COS 495 - Lecture 19
Autonomous Robot Navigation

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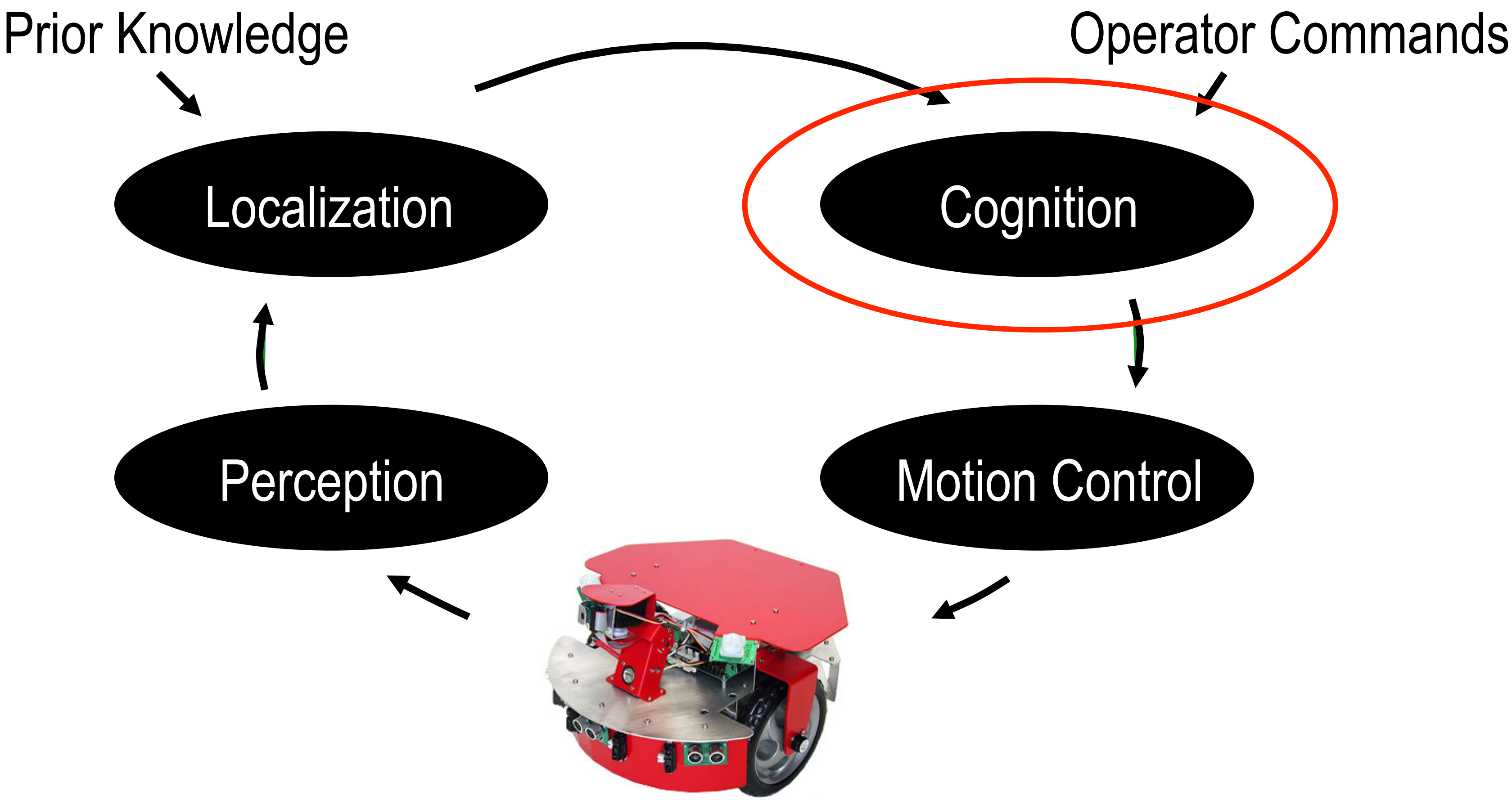
Figures courtesy of Siegwart & Nourbakhsh

