COS 495 - Lecture 14
Autonomous Robot Navigation

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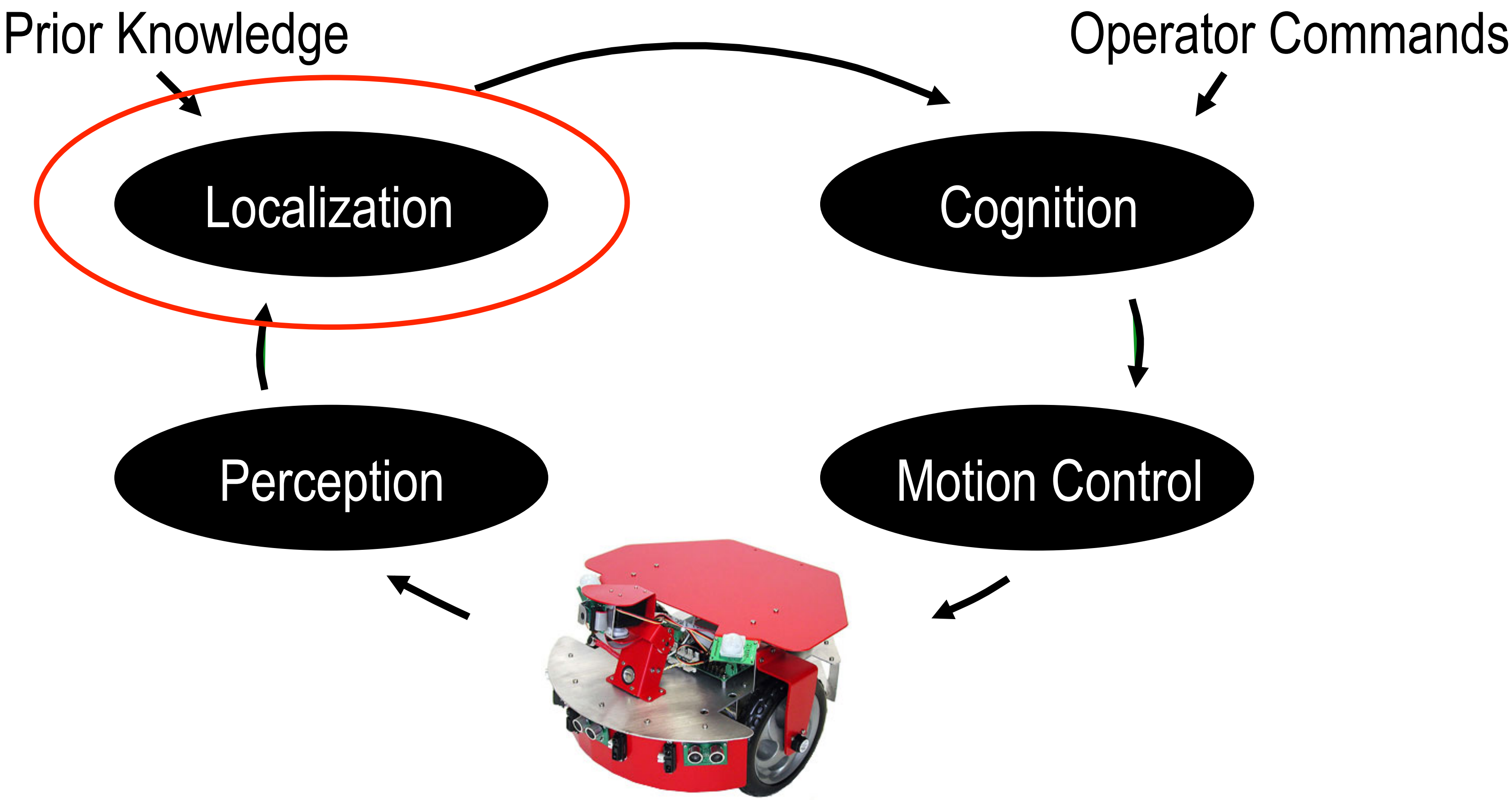
Figures courtesy of Siegwart & Nourbakhsh

