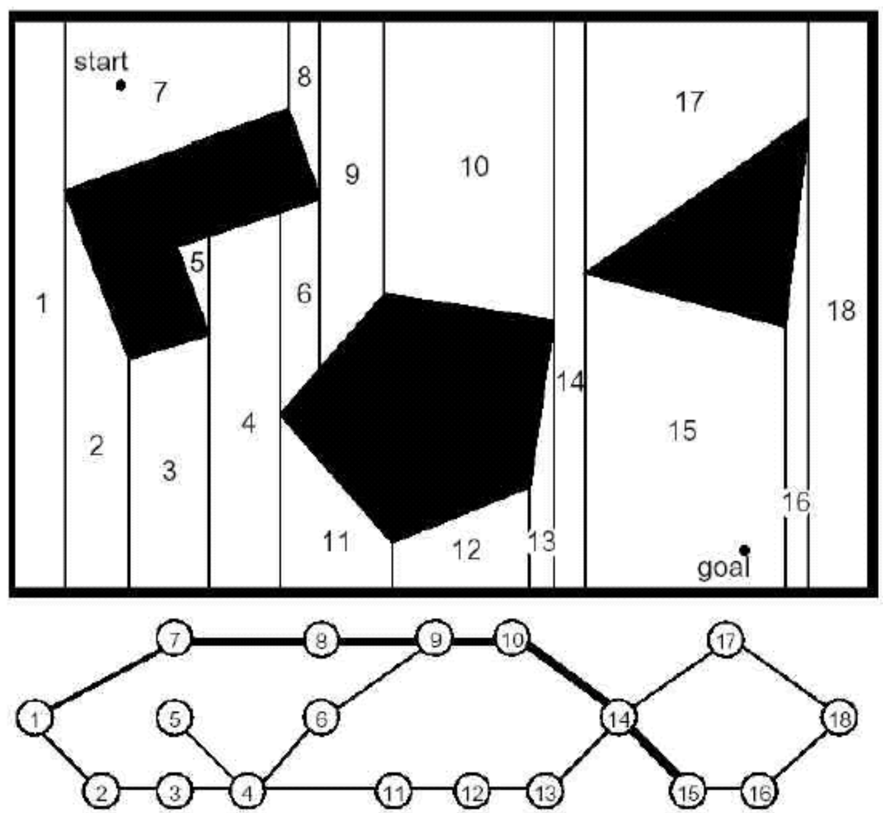
- Discretizations
  1. Probabilistic Road Maps
  2. Potential Fields
  3. Cell Decompositions