Control Structure



Discretizations: Outline

1. General Approach to MP
2. Discretization Types
3. Probabilistic Road Maps

Motion Planning: General Approach

- Motion planning is usually done with three steps:

Defining the configuration space

↓

Discretization of the configuration space

↓

Searching the configuration space

Discretizations: Outline

1. General Approach to MP
2. Discretization Types
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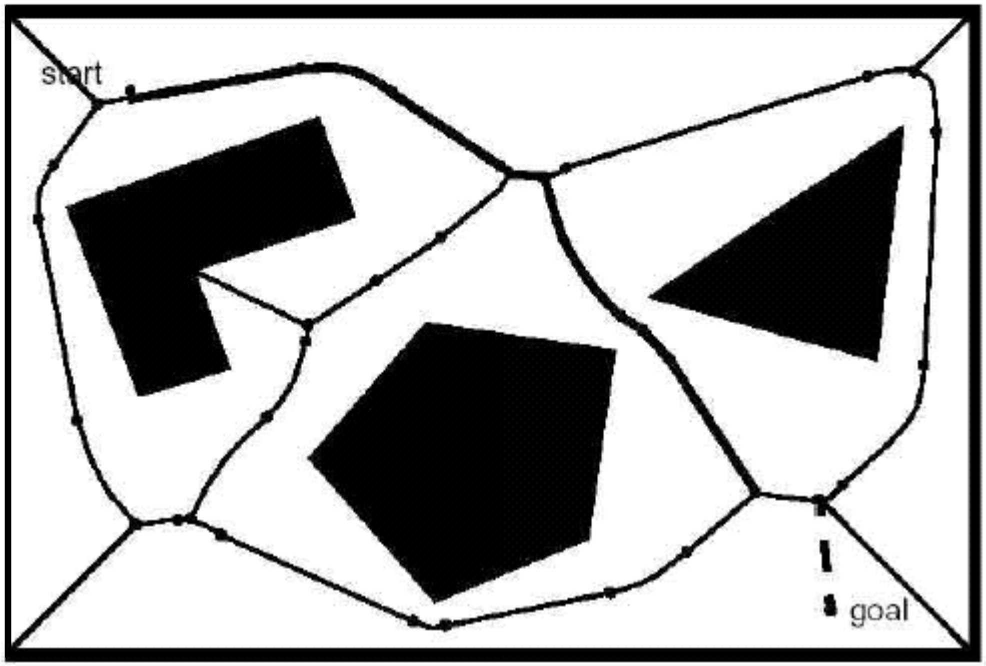
Motion Planning: Discretizations

1. Roadmap
   - Represent the connectivity of the free space by a network of 1-D curves
2. Cell decomposition
   - Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells
3. Potential field
   - Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent

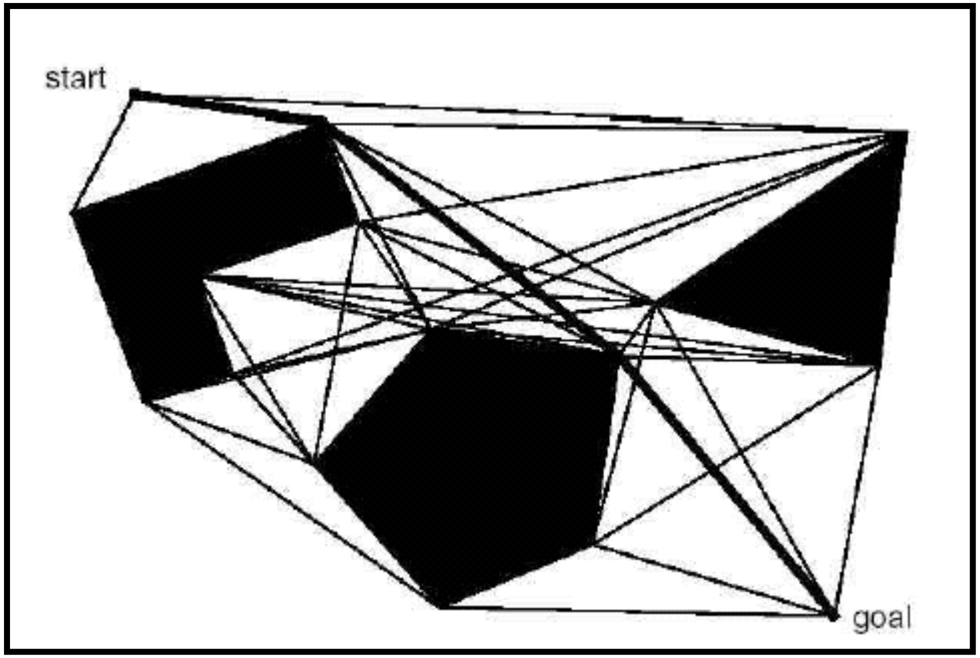
Motion Planning: RoadMaps

- Voronoi Diagram
  - Compute maximal distances from objects



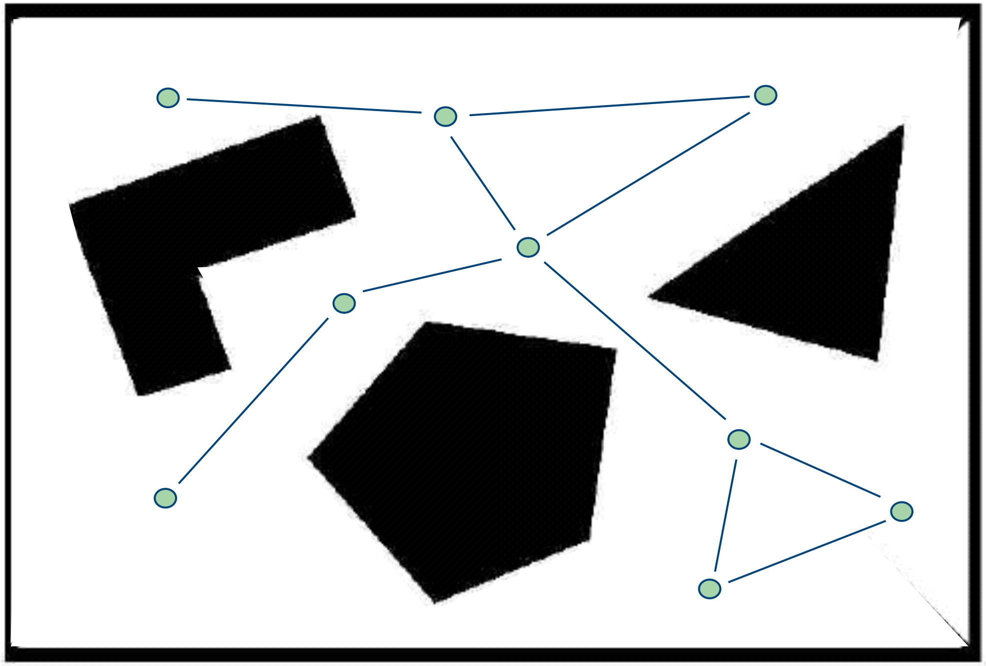
Motion Planning: RoadMaps

- Visibility Diagram
  - Introduced in Shakey. Can produce shortest paths in 2-D configuration spaces



Motion Planning: RoadMaps

- Probabilistic Road Maps (PRMs)
  - Randomly add nodes to the roadmap and connect them.



Discretizations: Outline

1. General Approach to MP
2. Discretization Types
3. Probabilistic Road Maps
    1. Introduction to PRMs
    2. Multi-Query PRMs

Probabilistic Road Maps

- Definition:
  - A probabilistic road map is a discrete representation of a continuous configuration space generated by randomly sampling the free configurations of the *C*-space and connecting those points into a graph.