Control Structure



Outline

1. Markov Localization Algorithm
    1. Overview
    2. Prediction Step
    3. Correction Step
2. ML Example

Markov Localization

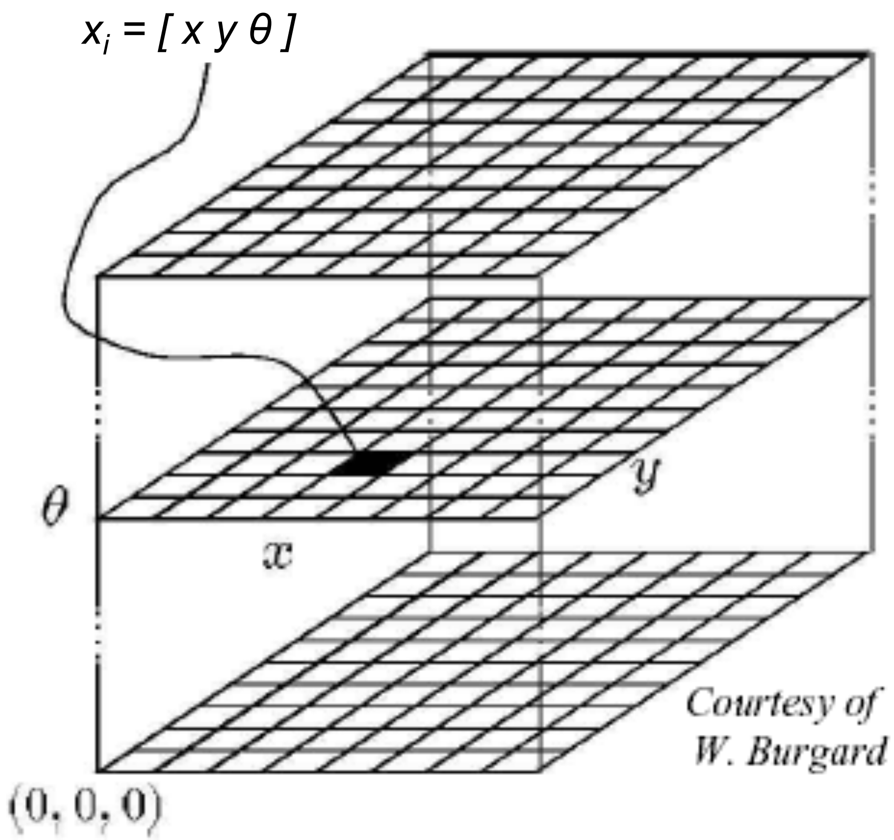
- Markov localization uses an explicit, **discrete** representation for the probability of all positions in the state space.
- Usually represent the environment by a finite number of (states) positions:
  - Grid
  - Topological Map
- At each iteration, the probability of each state of the entire space is updated

Markov Localization Grid Based Example

- Use a fixed decomposition grid by discretizing each dof:

  (x, y, θ)

  - For each location $x_i = [x\ y\ \theta]$ in the configuration space:
  - Determine probability $P(x_i)$ of robot being in that state.



Markov Localization

- We assume in localization the Markov Property holds true…
- Markov Property
  - aka memorylessness,
  - A stochastic Process satisfies the Markov Property if it is conditional *only on the present state of the system, and its future and past are independent*

Markov Localization

- Algorithm PseudoCode to update all *n* states

for $i = 1{:}n$
    $P(x_i) = 1/n$

while (true)
    o = getOdometryMeasurements
    z = getRangeMeasurements
    for $i = 1{:}n$
        $P(x_i')$ = predictionStep($P(x_j)$, o)
    for $i = 1{:}n$
        $P(x_i)$ = correctionStep($P(x_j')$, z)

Markov Localization
Applying Probability Theory

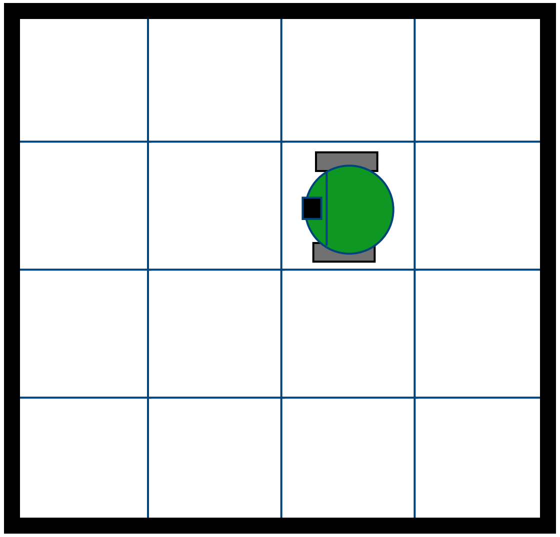
1. PREDICTION Step: Updating the belief state

$$P(x'_{i,t}) = P(x_{i,t} \mid o_t)$$
$$= \sum_{j=1}^{n} P(x_{i,t} \mid x_{j,t-1}, o_t)\, P(x_{j,t-1})$$

- Map from a belief state $P(x_{j,t-1})$ and action o_t to a new predicted belief state $P(x'_{i,t})$
- Sum over all possible ways (i.e. from all states $x_{j,t-1}$) in which the robot may have reached $x'_{i,t}$
- This assumes that update only depends on previous state and most recent actions/perception

