COS 495 – Lecture 20
Autonomous Robot Navigation

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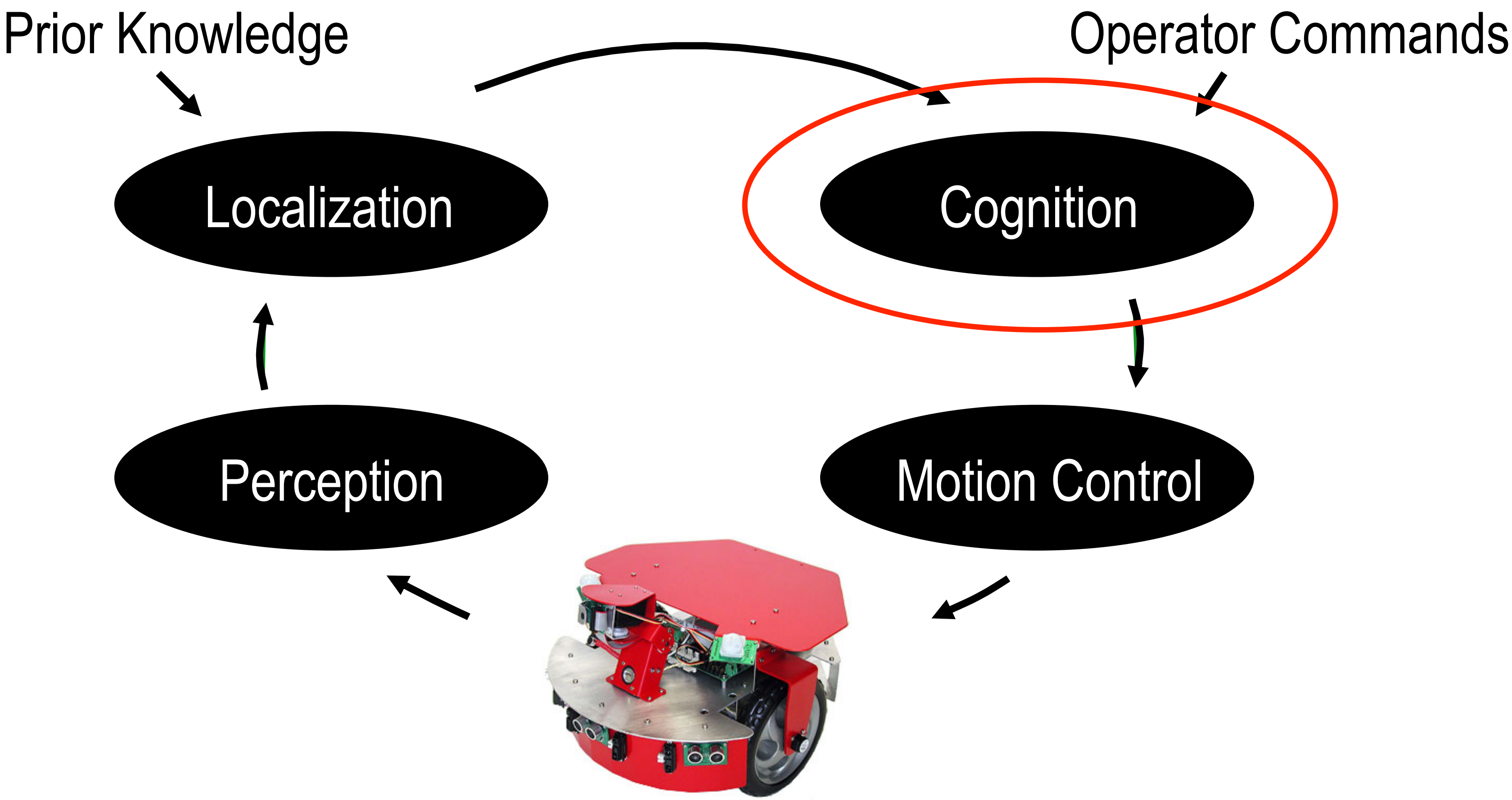
Semester: Fall 2011

Figures courtesy of Siegwart & Nourbakhsh

Control Structure



Single Query PRMs: Outline

1. Introduction
2. Algorithm Overview
3. Sampling strategies

Motion Planning: Probabilistic Road Maps

- Single-Query PRMs (a.k.a. Rapidly Exploring Random Trees - RRTs)
  - Try to only sample a subspace of *F* that is relevant to the problem.
  - Probabilistically complete assuming *C* is *expansive* [Hsu et. al. 2000].
  - Very fast for many applications (allow for on-the-fly planning).