Probabilistic Road Maps

- Goal of PRMs:
  - *Quickly* generate a *small* roadmap of the Free Space *F* that has good *coverage* and *connectivity*
- PRMS have proven to useful in mapping free spaces that are difficult to model, or have many degrees of freedom.
  - This can facilitate fast planning for these situations
- Sacrifice completeness for speed

Probabilistic Road Maps

Moving Objects, Kindel

Probabilistic Road Maps

- Two Main Strategies:

1. Multi-Query:
   - Generate a single roadmap of *F* which can be used many times.
2. Single-Query:
   - Use a new roadmap to characterize the subspace of F which is relevant to the search problem.

Discretizations: Outline

1. General Approach to MP
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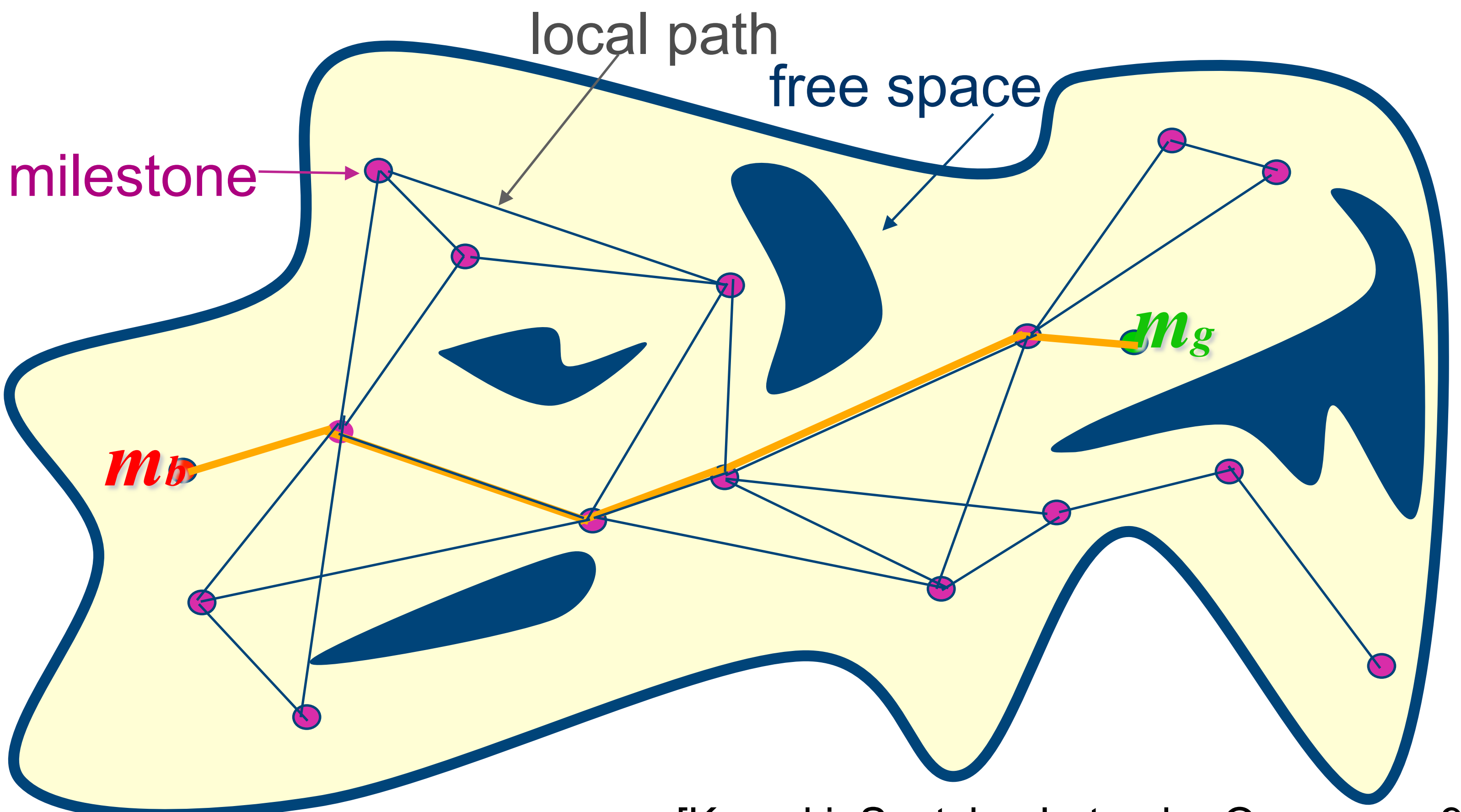
    1. Introduction to PRMs
    2. Multi-Query PRMs