Markov Localization
Grid Based Example

- Example Problem:
  - *Consider a robot equipped with encoders and a perfect compass moving in a square room that is discretized into a map of 16 cells:*



Markov Localization
Grid Based Example

- Example Problem:
  - *What is the probability of being in position (2,3) given odometry $o_t = (\Delta x, \Delta y) = (-1.0 \text{ cells}, 0.0 \text{ cells})$, and starting from the following distribution?*

.02	.05	.05	.05
.02	.05	.18	.05
.05	.05	.18	.05
.05	.05	.05	.05

Markov Localization
Grid Based Example

- Example Solution:
  - *We must have a model of how well our odometry works. For example, we could use a model for $o_t = (\Delta x, \Delta y) = (-1.0, 0.0)$ that looks like:*

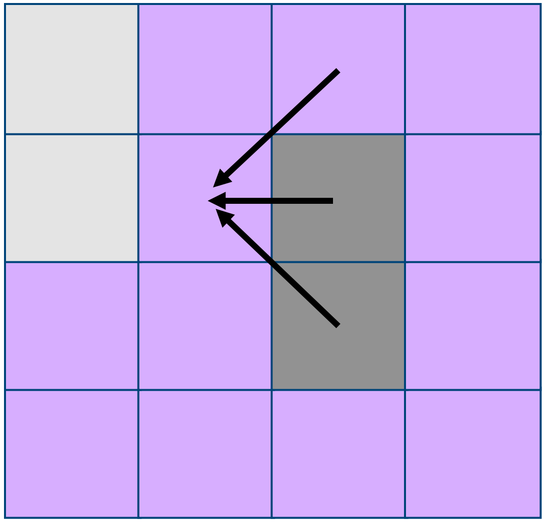
.00	.00	.00
.00	.00	1.0
.00	.00	.00

$(\Delta x, \Delta y)$ →

.00	.20	.00
.00	.50	.10
.00	.20	.00

Markov Localization
Grid Based Example

- Example Solution:
  - *Now apply this model to the initial state. We must consider the following possible scenarios for getting to position (2,3):*
    - *(3,3) → (2,3)*
    - *(2,3) → (2,3)*
    - *(3,2) → (2,3)*
    - *(3,4) → (2,3)*



Markov Localization
Grid Based Example

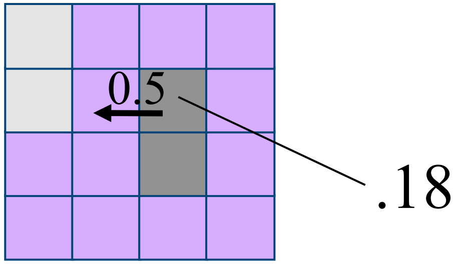
- Example Solution:
  - *Consider the first possibility:*
    - *(3,3) → (2,3)*
  - *We can calculate the probability of this happening*

$P\,(\,x_{i,t} \mid x_{j,t-1}\,,\,o_t\,)\,P\,(x_{j,t-1}\,)$

$= P\,(\,x_t{=}(2,3) \mid x_{t-1}{=}(3,3)\,,\,o_t{=}(-1,0)\,)\;\;P(x_{t-1}{=}\,(3,3))$

$= (0.5)\,(0.18)$

$= 0.09$



Markov Localization Grid Based Example

- Example Solution:
  - *Similarly, we can calculate the probability of all other possible ways to get to (2,3).*

$P(\, x_t=(2,3) \mid x_{t-1}=(2,3)\, ,\, o_t=(-1,0)\,)\;\; P(x_{t-1}=(2,3))$

$= 0.005$

$P(\, x_t=(2,3) \mid x_{t-1}=(3,2)\, ,\, o_t=(-1,0)\,)\;\; P(x_{t-1}=(3,2))$

$= 0.036$

$P(\, x_t=(2,3) \mid x_{t-1}=(3,4)\, ,\, o_t=(-1,0)\,)\;\; P(x_{t-1}=(3,4))$

$= 0.01$

Markov Localization
Grid Based Example

- Example Solution:
  - *So the probability of being at position (2,3) given the odometry is the total probability of moving there from each possible position:*

$$P(x_{it}=(2,3) \mid o_t=(-1,0)) = \sum P(x_t=(2,3) \mid x_{j,t-1}, o_t=(-1,0))\, P(x_{j,t-1})$$

$$= 0.09 + 0.005 + 0.036 + 0.01$$

$$= 0.141$$

Markov Localization
Applying Probability Theory

2. CORRECTION Step: refine the belief state

$$P(x_{i,t} \mid z_t) = \frac{P(z_t \mid x'_{i,t})\, P(x'_{i,t})}{P(z_t)}$$

- $P(x'_{i,t})$: the belief state before the perceptual update i.e. $P(x_{i,t} \mid o_t)$
- $P(z_t \mid x'_{i,t})$: the probability of getting measurement z_t from state x'_t
- $P(z_t)$: the probability of a sensor measurement z_t. Calculated so that the sum over all states $x_{i,t}$ from equals 1.