Motion Planning: Probabilistic Road Maps

- Two approaches:
  1. Single Directional:
     - Grow a milestone tree from start configuration until the tree reaches the goal configuration
  2. Bi-Directional:
     - Grow two trees, one from the start configuration and one from the goal configuration, until the two trees meet.
     - Can’t consider time in the configuration space

Single Query PRMs: Outline

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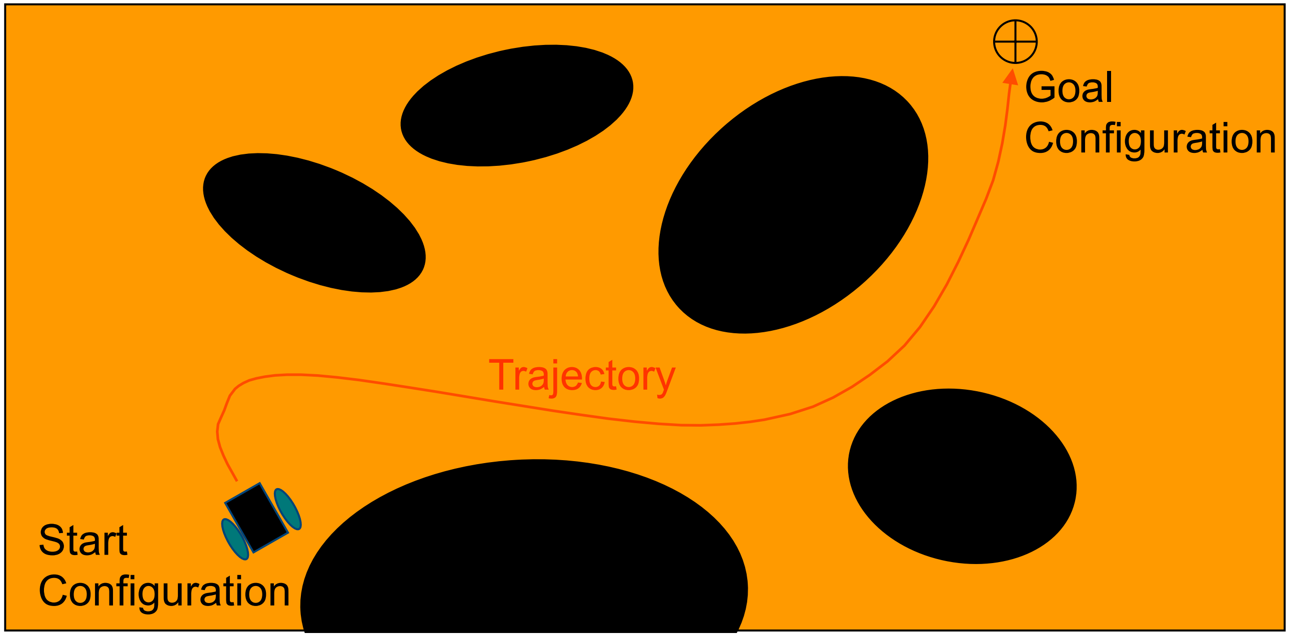
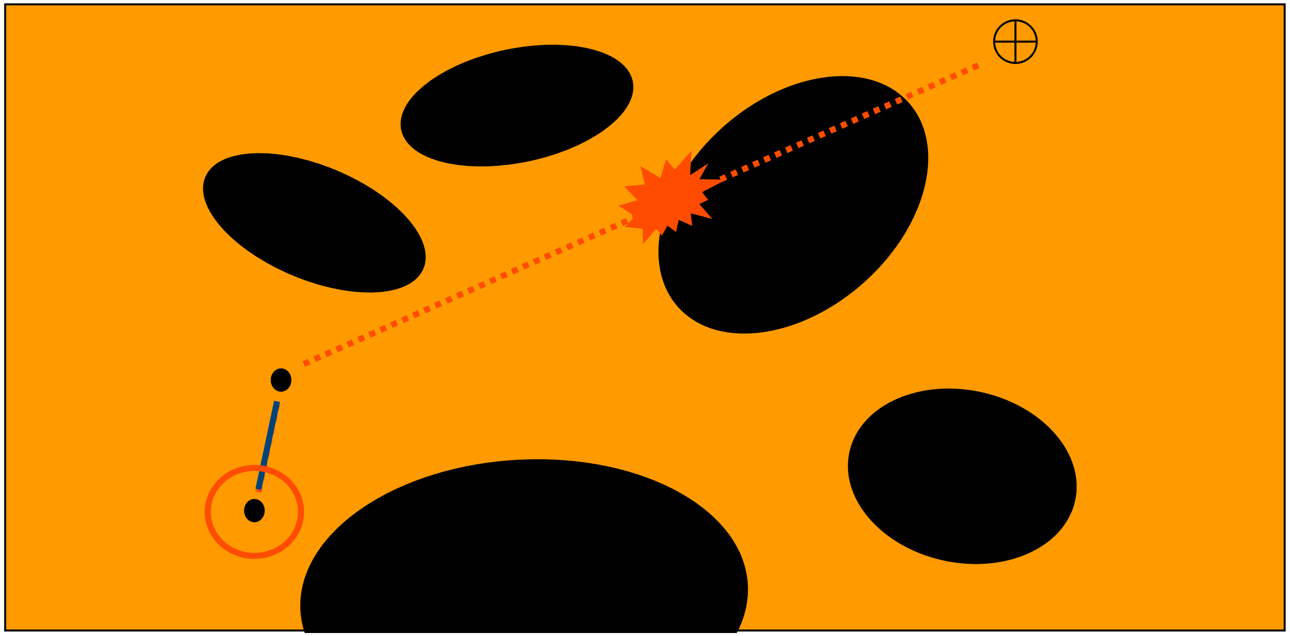
Motion Planning