Multi-Query PRMs

- Multi-Query Strategy
  1. Learning Phase:
     - Generate the PRM with two steps:
       - Construction
       - Expansion
     - Can take considerable time
  2. Query Phase:
     - Connect start and goal configurations to PRM
     - Perform a graph search to find path
     - Very fast
     - Smooth path?

Multi-Query PRMs



[Kavraki, Svetska, Latombe,Overmars, 95]

Multi-Query PRMs
Learning Phase

- Nomenclature

$R=(N,E)$	RoadMap
N	Set of Nodes
E	Set of edges
c	Configuration

Multi-Query PRMs
Learning Phase

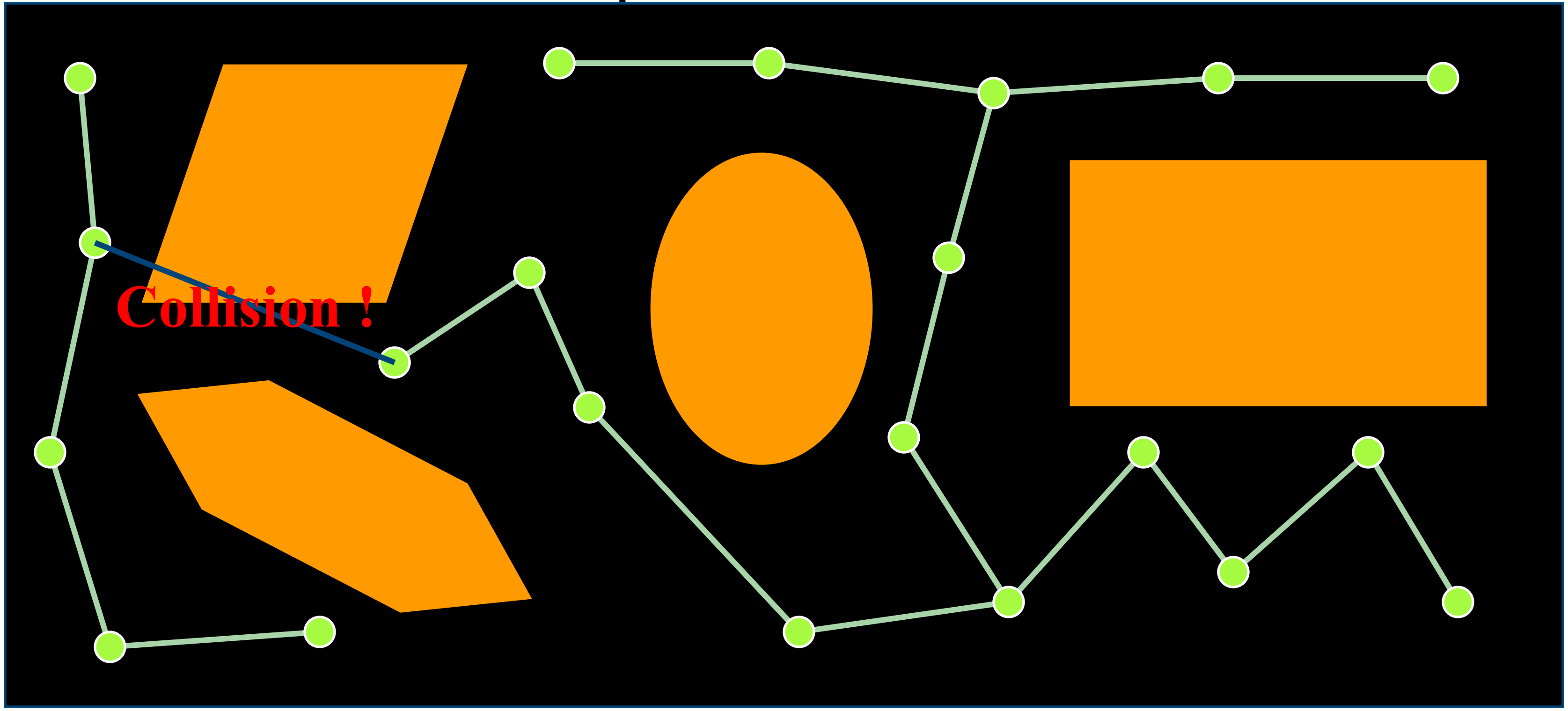
- Construction Step Algorithm
  1. Start with empty *R=(N,E)*
  2. Generate a random free config *c* and add to *N*
  3. Choose a subset N_c of candidate neighbors around *c* from *N*
  4. Try to connect *c* to each node in N_c in the order of increasing distance from *c* (w/ local planner)
  5. Add the edge found to *E*
  6. Repeat the above until satisfied