Markov Localization

- Critical challenge is calculation of $P(z \mid x)$
  - The number of possible sensor readings and geometric contexts is extremely large
  - $P(z \mid x)$ is computed using a model of the robot's sensor behavior, its position x, and the local environment metric map around x.
  - Assumptions
    - Measurement error can be described by a distribution with a mean
    - Non-zero chance for any measurement

Markov Localization
Grid Based Example

- Example Problem:
  - *What is the probability of being in state x= (2,3) given we have range measurement z=1.2m ?*

$$P(x_t = (2,3)) | z_t = 1.2)$$

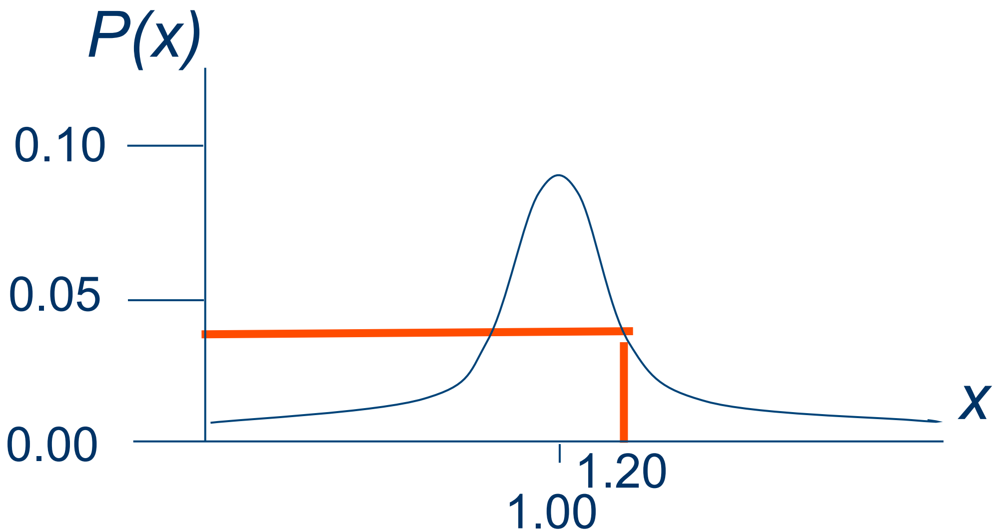
$$= \frac{P(z_t = 1.2 | x'_t = (2,3)) \; P(x'_t = (2,3))}{P(z_t = 1.2)}$$

Markov Localization Grid Based Example

- Example Solution:
  - *We can use the probability $P(x'_t=(2,3)) = 0.141$ from the previous example.*
  - *The interesting term is $P(z_t=1.2 | x'_t=(2,3))$.*
    - *Using the map, we can calculate the expected value of the range sensor measurement.*
    - *If the robot is at (2,3) and facing to the left, it should get a range measurement of 1m.*
    - *Recall that we can use the probability density function representing the sensor characteristics, and that the expected value is*

Markov Localization
Grid Based Example

- Example Solution:
  - *For Ultrasound, P(z|x) can be taken from the following distribution:*



$P(z|x) = 0.04$

Markov Localization Grid Based Example

- Example Solution:
  - *Now we can calculate the numerator for*

$$p\,(x_t = (2,3))|\ z_t = 1.2)$$

$$= \frac{p\,(z_t = 1.2|\ x'_t = (2,3))\ \ p\,(x'_t = (2,3))}{p\,(z_t = 1.2)}$$

$$= \frac{(0.04)\ (0.141)}{p\,(z_t = 1.2)}$$

Markov Localization
Grid Based Example

- Example Solution:
  - *Finally, we can calculate the denominator by ensuring the sum of all probabilities is 1.*