Motion Planning: Cell Decomposition

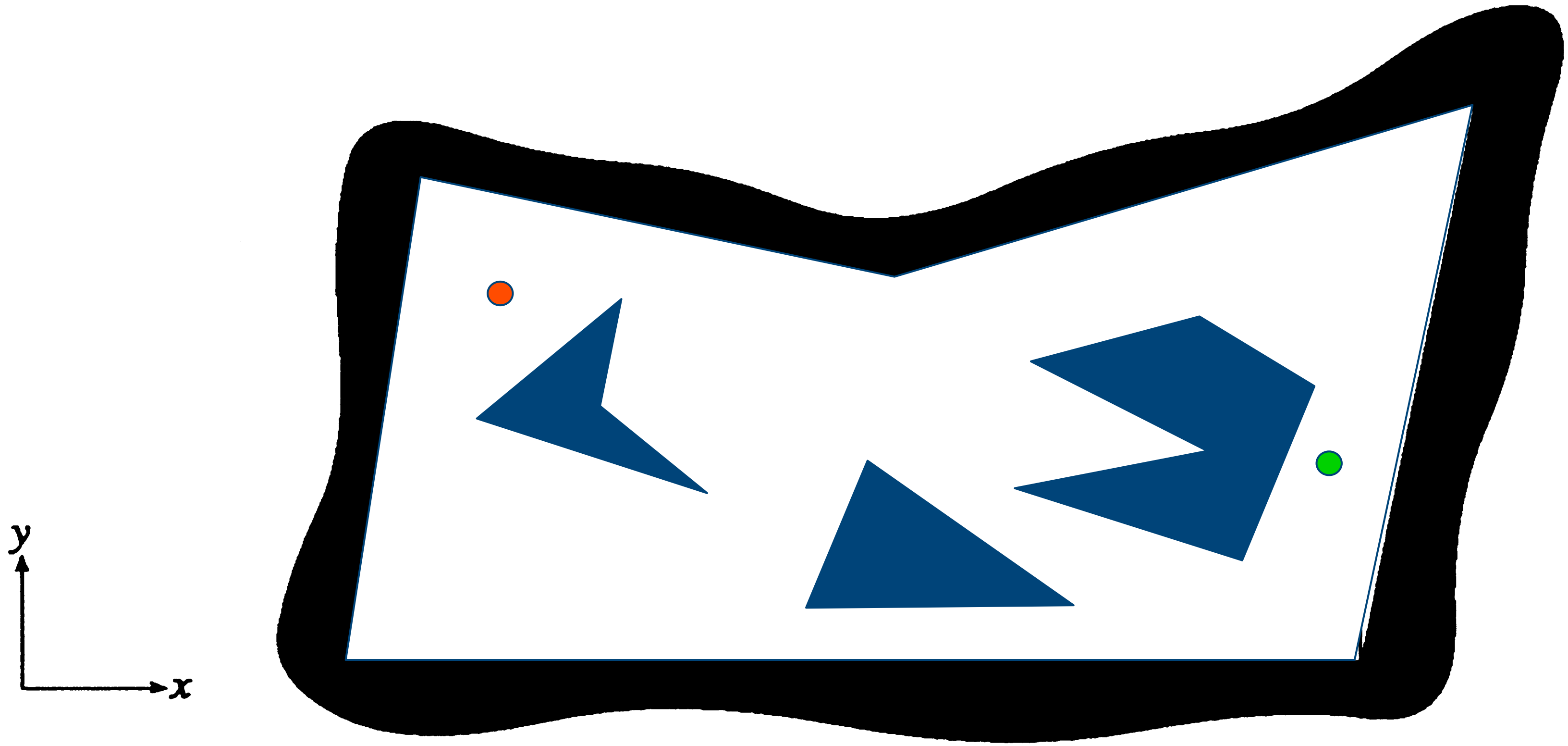
1. Divide space into simple, connected regions (i.e. cells)
2. Determine which open cells are adjacent and construct a connectivity graph
3. Find cells in which the initial and goal configuration (state) lie and search for a path in the connectivity graph to join them.
4. From the sequence of cells found with an appropriate search algorithm, compute a path within each cell.

Motion Planning: Cell Decomposition

- Exact Cell Decomposition – Trapezoidal Alg.