Multi-Query PRMs
Learning Phase

- Local Planner
  - Used to connect two nodes.
  - Must contain collision-check.
  - For good performance, the LP must be:
    1. Deterministic - This eliminates the need for storing local plans.
    2. Fast - To ensure quick planning queries.

Multi-Query PRMs
Learning Phase

- Construction Step



Courtesy of C. Allocco

Multi-Query PRMs Learning Phase

- Expansion Step Algorithm
  1. Find the nodes in 'difficult' regions using heuristic weight function *w(c)*
  2. Expand *c* using random-bounce walks
  3. Repeat as necessary

Multi-Query PRMs Learning Phase

- Expansion Weighting Function
  - Several options to define *w(c)*
    - Inversely proportional to the "number of nodes within some predefined distance from *c*"
    - Inversely proportional to the "distance from *c* to the nearest connected component not containing *c*"
    - Proportional to the "failure ratio of the local planner"

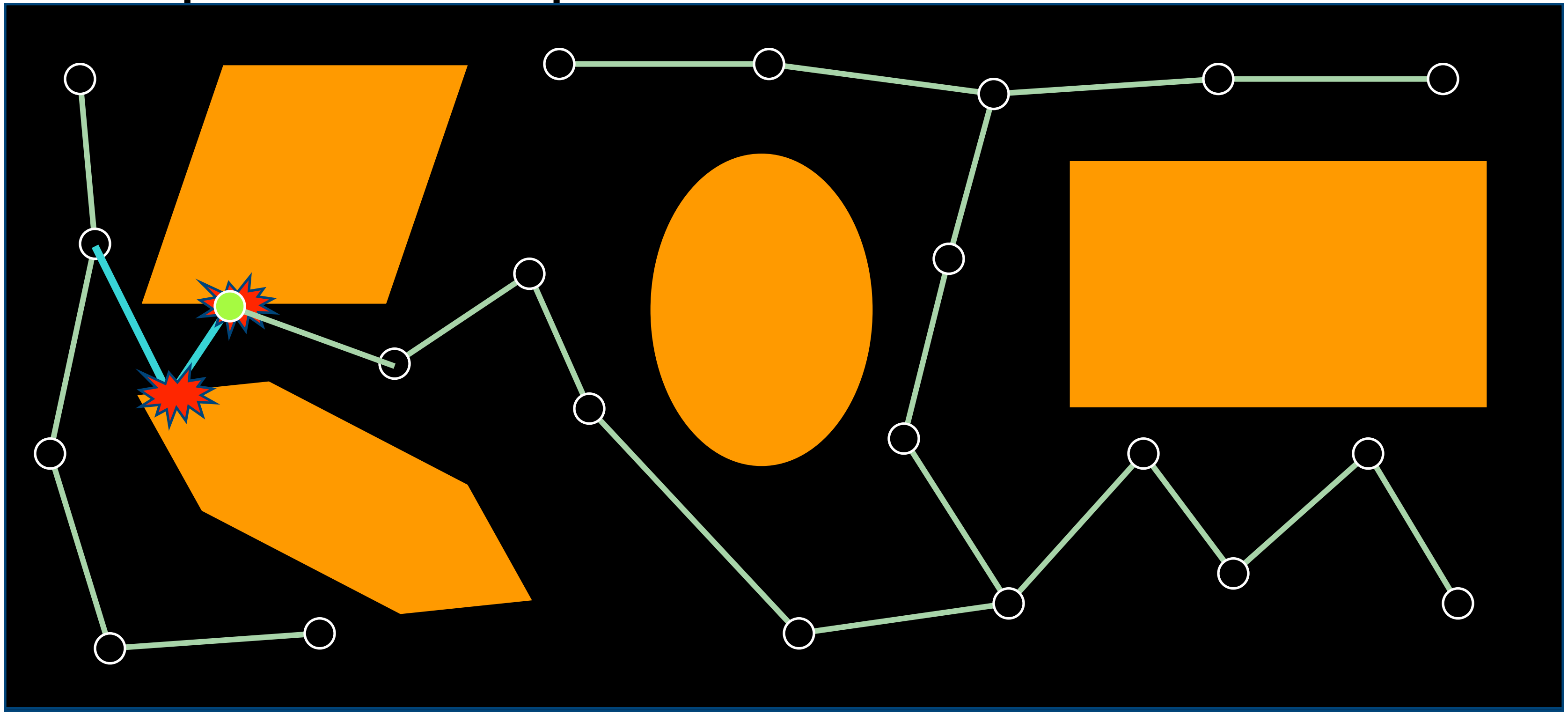
Multi-Query PRMs
Learning Phase

- Expansion Random-Bounce Walks

1. Loop
   1. Pick a random direction of motion in C-space
   2. Move in the direction until an obstacle is hit
   3. Check for connection with another node
   4. Repeat until the path can be connected to another node
2. Store the final config *n* and the edge *(c,n)* in *R*
3. Store the computed path (non-deterministic)
4. Record that *n* belongs to the same connected component as *c*