$$1 = \sum_{i=1}^{n} P(x_{i,t} \mid z_t = 1.2)$$

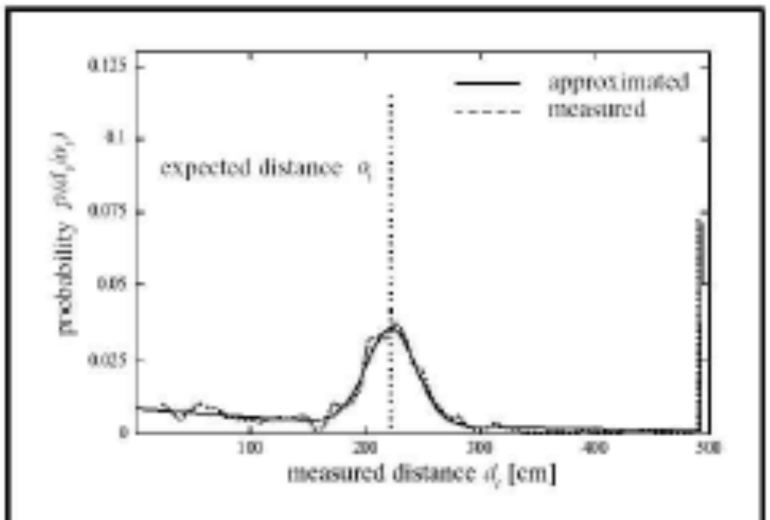
$$= \frac{\sum P(z_t = 1.2 \mid x'_{i,t})\, P(x'_{i,t})}{P(z_t = 1.2)}$$

Therefore:

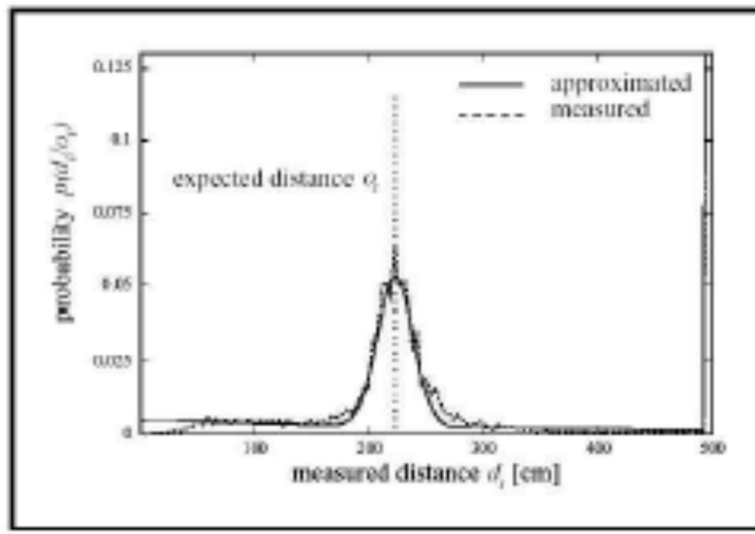
$$P(z_t = 1.2) = \sum P(z_t = 1.2 \mid x'_{i,t})\; P(x'_{i,t})$$

Markov Localization
Grid Based Example

- Here are some typical sensor distributions:



Ultrasound.



Laser range-finder.

Markov Localization: Outline

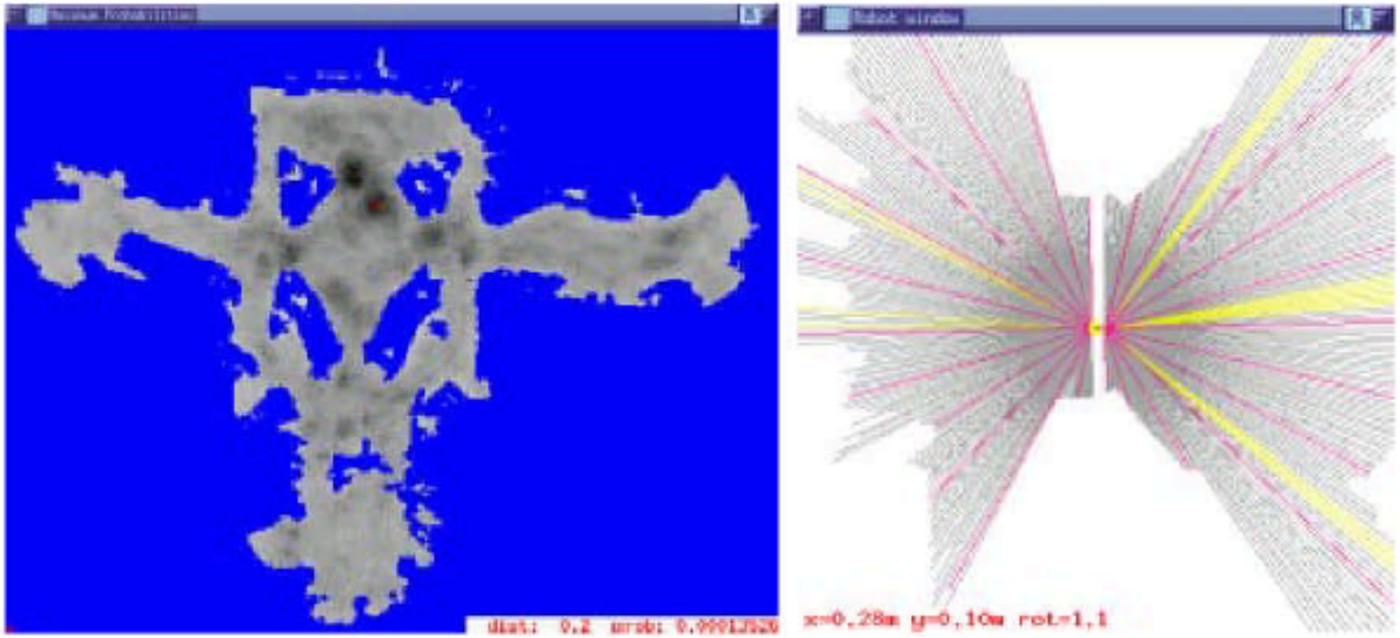
1. Markov Localization Algorithm
    1. Overview
    2. Prediction Step
    3. Correction Step
2. ML Example