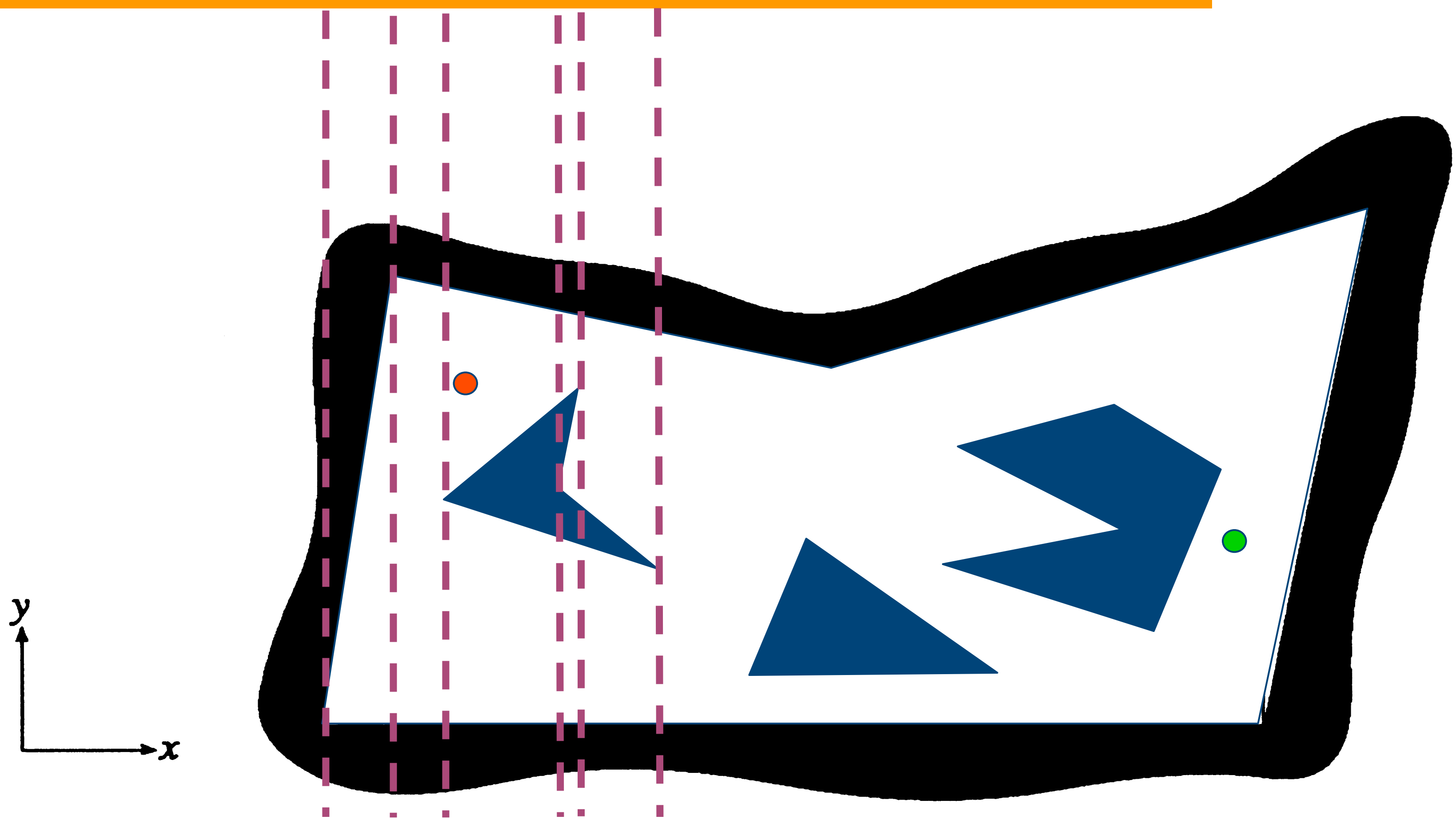


Motion Planning: Cell Decomposition



Courtesy of Latombe

Motion Planning: Cell Decomposition



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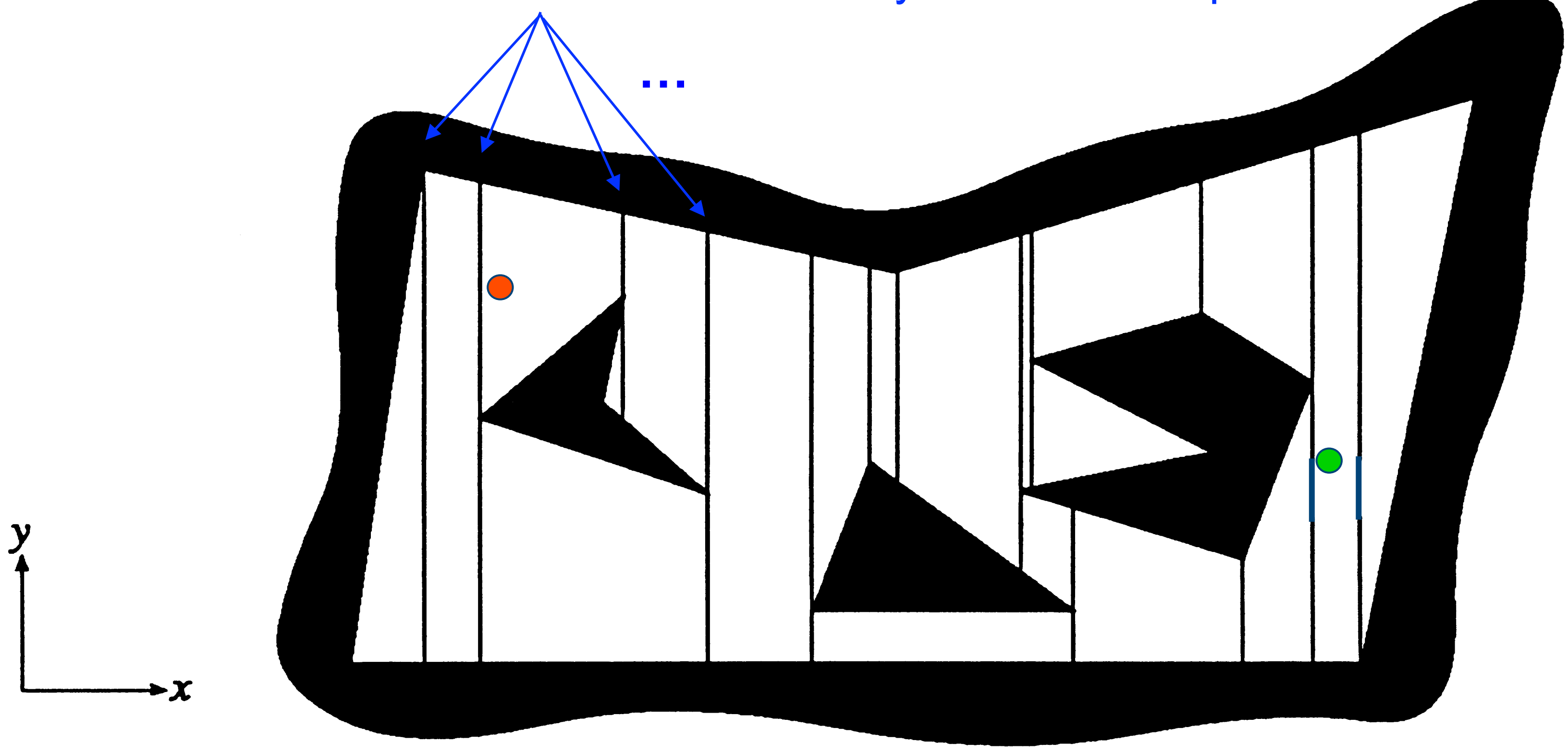
Motion Planning: Cell Decomposition