- Example:



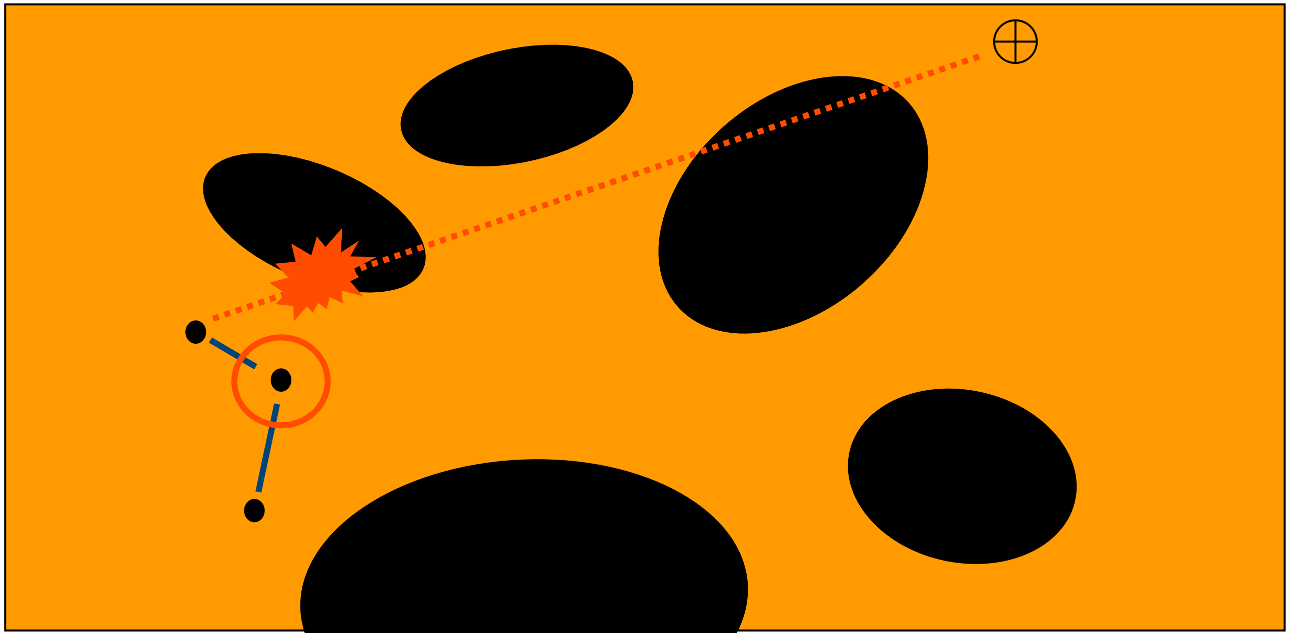
Motion Planning

- Example: Iteration 1



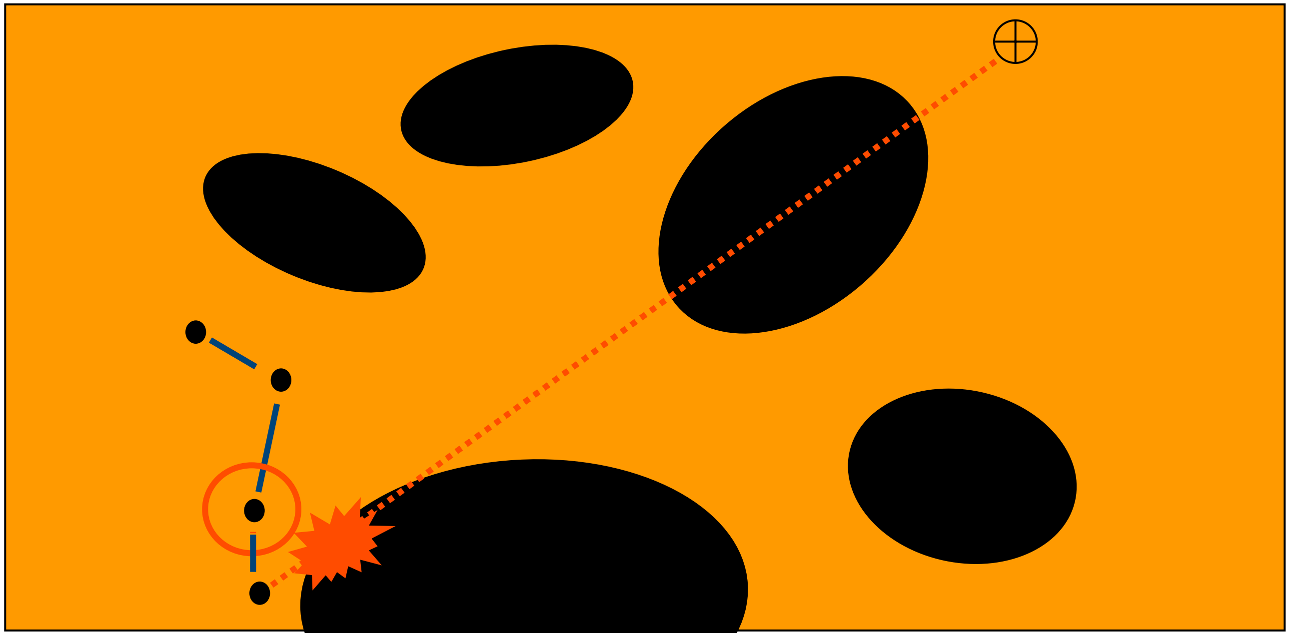
Motion Planning

- Example: Iteration 2



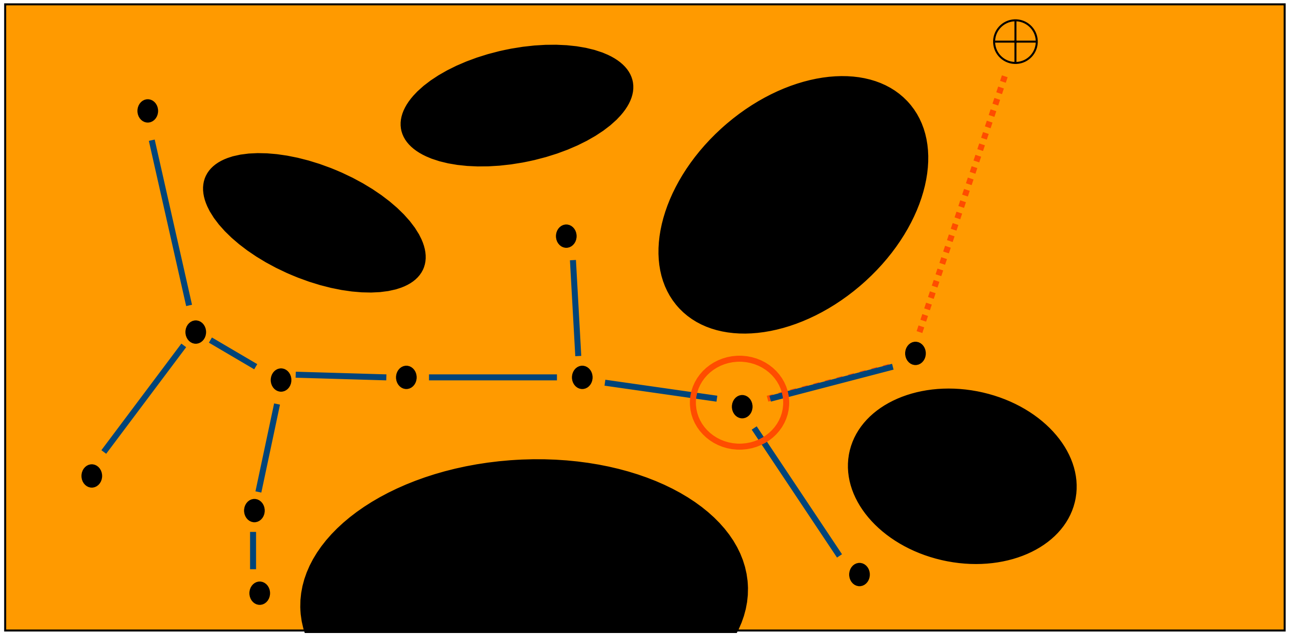
Motion Planning

- Example: Iteration 3



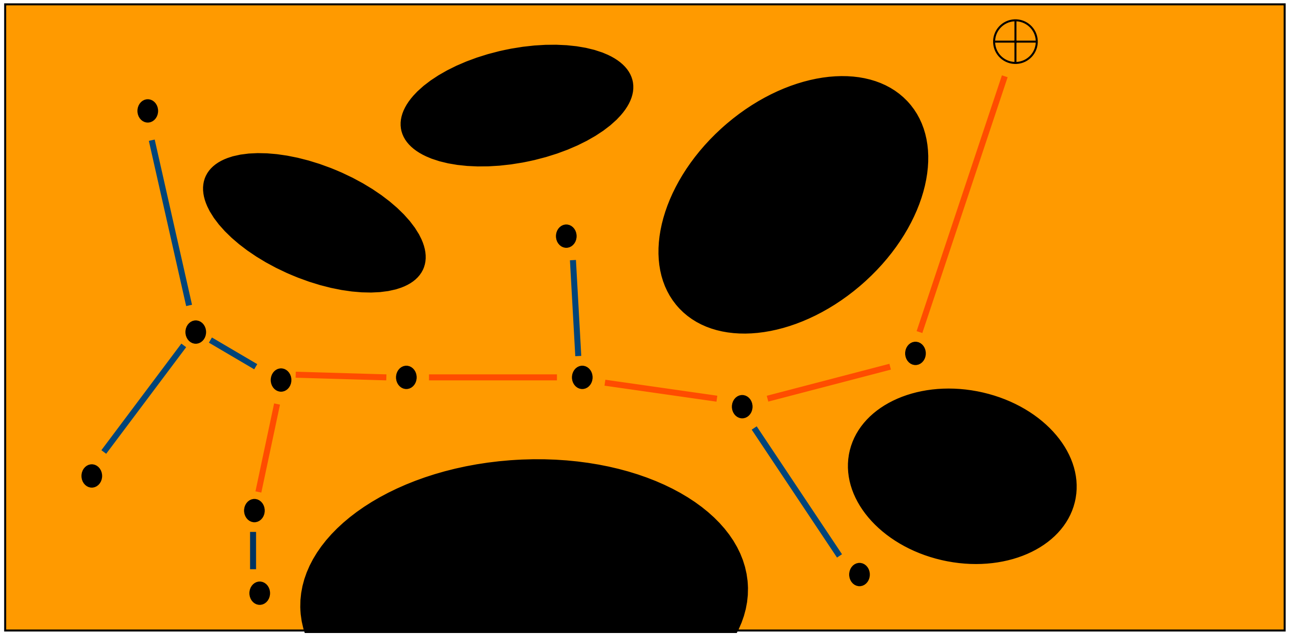
Motion Planning

- Example: Iteration 11



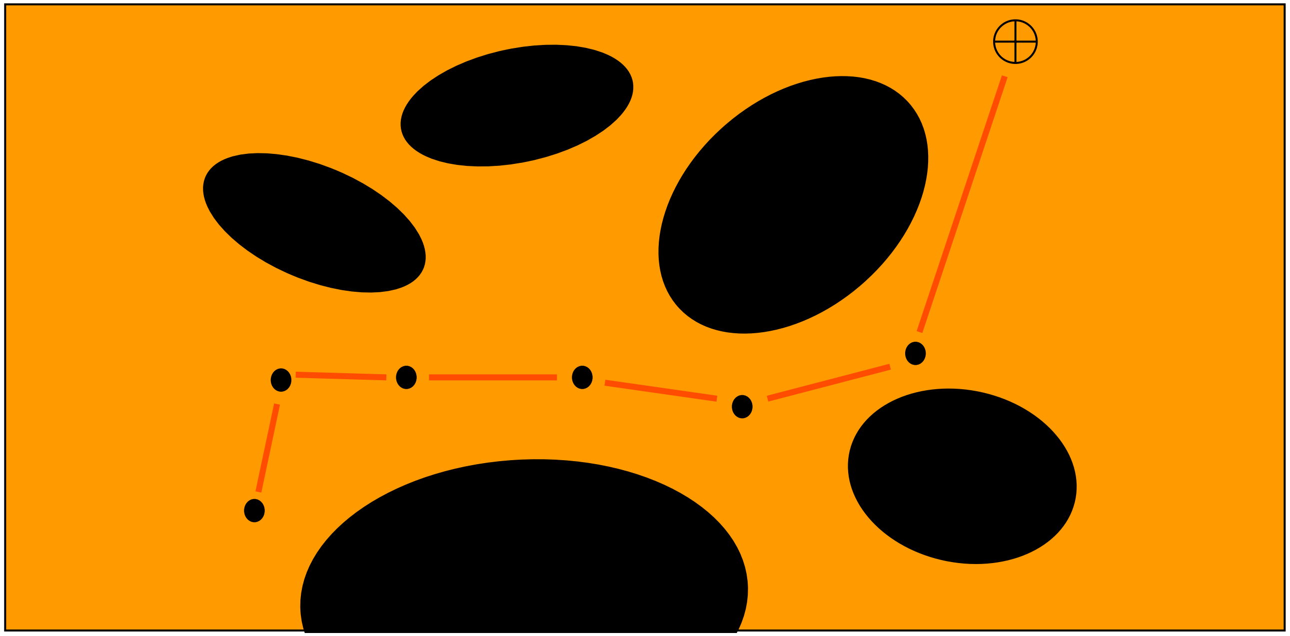
Motion Planning