Markov Localization: Outline

- Markov Localization Example
  - Time steps taken from ML example of the robot Minerva navigating around the Smithsonian.
  - In the following figures:
    - Left side shows belief state. Darker means higher probability.
    - Right side shows actual robot position and sensor measurements.

Markov Localization
Grid Based Example

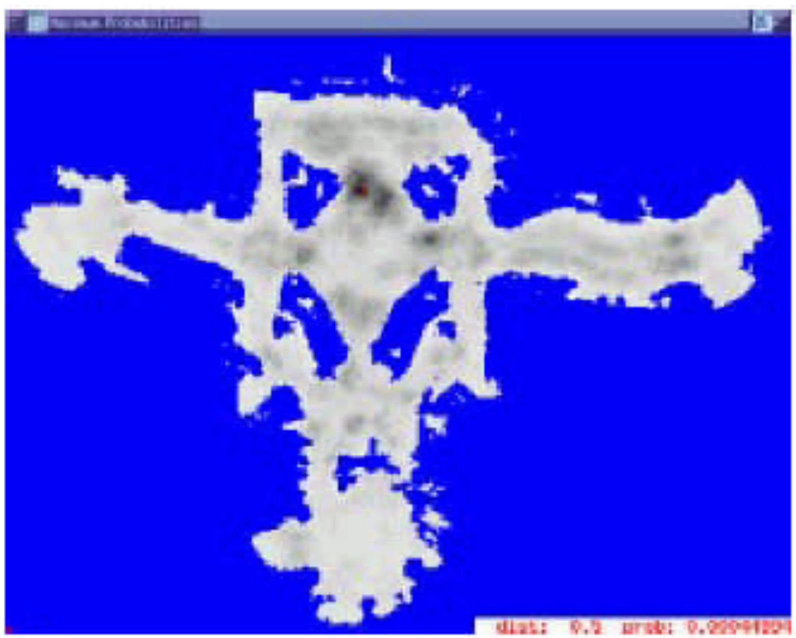
- Laser Scan 1 of Museum

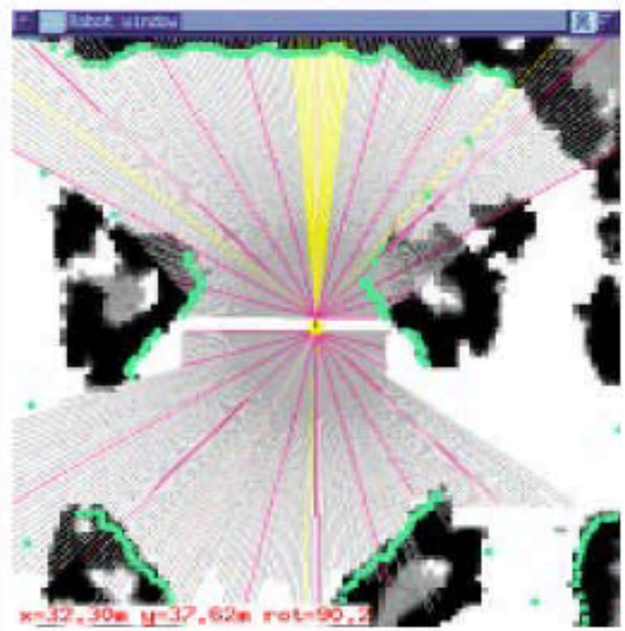


Figures courtesy of W. Burgard

Markov Localization
Grid Based Example

- Laser Scan 2 of Museum





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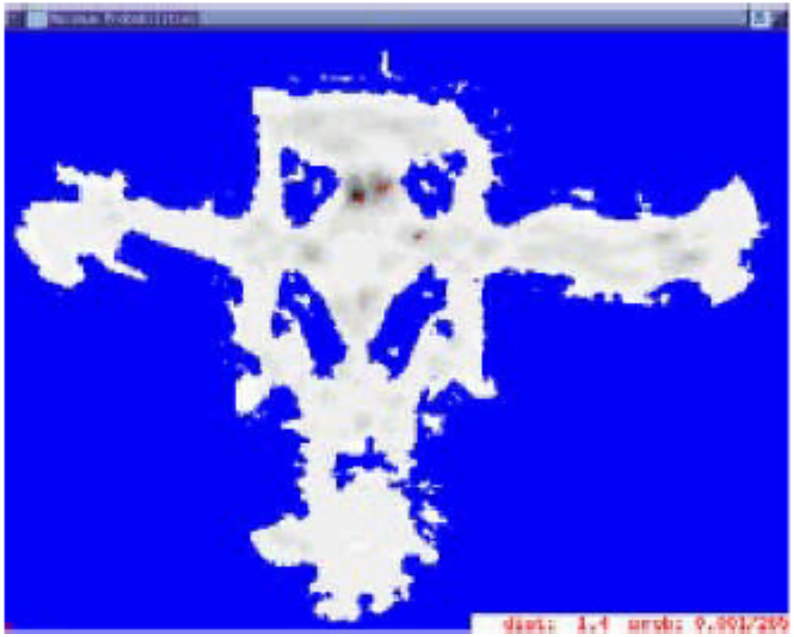
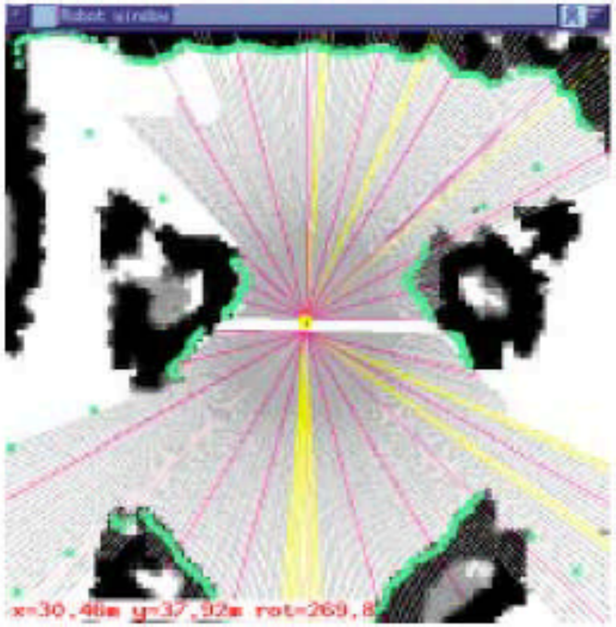
Markov Localization
Grid Based Example