Multi-Query PRMs
Learning Phase

- Expansion Step

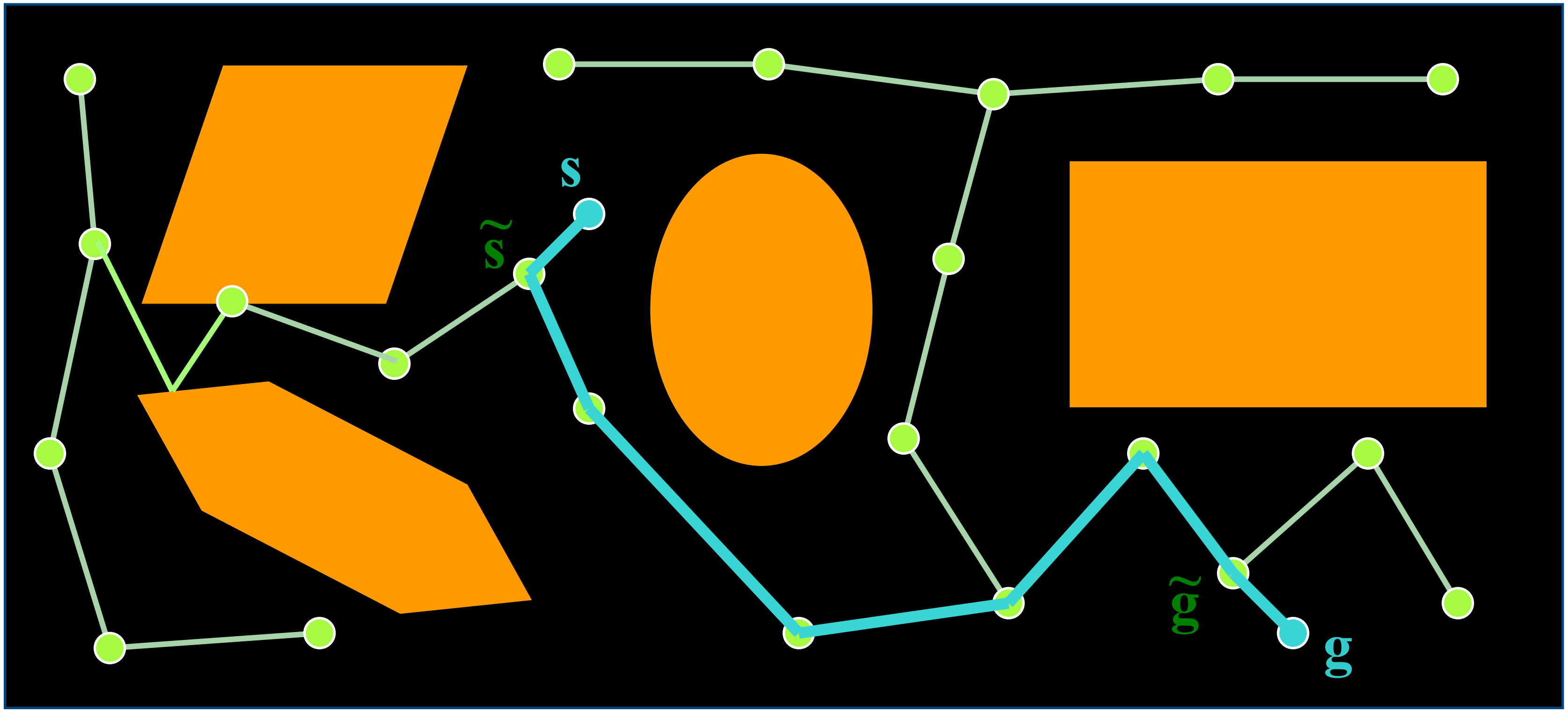


Courtesy of C. Allocco

Multi-Query PRMs
Query Phase

- Query Phase Algorithm
  1. Given the start and goal configurations *s* and *g*, calculate feasible paths P_s and P_g to the nodes *s* and *g* on the roadmap (w/ LP)
  2. Recalculate the path *P* from *s* to *g* using the roadmap
  3. Return the total path: $P_s – P – P_g^{-1}$

Multi-Query PRMs
Query Phase



Courtesy of C. Allocco

Probabilistic Road Maps

- Two Tenets:
  1. Checking sampled configurations and connections between samples for collision can be done efficiently.

  2. A relatively small number of milestones and local paths are sufficient to capture the connectivity of the free space.

     → Exponential convergence in expansive free space (probabilistic completeness)

Probabilistic Road Maps

- Probabilistically Complete
  - If a solution exists, the probability that the planner will find a solution is a (fast growing) function that goes to 1 as running time increases.
  - Example:
    - If a solution exists, the probability of failure decays exponentially to zero with the number of milestones added to the PRM.
  - This is less reliable than a complete algorithm, but is the trade-off for speed.

Probabilistic Road Maps: Discrete and Continous Planning

Courtesy of T. Bretl