critical events → criticality-based decomposition



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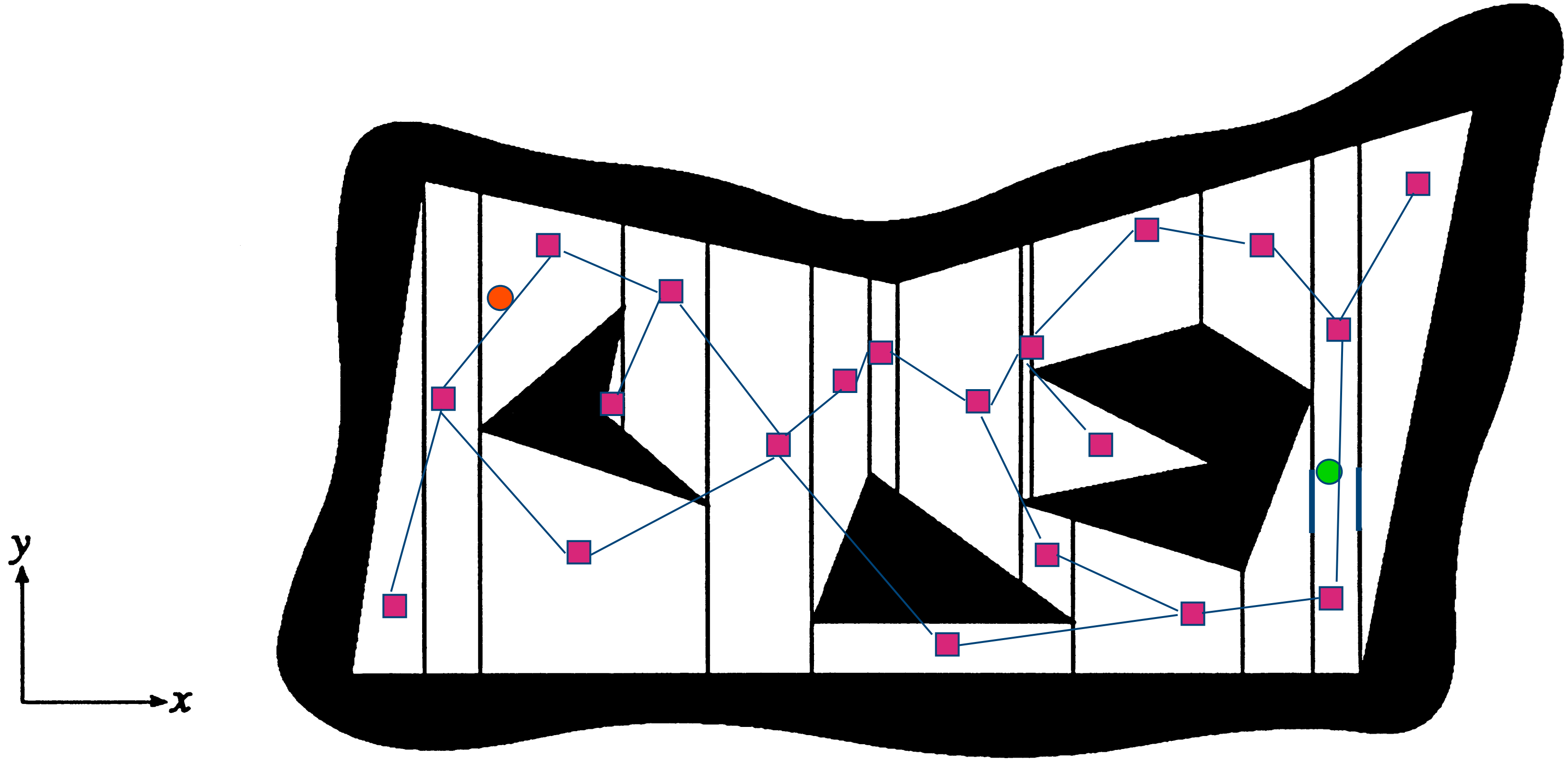
Motion Planning: Cell Decomposition