- Laser Scan 3 of Museum





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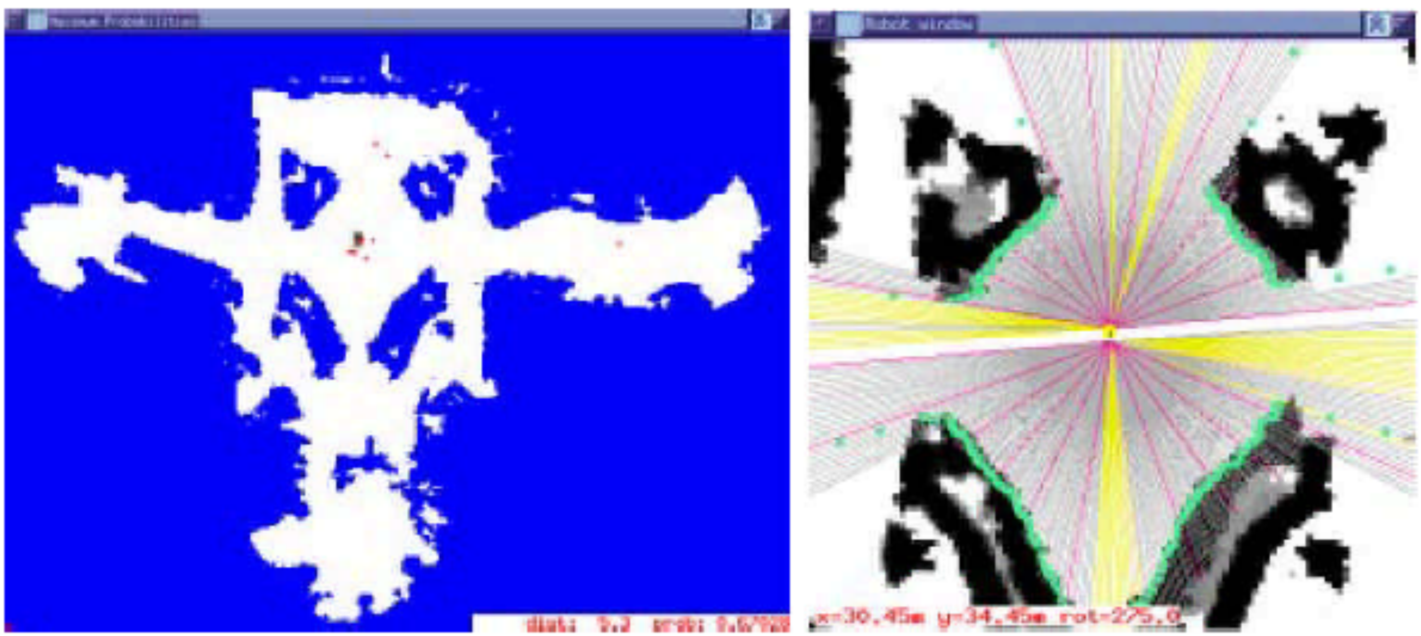
Markov Localization
Grid Based Example

- Laser Scan 13 of Museum



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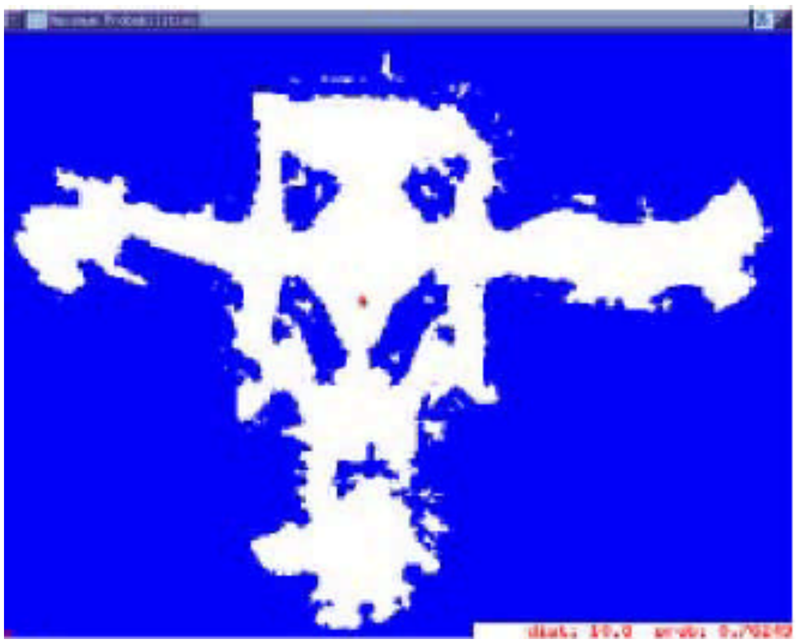
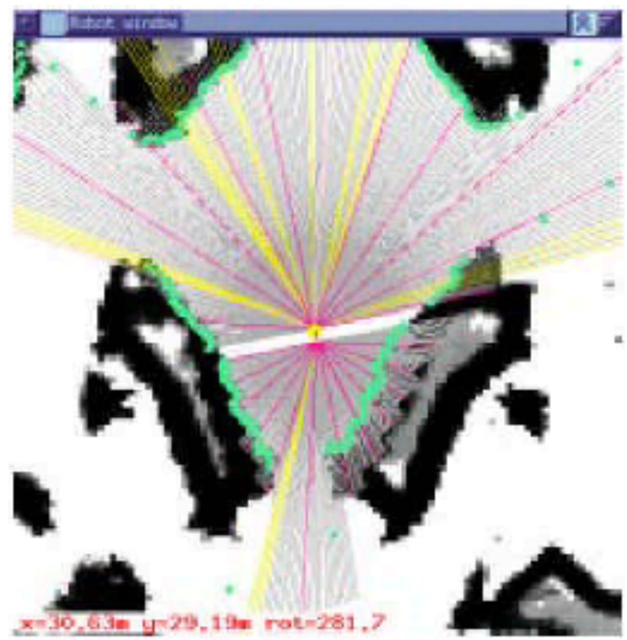
Markov Localization
Grid Based Example

- Laser Scan 21 of Museum





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