- Example: Construct Path



Motion Planning

- Example: Construct Path



Probabilistic Road Maps: Learning Phase

- Nomenclature

$R=(N,E)$	RoadMap
N	Set of Nodes
E	Set of edges
c	Configuration
e	edge

Motion Planning: Probabilistic Road Maps

- Algorithm
  1. Add start configuration c_{start} to $R(N,E)$
  2. Loop
  3. Randomly Select New Node *c* to expand
  4. Randomly Generate new Node *c'* from *c*
  5. If edge *e* from *c* to *c'* is collision-free
  6. Add *(c', e)* to *R*
  7. If c' belongs to endgame region, return path
  8. Return if stopping criteria is met

Single Query PRMs: Outline

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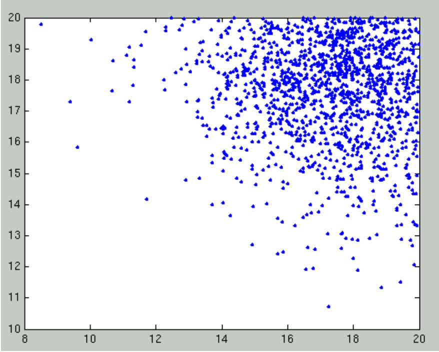
   - Node Selection (step 3)
   - Node Generation (step 4)
   - Endgame Region (step 7)

Motion Planning: PRM Node Selection

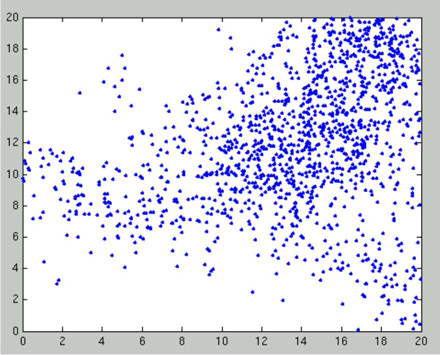
- One could pick the next node for expansion by picking from all nodes in the roadmap with equal probability.
  - This is easy to implement, but leads to poor expansion → *Clustering*
  - Method is to weight the random selection of nodes to expand, this can greatly affect the roadmap coverage of the configuration space.
  - Want to pick nodes with probability proportional to the inverse of node density.

Motion Planning: PRM Node Selection

- Example:
  - Presented is a 2DOF configuration space where the initial node in the roadmap is located in the upper right corner.
  - After X iterations, the roadmap produced from an unweighted expansion has limited coverage.



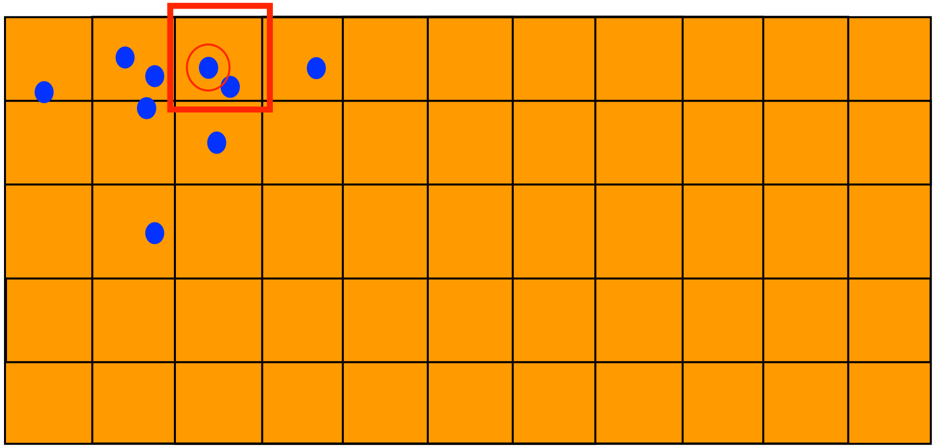
Unweighted



Weighted

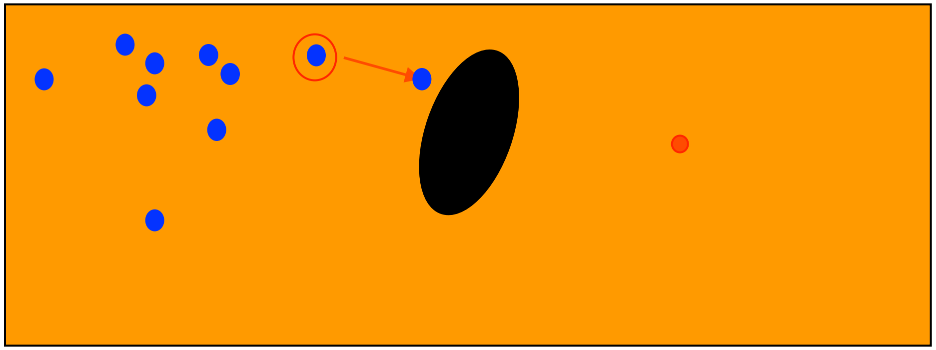
Motion Planning:
PRM Node Selection Technique 1

- The workspace was divided up into cells to form a grid [Kindel 2000].
  - Algorithm:
    1. Randomly pick an occupied cell from the grid.
    2. Randomly pick a milestone in that cell.



Motion Planning: PRM Node Selection Technique 2

- Commonly used in Rapidly exploring Random Trees (RRTs) [Lavalle]
  - Algorithm:
    1. Randomly pick configuration c_{rand} from C.
    2. Find node c from R that is closest to node c_{rand}
    3. Expand from c in the direction of c_{rand}



Single Query PRMs: Outline

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Motion Planning: PRM Milestone Generation

- Use random control inputs to propagate robot from previous node *c* to new configuration *c'*

  - Algorithm:
    1. Randomly select controls *u* and *Δt*
    2. Use known dynamics/kinematics equation *f* of robot to generate new configuration

       $$c' = f(c, u, \Delta t)$$

    3. If path from *c* to *c'* is collision-free, then add *c'* to *R*

Motion Planning: PRM Milestone Generation

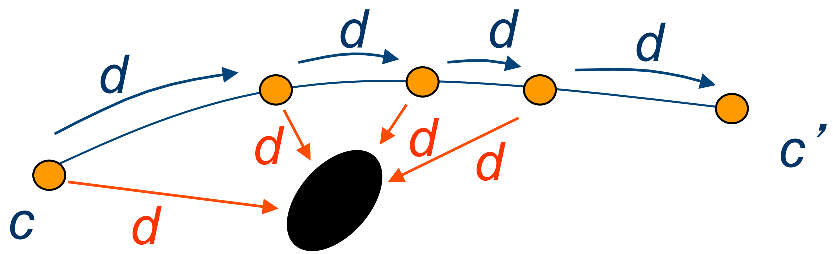
- Example: Differential drive robot
  1. Randomly select controls $\dot{\varphi}_{left}$, $\dot{\varphi}_{right}$ and Δt
  2. Propagate:
     1. Get Δs_{left} and Δs_{right} from $\Delta s = v\Delta t$
     2. Calculate new state c' with:

$$c' = f(x, y, \theta, \Delta s_r, \Delta s_l) = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta s_r + \Delta s_l}{2} \cos\left(\theta + \frac{\Delta s_r - \Delta s_l}{2b}\right) \\ \frac{\Delta s_r + \Delta s_l}{2} \sin\left(\theta + \frac{\Delta s_r - \Delta s_l}{2b}\right) \\ \frac{\Delta s_r - \Delta s_l}{b} \end{bmatrix}$$

     3. Use iterative search to check for collisions on path.

Motion Planning: PRM Milestone Generation

- Example: Differential drive robot (cont')
  - Iterative Collision checking is simple but not always efficient:
  - Algorithm:
    1. Calculate distance *d* to nearest obstacle
    2. Propagate forward distance *d* along path from *c* to *c'*
    3. If *d* is too small, return *collision*
    4. If *c* reaches or surpasses *c'*, return *collision-free*



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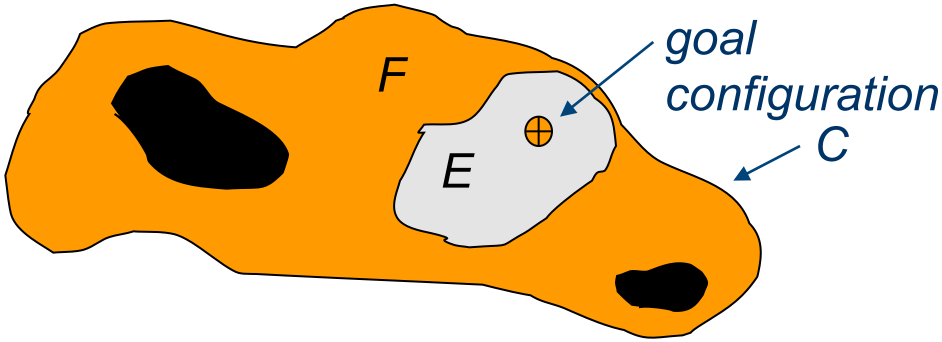
Motion Planning: PRM Endgame Region

- We define the endgame region *E*, to be the set of configurations that have a *simple* connection to the goal configuration.
- For each planning problem, we can define a unique method of making *simple* connections.
- This method will inherently define *E*.



Motion Planning: PRM Endgame Region

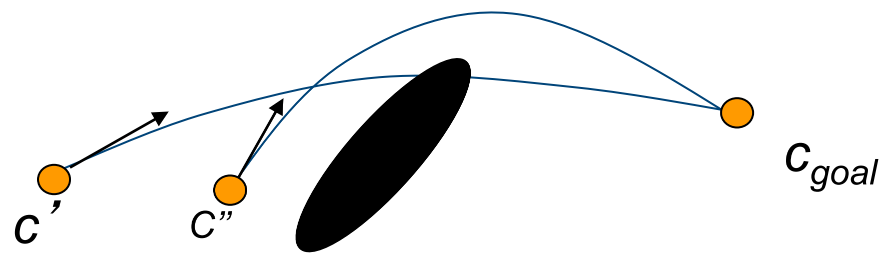
- Given the complexity of most configuration spaces, it is very difficult to model *E*.
- In practice, we develop a simple admissibility test to calculate if a configuration *c'* belongs to the *E*
- At every iteration of the algorithm, this test is used to determine if newly generated configurations are connected to the goal configuration.

Motion Planning: PRM Endgame Region

- In defining *E*, we need two things for good performance:
  1. The region *E* should be large: this increases the chance that a newly generated milestone will belong to *E* and provide us a solution.
  2. The admissibility test to be as fast as possible. This test is conducted at every iteration of the algorithm and will greatly affect the algorithm running time.

Motion Planning: PRM Endgame Region

- Several endgame definitions exist:
  1. The set of all configurations within some radius *r* of the goal configuration
  2. The set of all configurations that have “simple”, collision-free connection with the goal configuration.
     - Example: Use circular arc for differential drive robots.



Motion Planning: Probabilistic Road Maps

- Video example