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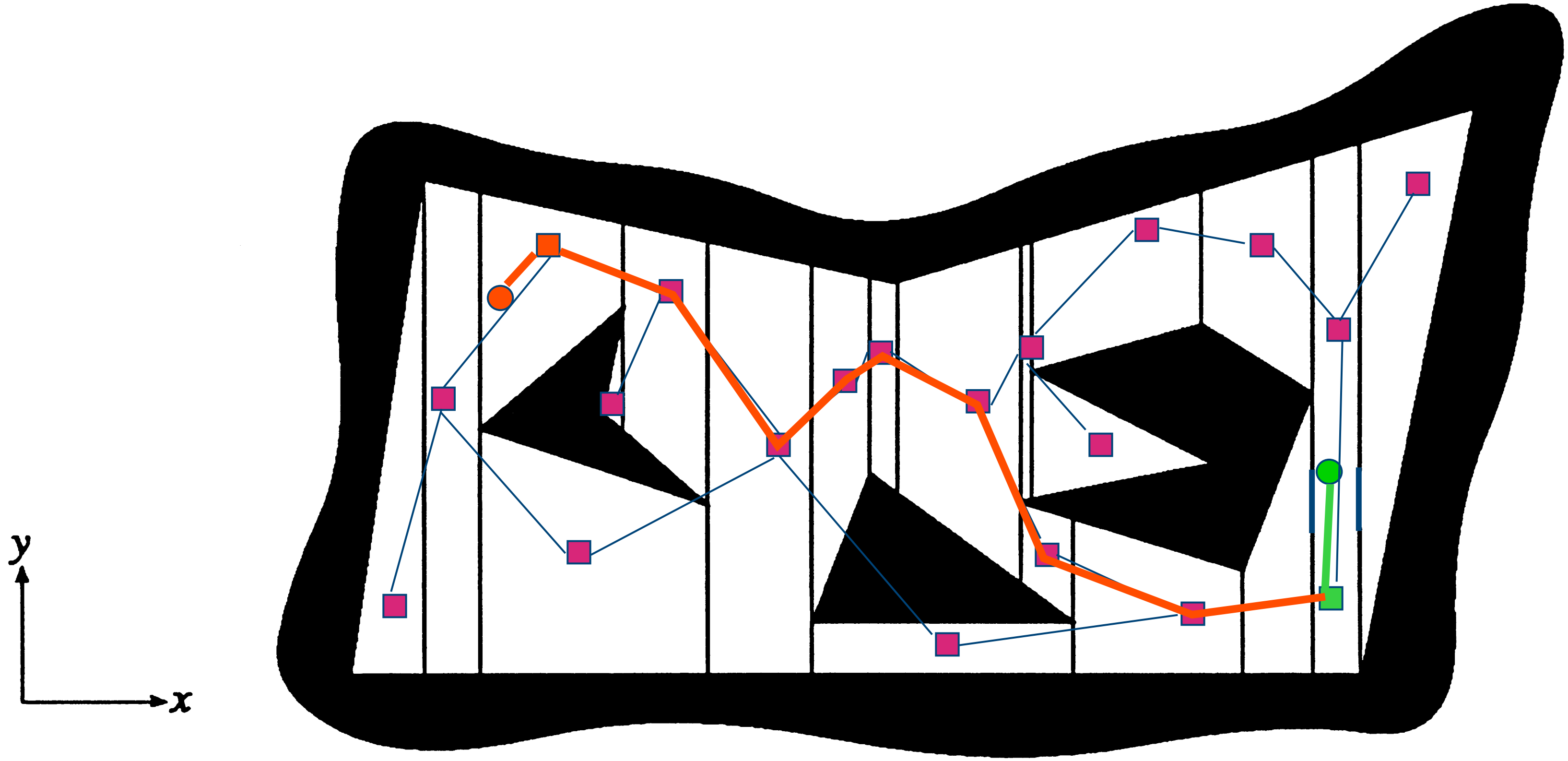
Motion Planning: Cell Decomposition



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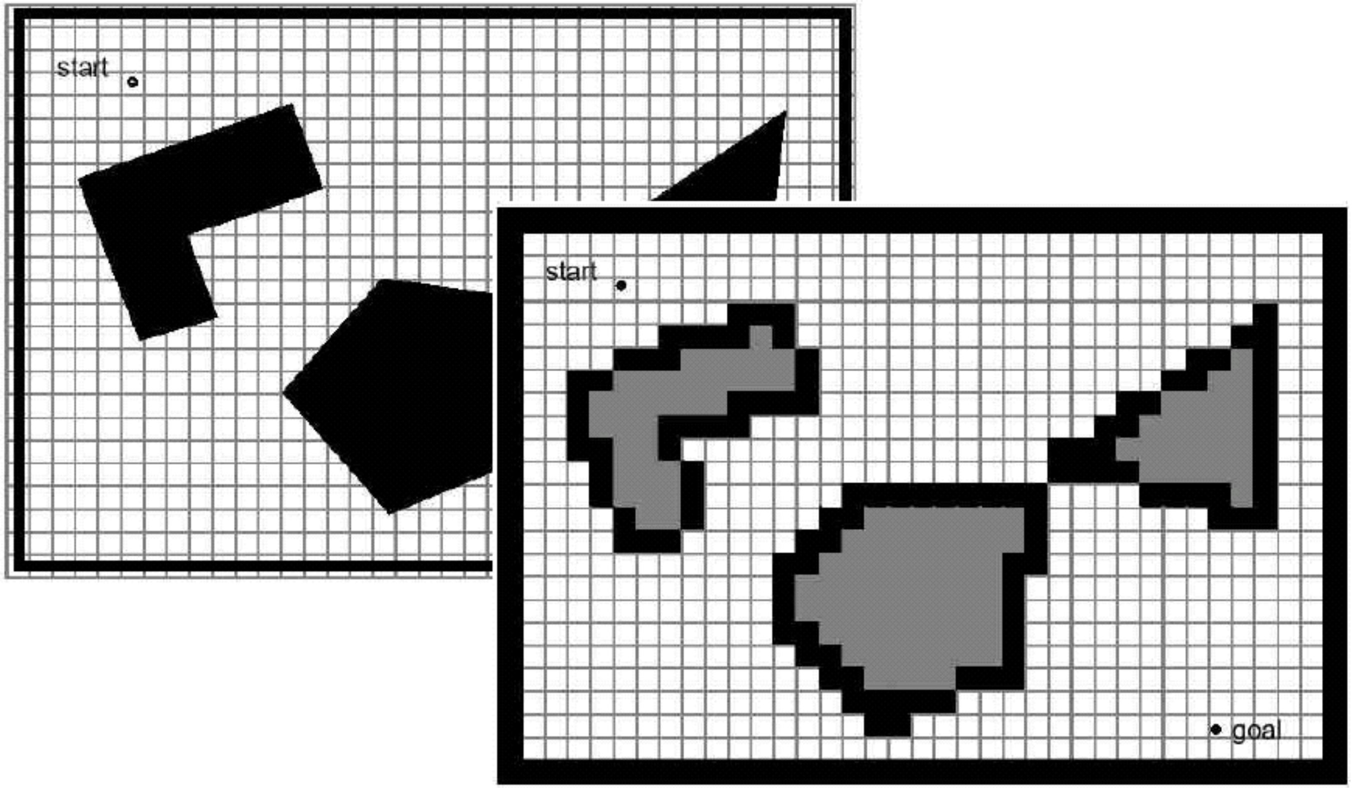
Motion Planning: Cell Decomposition

- Approximate Cell Decomposition



Motion Planning: Cell Decomposition

- Approximate Cell Decomposition
  - Example: