ALGORITHMS AND DATA STRUCTURES

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http://www.princeton.edu/~cos226

Course Overview

- outline
- why study algorithms?
- usual suspects
- coursework
- resources

What is COS 226?

- Intermediate-level survey course.
- Programming and problem solving, with applications.
- Algorithm: method for solving a problem.
- Data structure: method to store information.

topic	data structures and algorithms
data types	stack, queue, bag, union-find, priority queue
sorting	quicksort, mergesort, heapsort, radix sorts
searching	BST, red-black BST, hash table
graphs	BFS, DFS, Prim, Kruskal, Dijkstra
strings	KMP, regular expressions, TST, Huffman, LZW
advanced	B-tree, suffix array, maxflow, simplex

Why study algorithms?

Their impact is broad and far-reaching.

Internet. Web search, packet routing, distributed file sharing, ...

Biology. Human genome project, protein folding, ...

Computers. Circuit layout, file system, compilers, ...

Computer graphics. Movies, video games, virtual reality, ...

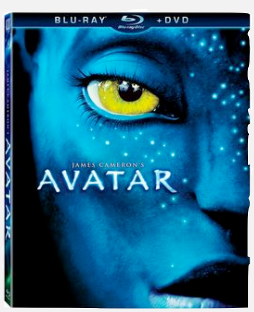
Security. Cell phones, e-commerce, voting machines, ...

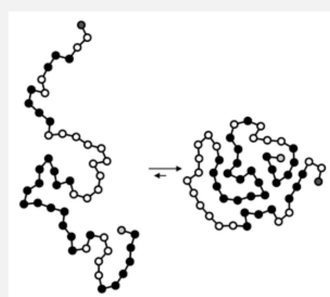
Multimedia. MP3, JPG, DivX, HDTV, face recognition, ...

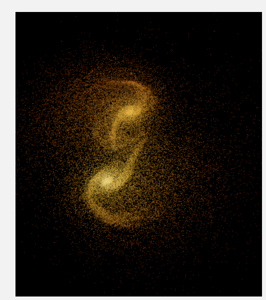
Social networks. Recommendations, news feeds, advertisements, ...

Physics. N-body simulation, particle collision simulation, ...

...



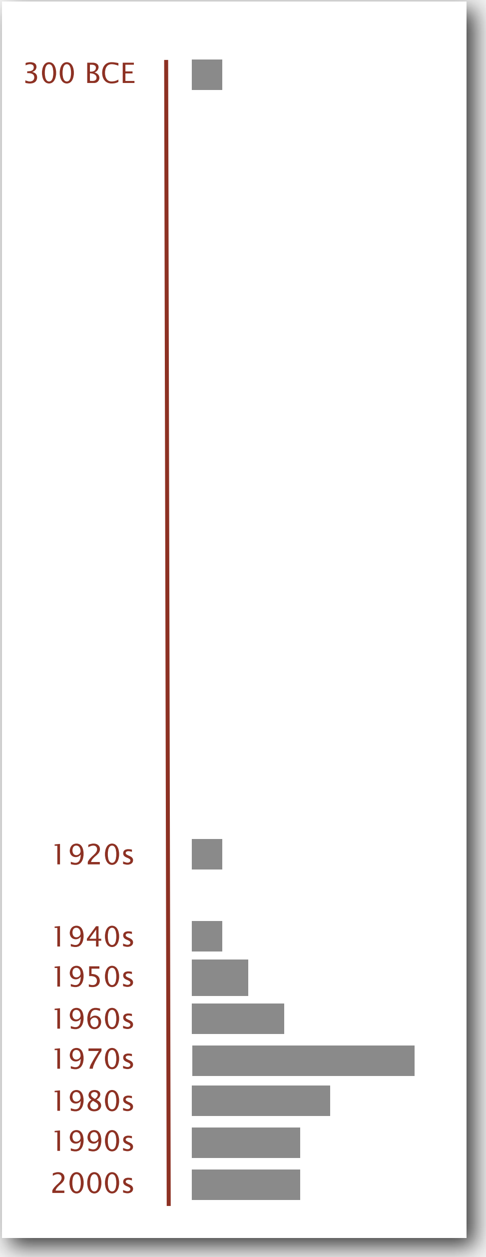





Why study algorithms?

Old roots, new opportunities.

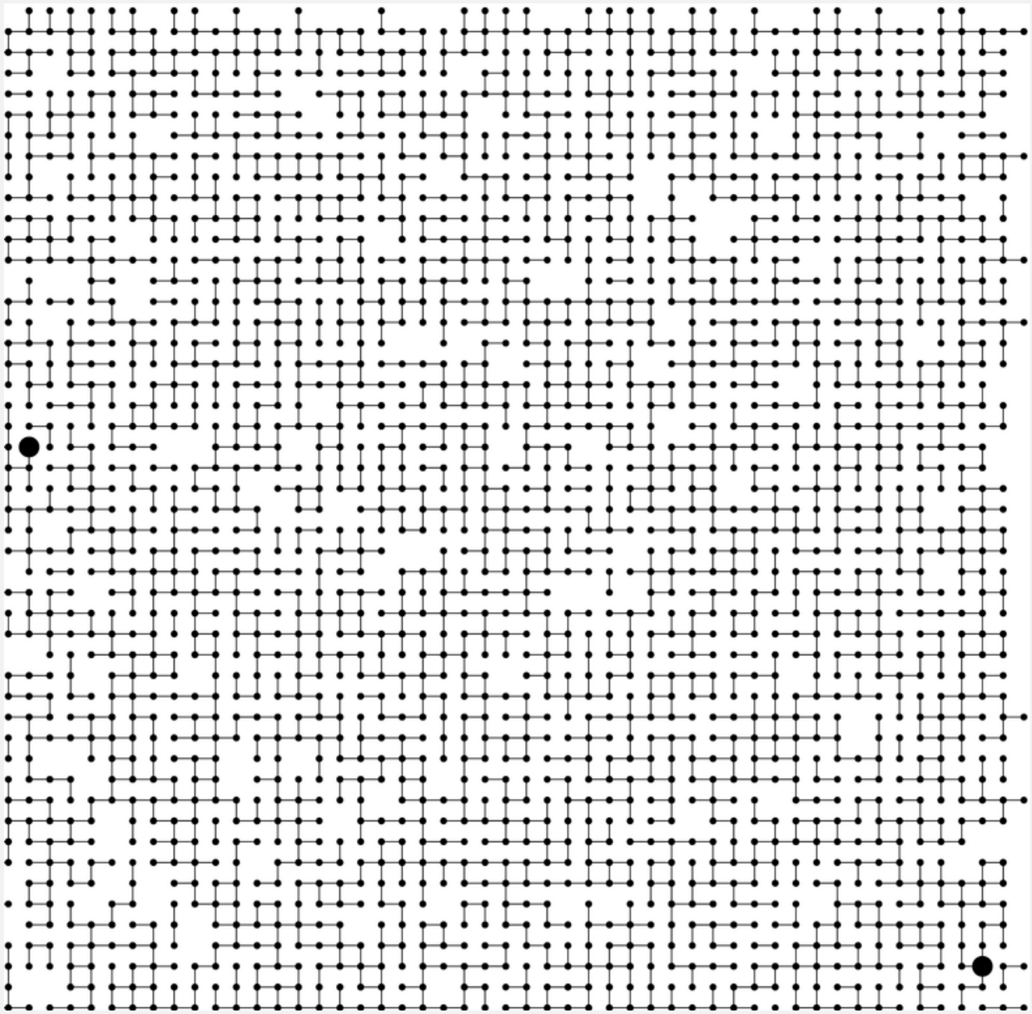
- Study of algorithms dates at least to Euclid.
- Some important algorithms were discovered by undergraduates in a course like this!



Why study algorithms?

To solve problems that could not otherwise be addressed.

Ex. Network connectivity. [stay tuned]

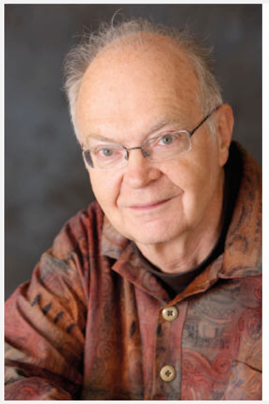


Why study algorithms?

For intellectual stimulation.

> " *For me, great algorithms are the poetry of computation. Just like verse, they can be terse, allusive, dense, and even mysterious. But once unlocked, they cast a brilliant new light on some aspect of computing.* " — *F. Sullivan*

> " *An algorithm must be seen to be believed.* " — *D. E. Knuth*



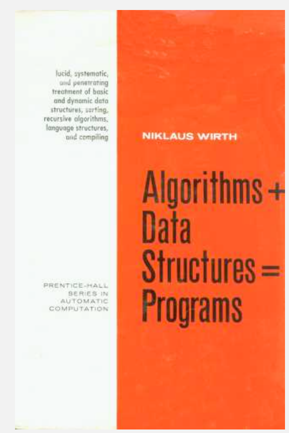
Why study algorithms?

To become a proficient programmer.

" I will, in fact, claim that the difference between a bad programmer and a good one is whether he considers his code or his data structures more important. Bad programmers worry about the code. Good programmers worry about data structures and their relationships. "

— Linus Torvalds (creator of Linux)

" Algorithms + Data Structures = Programs. " *— Niklaus Wirth*



Why study algorithms?

They may unlock the secrets of life and of the universe.

Computational models are replacing mathematical models in scientific inquiry.

$$E = mc^2$$

$$F = ma$$

$$F = \frac{G m_1 m_2}{r^2}$$

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \Psi(r) = E \, \Psi(r)$$

20th century science
(formula based)

```
for (double t = 0.0; true; t = t + dt)
   for (int i = 0; i < N; i++)
   {
      bodies[i].resetForce();
      for (int j = 0; j < N; j++)
         if (i != j)
            bodies[i].addForce(bodies[j]);
   }
```

21st century science
(algorithm based)

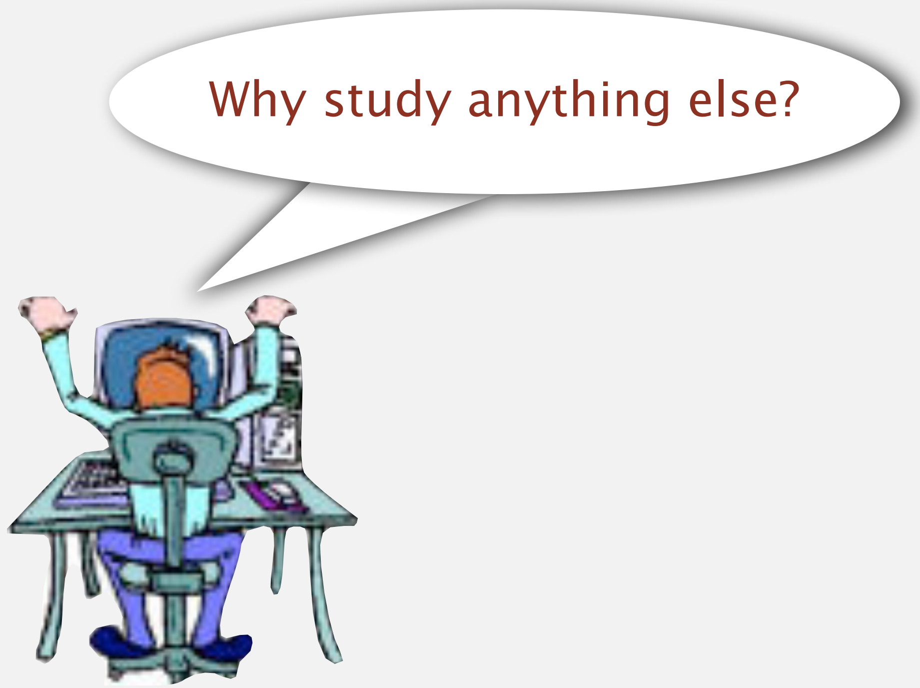
" *Algorithms: a common language for nature, human, and computer.* " — *A. Wigderson*

Why study algorithms?

For fun and profit.

Why study algorithms?

- Their impact is broad and far-reaching.
- Old roots, new opportunities.
- To solve problems that could not otherwise be addressed.
- For intellectual stimulation.
- To become a proficient programmer.
- They may unlock the secrets of life and of the universe.
- For fun and profit.



The usual suspects

Lectures. Introduce new material.

Precepts. Discussion, problem-solving, background for programming assignment.

What	When	Where	Who	Office Hours
L01	TTh 11–12:20	CS 104	Kevin Wayne	see web
P01	F 11–11:50	Friend 112	Maia Ginsburg †	see web
P01A	F 11–11:50	Friend 108	Aman Dhesi	see web
P02	F 12:30–1:20	Friend 112	Joey Dodds	see web
P02A	F 12:30–1:20	Friend 108	Steven Liu	see web
P03	F 1:30–2:20	Friend 112	Maia Ginsburg †	see web
P03A	F 1:30–2:20	Friend 108	Sasha Koruga	see web

† lead preceptor

Where to get help?

Piazza. Online discussion forum.

- Short questions.
- Clarifications on lectures, readings, and assignments.

http://www.piazza.com/class#cos226fall2011

Email. For personal (or solution-revealing) questions.

Office hours. For longer questions.

Computing laboratory. Undergrad TAs in Friend 016. See web for schedule.

Coursework and grading

Programming assignments. 45%

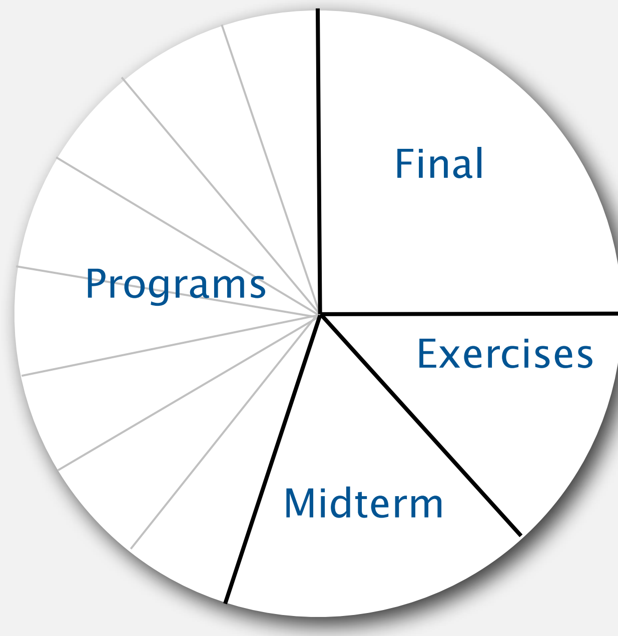
Due at 11pm via electronic submission.

Written exercises. 15%

Due at 11am in lecture.

Exams. 15% + 25%

- Closed-book with cheatsheet.
- Midterm (in class on Tuesday, October 25).
- Final (scheduled by Registrar).



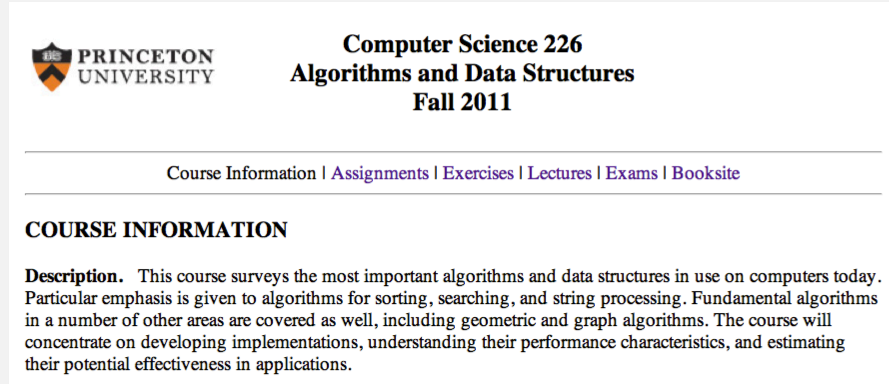
Staff discretion. To adjust borderline cases.

- Report errata.
- Contribute to Piazza discussions.
- Attend and participate in precept/lecture.

Resources (web)

Course content.

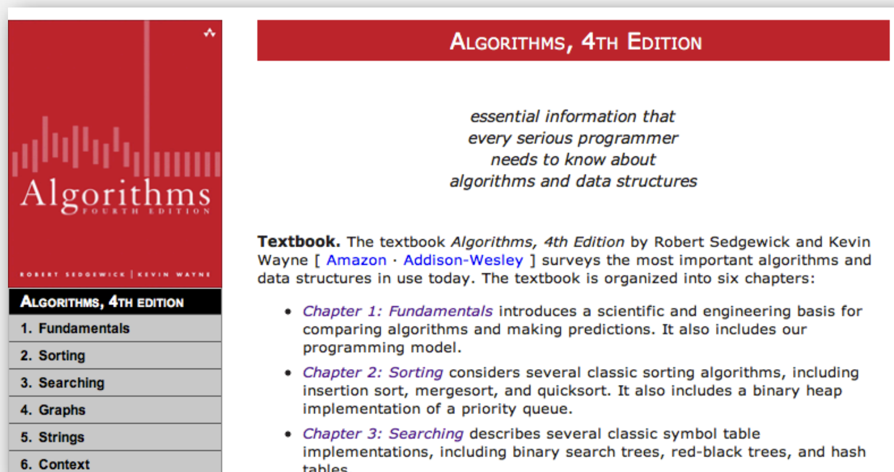
- Course info.
- Programming assignments.
- Exercises.
- Lecture slides.
- Exam archive.
- Submit assignments.



http://www.princeton.edu/~cos226

Booksites.

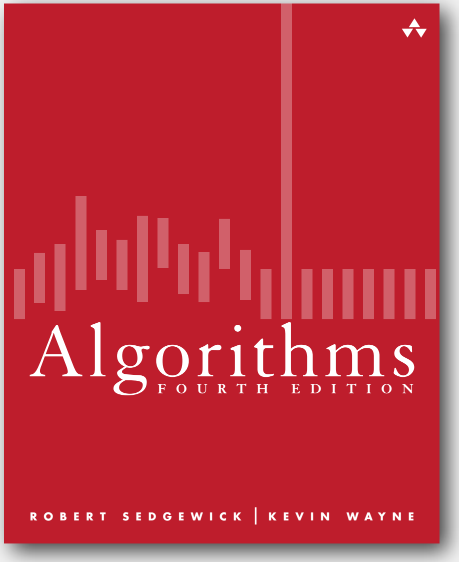
- Brief summary of content.
- Download code from lecture.



http://www.algs4.princeton.edu

Resources (textbook)

Required readings: Algorithms 4th edition by R. Sedgewick and K. Wayne, Addison-Wesley Professional, 2011, ISBN 0-321-57351-X.



Available in hardcover and Kindle.

- Online: Amazon ($60 to buy), Chegg ($40 to rent), ...
- Brick-and-mortar: Labyrinth Books (122 Nassau St).
- On reserve: Engineering library.

What's ahead?

Lecture 1. Union find. ← today

Precept 1. Meets tomorrow.

Lecture 2. Analysis of algorithms.

Exercise 1. Due via hardcopy in lecture at 11am on Tuesday.

Assignment 1. Due via electronic submission at 11pm on Wednesday.

Right course? See me.

Placed out of COS 126? Review Sections 1.1–1.2 of Algorithms, 4th edition (includes command-line interface and our I/O libraries).

Not registered? Go to any precept tomorrow.

Change precept? Use SCORE. ← see Colleen Kenny-McGinley in CS 210 if the only precept you can attend is closed

1.5 UNION FIND

- dynamic connectivity
- quick find
- quick union
- improvements
- applications

Subtext of today's lecture (and this course)

Steps to developing a usable algorithm.

- Model the problem.
- Find an algorithm to solve it.
- Fast enough? Fits in memory?
- If not, figure out why.
- Find a way to address the problem.
- Iterate until satisfied.

The scientific method.

Mathematical analysis.

- dynamic connectivity
- quick find
- quick union
- improvements
- applications

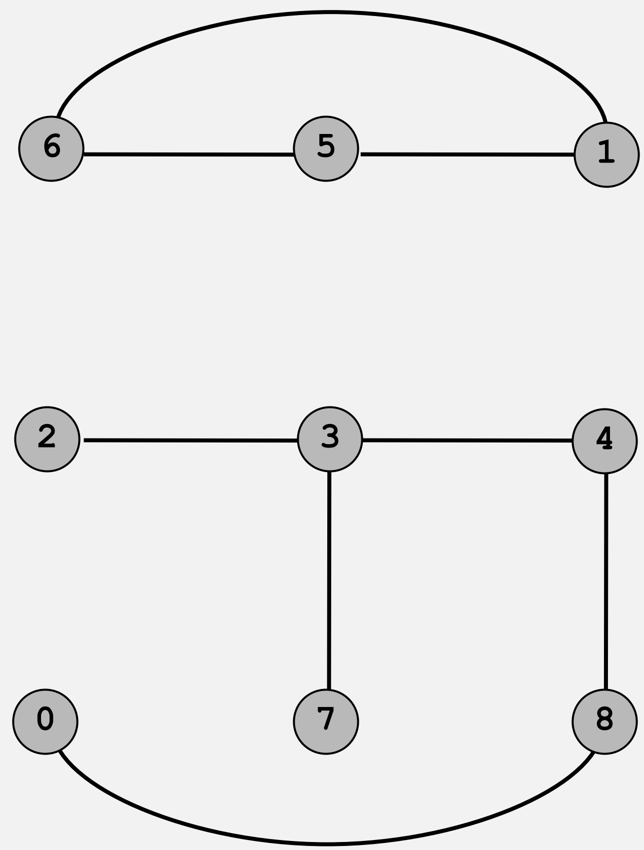
Dynamic connectivity

Given a set of objects

- Union: connect two objects.
- Connected: is there a path connecting the two objects?

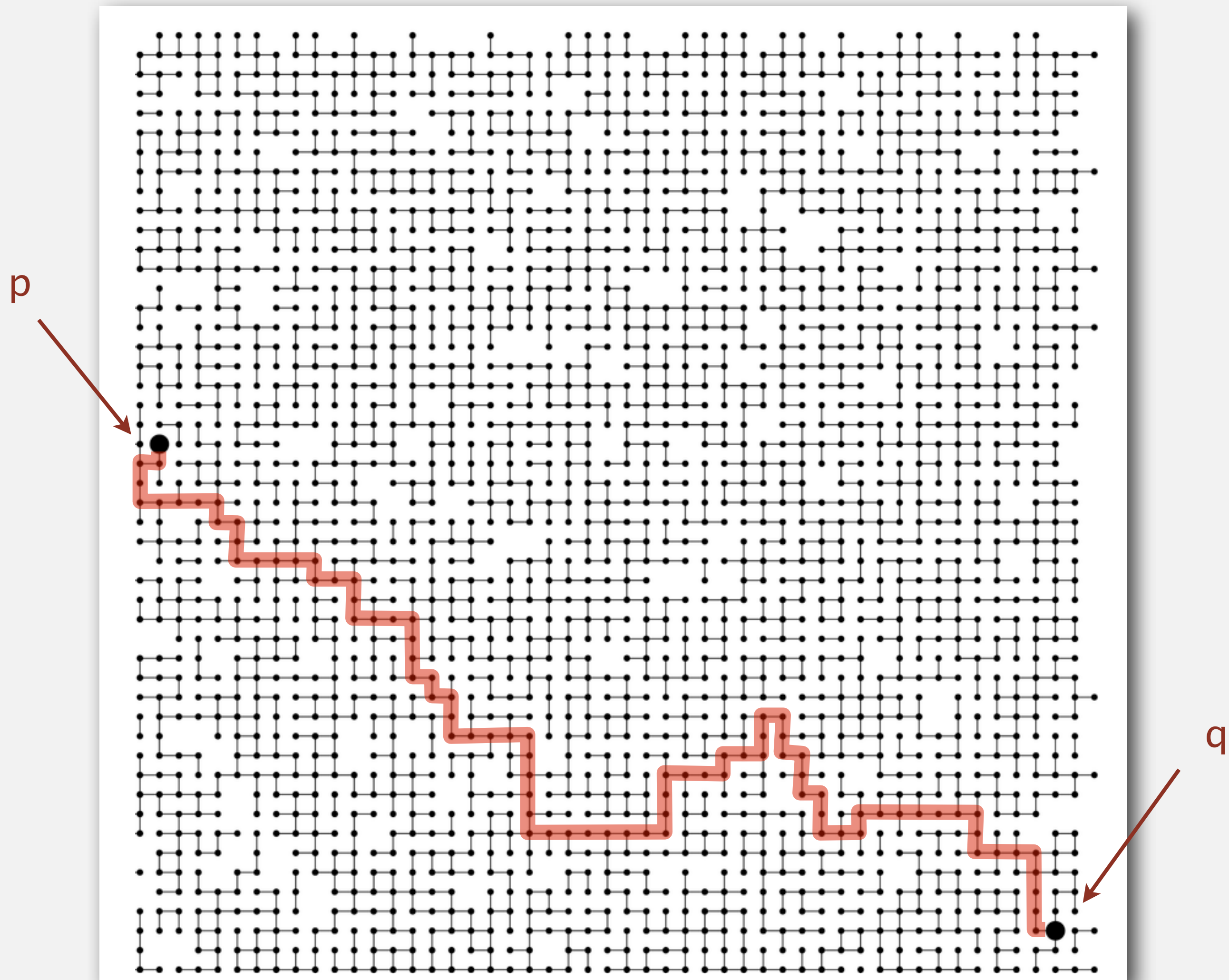
more difficult problem: find the path

```
union(3, 4)
union(8, 0)
union(2, 3)
union(5, 6)
connected(0, 2)    no
connected(2, 4)    yes
union(5, 1)
union(7, 3)
union(1, 6)
union(4, 8)
connected(0, 2)    yes
connected(2, 4)    yes
```



Connectivity example

Q. Is there a path from p to q?



A. Yes.

Modeling the objects

Dynamic connectivity applications involve manipulating objects of all types.

- Pixels in a digital photo.
- Computers in a network.
- Variable names in Fortran.
- Friends in a social network.
- Transistors in a computer chip.
- Elements in a mathematical set.
- Metallic sites in a composite system.

When programming, convenient to name sites 0 to N-1.

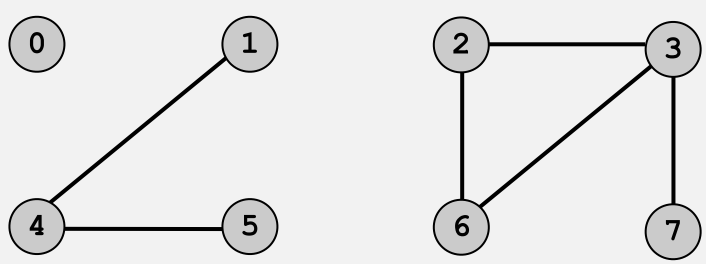
- Use integers as array index.
- Suppress details not relevant to union-find.

can use symbol table to translate from site names to integers: stay tuned (Chapter 3)

We assume "is connected to" is an equivalence relation:

- Reflexive: p is connected to p.
- Symmetric: if p is connected to q, then q is connected to p.
- Transitive: if p is connected to q and q is connected to r, then p is connected to r.

Connected components. Maximal set of objects that are mutually connected.



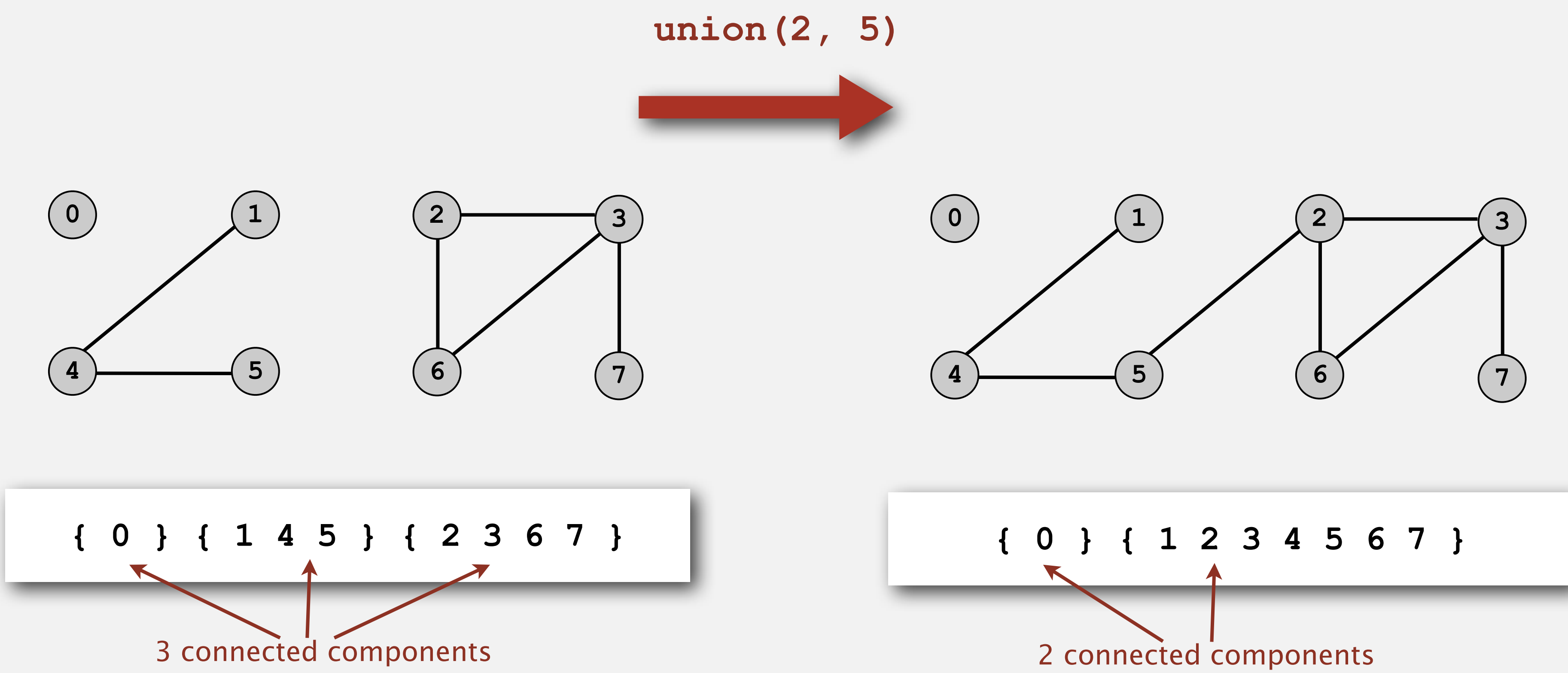
{ 0 } { 1 4 5 } { 2 3 6 7 }

3 connected components

Implementing the operations

Find query. Check if two objects are in the same component.

Union command. Replace components containing two objects with their union.



Union-find data type (API)

Goal. Design efficient data structure for union-find.

- Number of objects N can be huge.
- Number of operations M can be huge.
- Find queries and union commands may be intermixed.

public class UF		
	`UF(int N)`	*initialize union-find data structure with N objects (0 to N-1)*
`void`	`union(int p, int q)`	*add connection between p and q*
`boolean`	`connected(int p, int q)`	*are p and q in the same component?*
`int`	`find(int p)`	*component identifier for p (0 to N-1)*
`int`	`count()`	*number of components*

Dynamic-connectivity client

- Read in number of objects N from standard input.
- Repeat:
  - read in pair of integers from standard input
  - write out pair if they are not already connected

```
public static void main(String[] args)
{
   int N = StdIn.readInt();
   UF uf = new UF(N);
   while (!StdIn.isEmpty())
   {
      int p = StdIn.readInt();
      int q = StdIn.readInt();
      if (uf.connected(p, q)) continue;
      uf.union(p, q);
      StdOut.println(p + " " + q);
   }
}
```

```
% more tiny.txt
10
4 3
3 8
6 5
9 4
2 1
8 9
5 0
7 2
6 1
1 0
6 7
```

- dynamic connectivity
- **quick find**
- quick union
- improvements
- applications

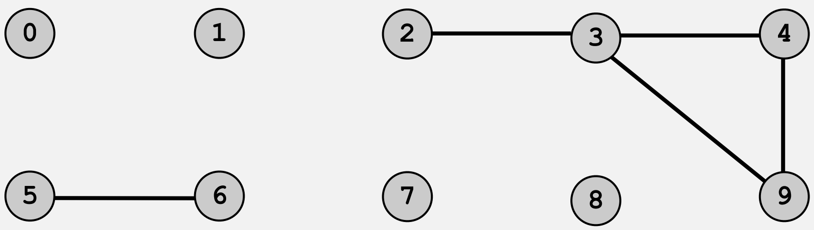
Quick-find [eager approach]

Data structure.

- Integer array `id[]` of size `N`.
- Interpretation: `p` and `q` are connected iff they have the same id.

i	0	1	2	3	4	5	6	7	8	9
id[i]	0	1	9	9	9	6	6	7	8	9

5 and 6 are connected
2, 3, 4, and 9 are connected



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Find. Check if `p` and `q` have the same id.

`id[3] = 9`; `id[6] = 6`
3 and 6 are not connected

Quick-find [eager approach]

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Find. Check if `p` and `q` have the same id.

`id[3] = 9; id[6] = 6`
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Union. To merge components containing `p` and `q`, change all entries whose `id[]` equals `id[p]` to `id[q]`.

i	0	1	2	3	4	5	6	7	8	9
id[i]	0	1	6	6	6	6	6	7	8	6

after union of 3 and 6

problem: many values can change

Quick-find example

						id[]					
p	q	0	1	2	3	4	5	6	7	8	9
4	3	0	1	2	3	4	5	6	7	8	9
		0	1	2	3	3	5	6	7	8	9
3	8	0	1	2	3	3	5	6	7	8	9
		0	1	2	8	8	5	6	7	8	9
6	5	0	1	2	8	8	5	6	7	8	9
		0	1	2	8	8	5	5	7	8	9
9	4	0	1	2	8	8	5	5	7	8	9
		0	1	2	8	8	5	5	7	8	8
2	1	0	1	2	8	8	5	5	7	8	8
		0	1	1	8	8	5	5	7	8	8
8	9	0	1	1	8	8	5	5	7	8	8
5	0	0	1	1	8	8	5	5	7	8	8
		0	1	1	8	8	0	0	7	8	8
7	2	0	1	1	8	8	0	0	7	8	8
		0	1	1	8	8	0	0	1	8	8
6	1	0	1	1	8	8	0	0	1	8	8
		1	1	1	8	8	1	1	1	8	8
1	0	1	1	1	8	8	1	1	1	8	8
6	7	1	1	1	8	8	1	1	1	8	8

`id[p]` *and* `id[q]` *differ, so* `union()` *changes entries equal to* `id[p]` *to* `id[q]` *(in red)*

`id[p]` *and* `id[q]` *match, so no change*

Quick-find: Java implementation

```
public class QuickFindUF
{
   private int[] id;

   public QuickFindUF(int N)
   {
      id = new int[N];
      for (int i = 0; i < N; i++)
         id[i] = i;
   }

   public boolean connected(int p, int q)
   {  return id[p] == id[q];  }

   public void union(int p, int q)
   {
      int pid = id[p];
      int qid = id[q];
      for (int i = 0; i < id.length; i++)
         if (id[i] == pid) id[i] = qid;
   }
}
```

set id of each object to itself
(N array accesses)

check whether `p` and `q`
are in the same component
(2 array accesses)

change all entries with `id[p]` to `id[q]`
(linear number of array accesses)

Quick-find is too slow

Cost model. Number of array accesses (for read or write).

algorithm	init	union	connected
quick-find	N	N	1

order of growth of number of array accesses

Quick-find defect.

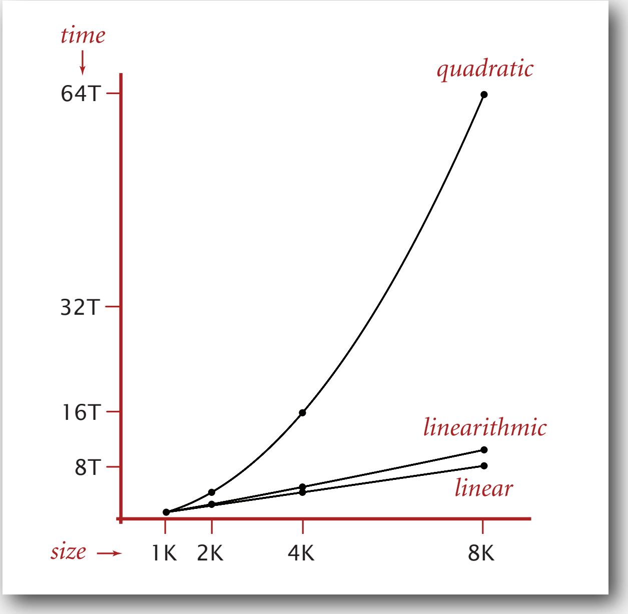
- Union too expensive.
- Trees are flat, but too expensive to keep them flat.
- Ex. Takes N^2 array accesses to process sequence of N union commands on N objects.

Quadratic algorithms do not scale

Rough standard (for now).

- 10^9 operations per second.
- 10^9 words of main memory.
- Touch all words in approximately 1 second.

a truism (roughly) since 1950!



Ex. Huge problem for quick-find.

- 10^9 union commands on 10^9 objects.
- Quick-find takes more than 10^{18} operations.
- 30+ years of computer time!

Paradoxically, quadratic algorithms get worse with newer equipment.

- New computer may be 10x as fast.
- But, has 10x as much memory so problem may be 10x bigger.
- With quadratic algorithm, takes 10x as long!

- dynamic connectivity
- quick find
- **quick union**
- improvements
- applications

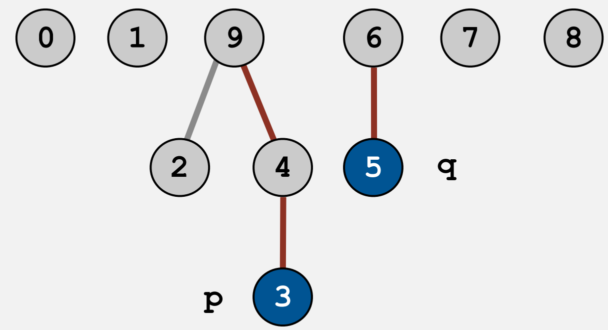
Quick-union [lazy approach]

Data structure.

- Integer array `id[]` of size `N`.
- Interpretation: `id[i]` is parent of `i`.
- Root of `i` is `id[id[id[...id[i]...]]]`.

keep going until it doesn't change

i	0	1	2	3	4	5	6	7	8	9
id[i]	0	1	9	4	9	6	6	7	8	9



3's root is 9; 5's root is 6

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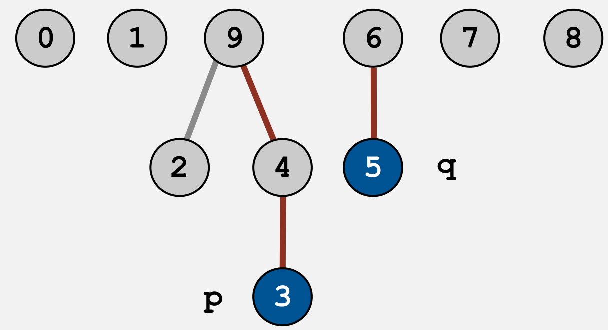

3's root is 9; 5's root is 6
3 and 5 are not connected

Find. Check if `p` and `q` have the same root.

Quick-union [lazy approach]

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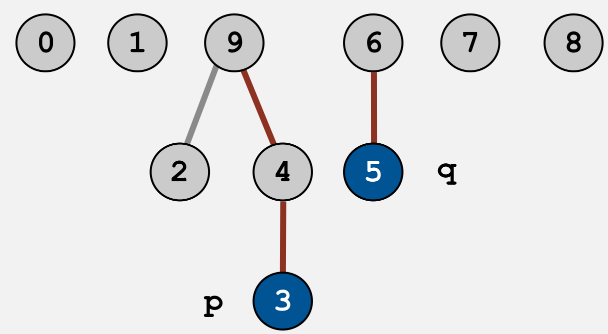


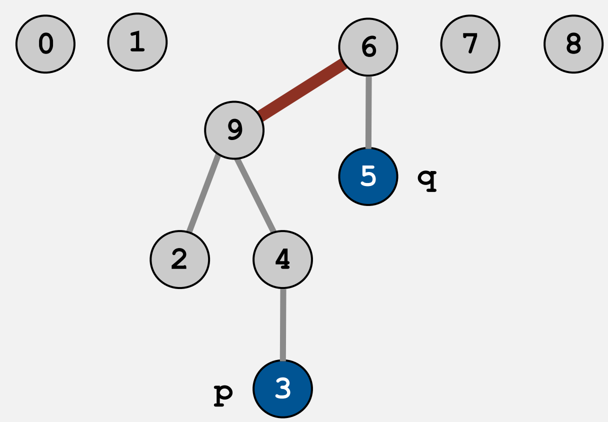
3's root is 9; 5's root is 6
3 and 5 are not connected

Find. Check if `p` and `q` have the same root.

Union. To merge components containing `p` and `q`, set the id of `p`'s root to the id of `q`'s root.

i	0	1	2	3	4	5	6	7	8	9
id[i]	0	1	9	4	9	6	6	7	8	6

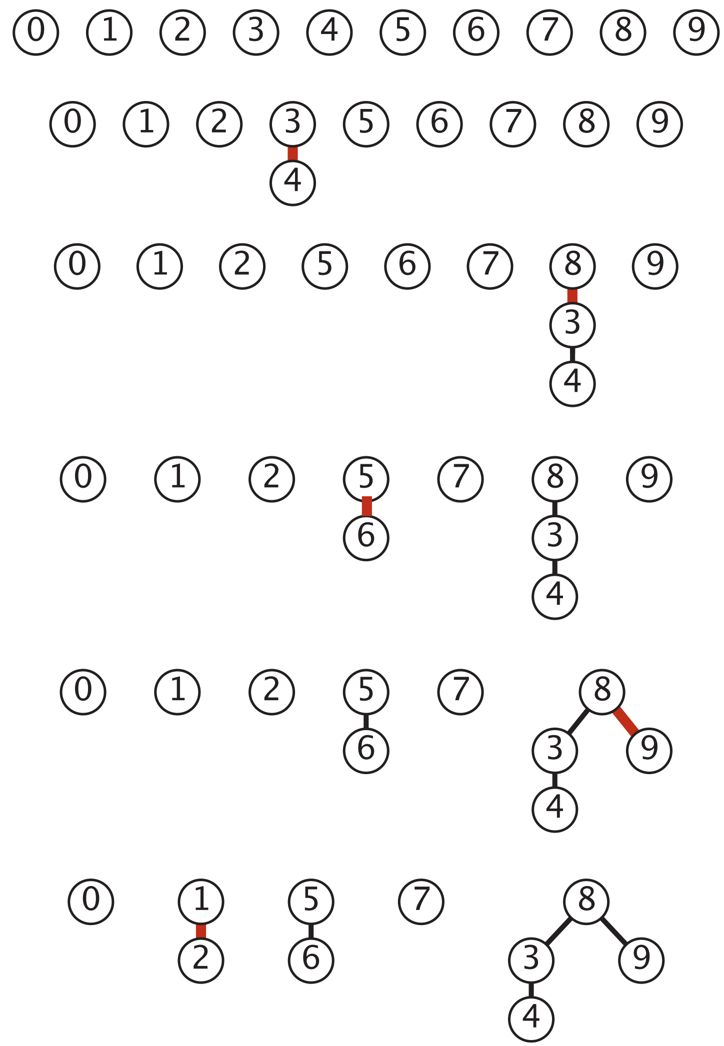
only one value changes



Quick-union demo

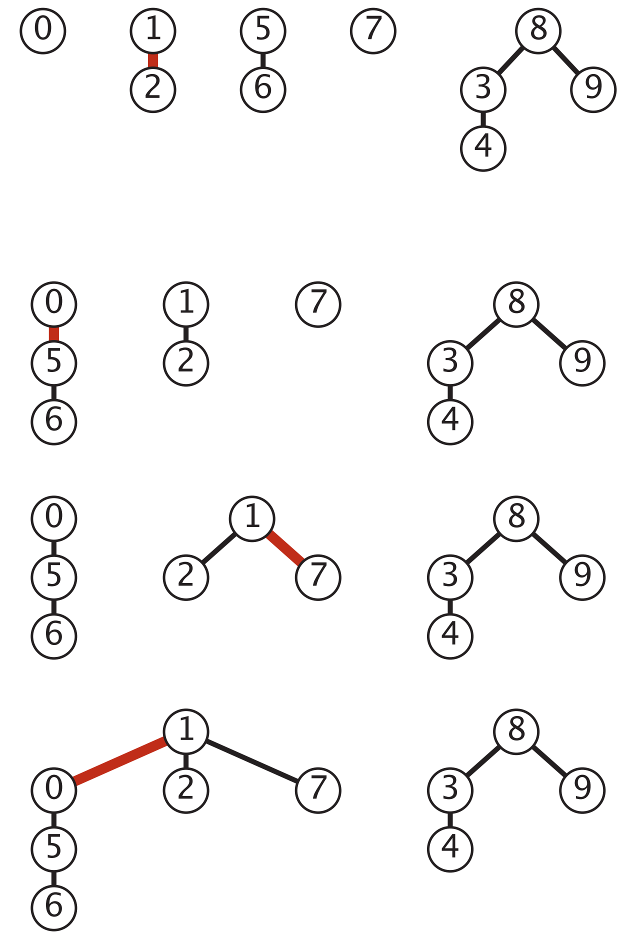
Quick-union example

p	q	0	1	2	3	4	5	6	7	8	9
4	3	0	1	2	3	4	5	6	7	8	9
		0	1	2	3	3	5	6	7	8	9
3	8	0	1	2	3	3	5	6	7	8	9
		0	1	2	8	3	5	6	7	8	9
6	5	0	1	2	8	3	5	6	7	8	9
		0	1	2	8	3	5	5	7	8	9
9	4	0	1	2	8	3	5	5	7	8	9
		0	1	2	8	3	5	5	7	8	8
2	1	0	1	2	8	3	5	5	7	8	8
		0	1	1	8	3	5	5	7	8	8



Quick-union example

p	q	0	1	2	3	4	5	6	7	8	9
8	9	0	1	1	8	3	5	5	7	8	8
5	0	0	1	1	8	3	5	5	7	8	8
		0	1	1	8	3	0	5	7	8	8
7	2	0	1	1	8	3	0	5	7	8	8
		0	1	1	8	3	0	5	1	8	8
6	1	0	1	1	8	3	0	5	1	8	8
		1	1	1	8	3	0	5	1	8	8
1	0	1	1	1	8	3	0	5	1	8	8
6	7	1	1	1	8	3	0	5	1	8	8



Quick-union: Java implementation

```
public class QuickUnionUF
{
   private int[] id;

   public QuickUnionUF(int N)
   {
      id = new int[N];
      for (int i = 0; i < N; i++) id[i] = i;
   }

   private int root(int i)
   {
      while (i != id[i]) i = id[i];
      return i;
   }

   public boolean connected(int p, int q)
   {
      return root(p) == root(q);
   }

   public void union(int p, int q)
   {
      int i = root(p);
      int j = root(q);
      id[i] = j;
   }
}
```

set id of each object to itself
(N array accesses)

chase parent pointers until reach root
(depth of i array accesses)

check if p and q have same root
(depth of p and q array accesses)

change root of p to point to root of q
(depth of p and q array accesses)

Quick-union is also too slow

Cost model. Number of array accesses (for read or write).

algorithm	init	union	connected
quick-find	N	N	1
quick-union	N	N †	N

← worst case

† includes cost of finding roots

Quick-find defect.

- Union too expensive (N array accesses).
- Trees are flat, but too expensive to keep them flat.

Quick-union defect.

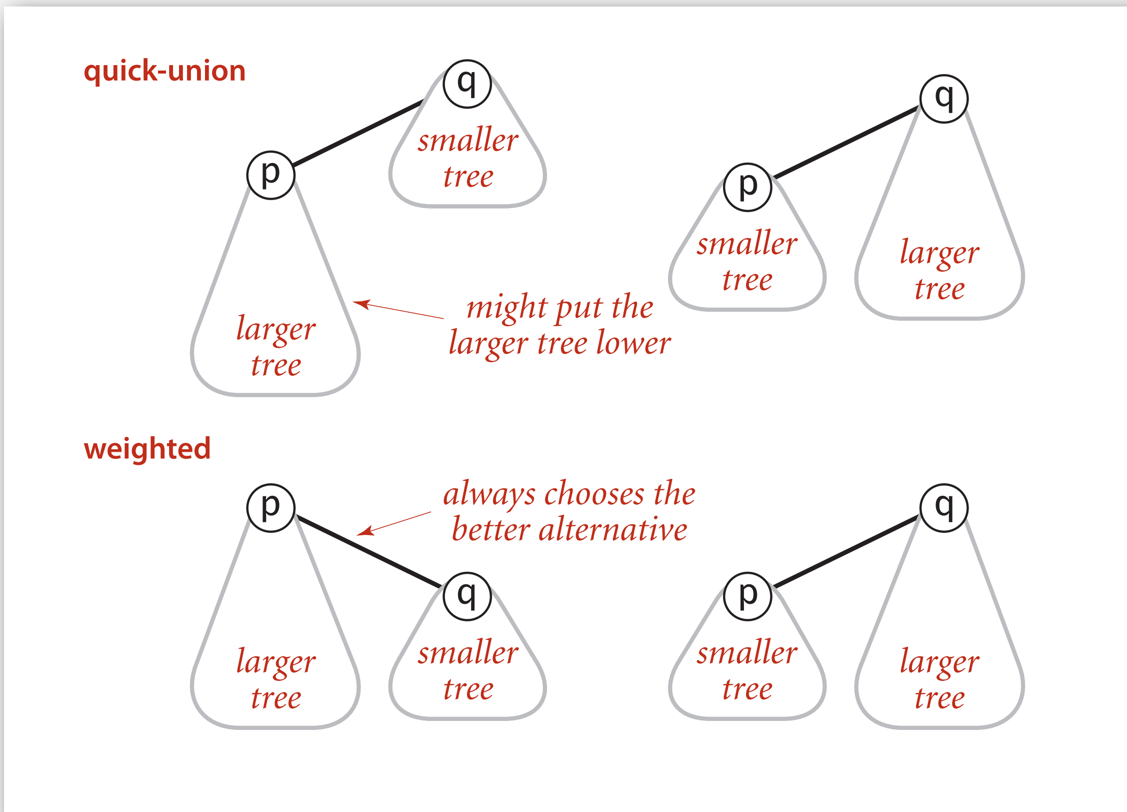
- Trees can get tall.
- Find too expensive (could be N array accesses).

- dynamic connectivity
- quick find
- quick union
- improvements
- applications

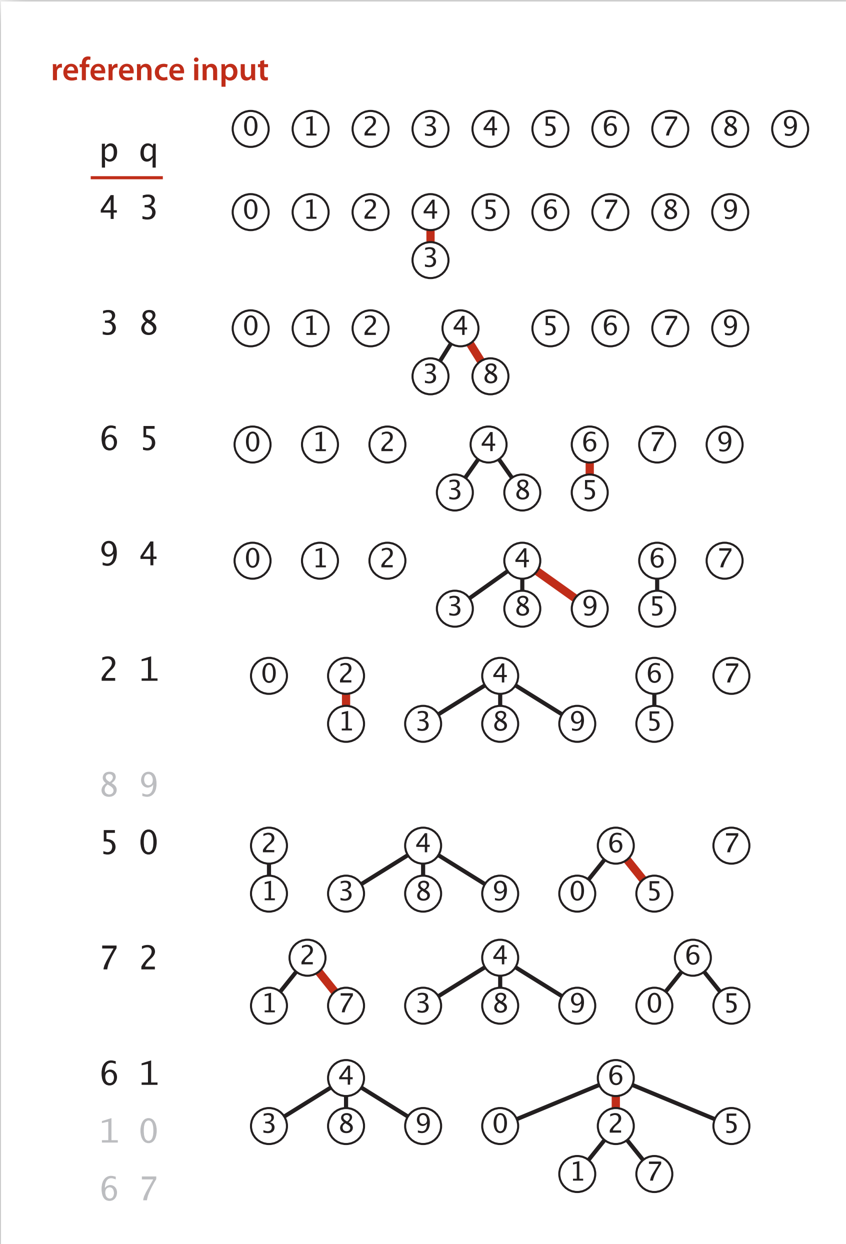
Improvement 1: weighting

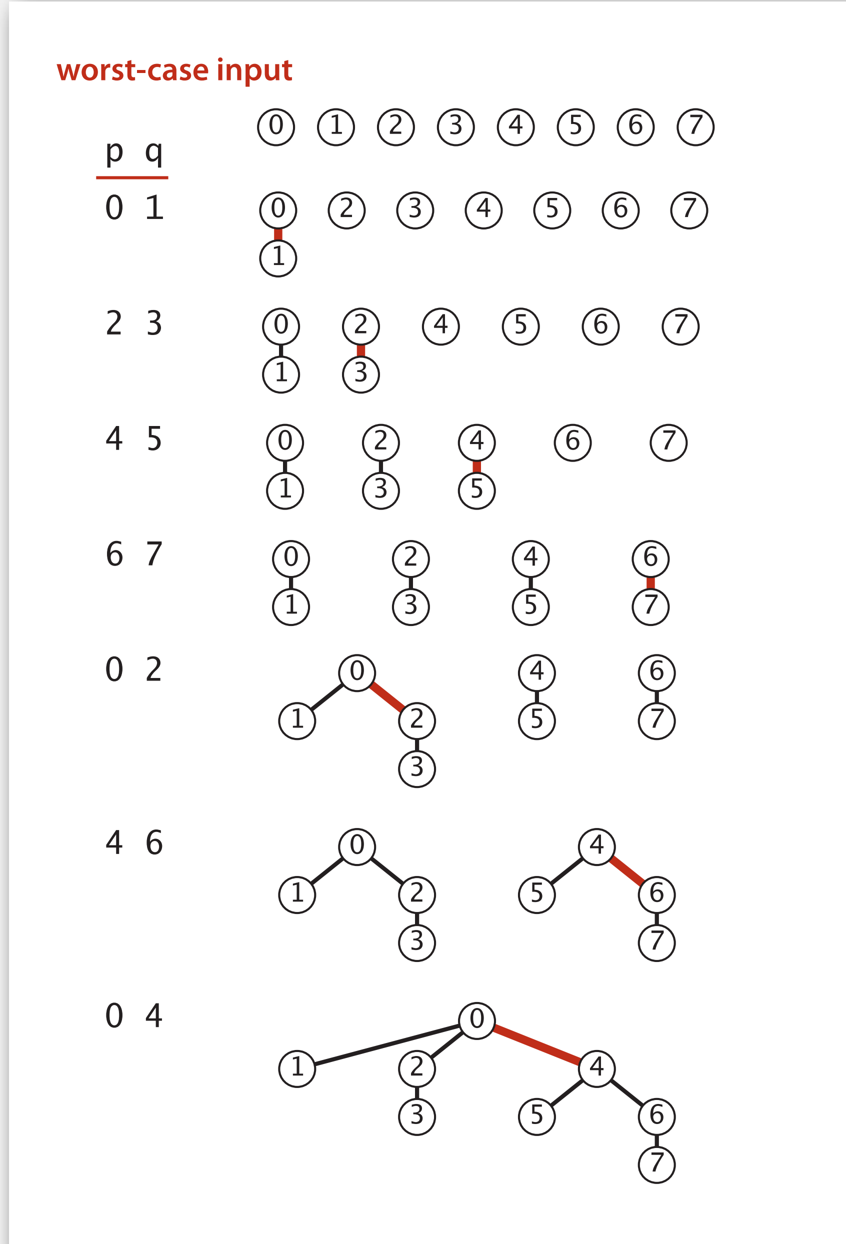
Weighted quick-union.

- Modify quick-union to avoid tall trees.
- Keep track of size of each tree (number of objects).
- Balance by linking small tree below large one.

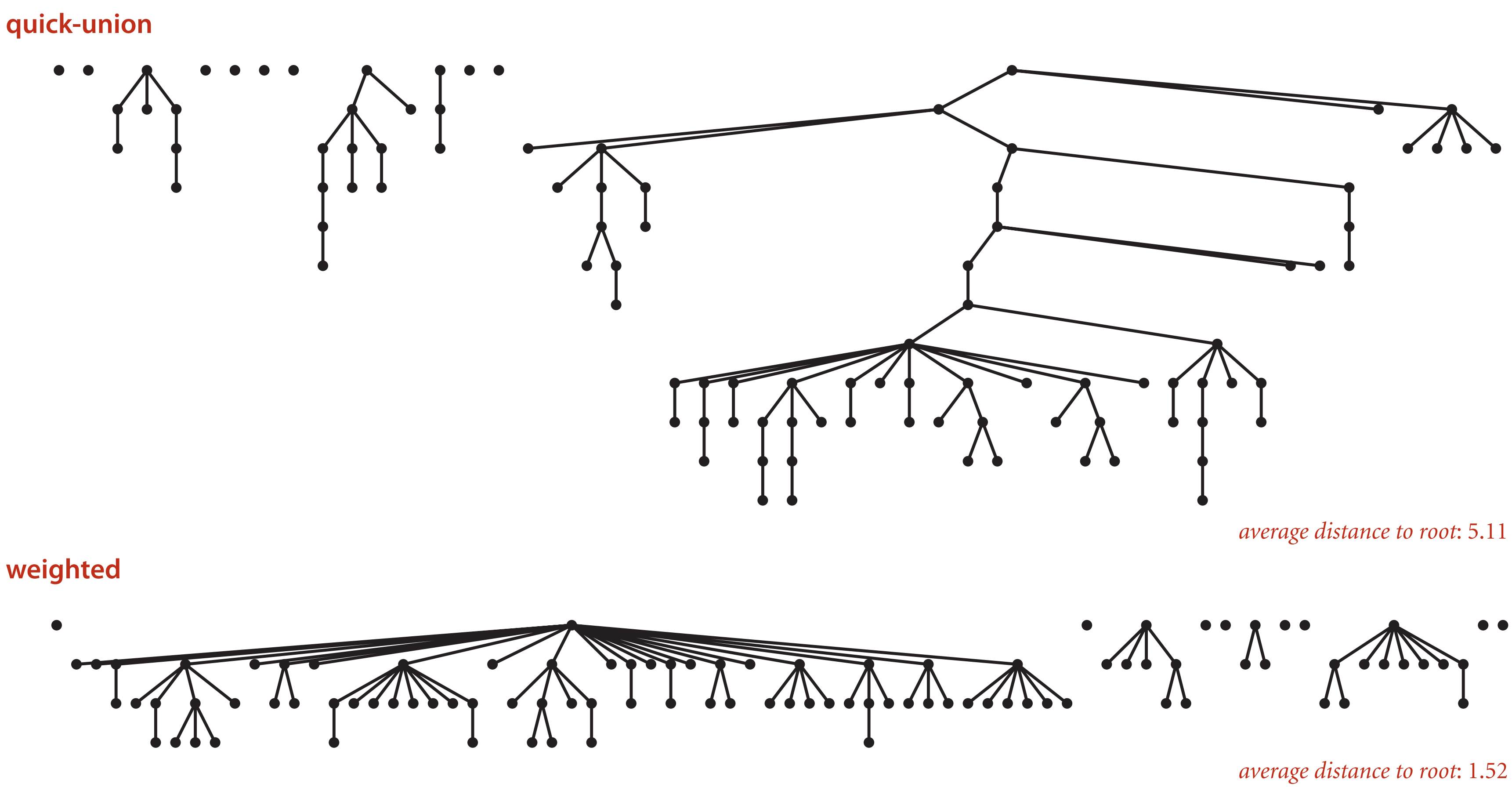


Weighted quick-union examples





Quick-union and weighted quick-union example



Quick-union and weighted quick-union (100 sites, 88 `union()` operations)

Weighted quick-union: Java implementation

Data structure. Same as quick-union, but maintain extra array `sz[i]` to count number of objects in the tree rooted at `i`.

Find. Identical to quick-union.

```
return root(p) == root(q);
```

Union. Modify quick-union to:

- Merge smaller tree into larger tree.
- Update the `sz[]` array.

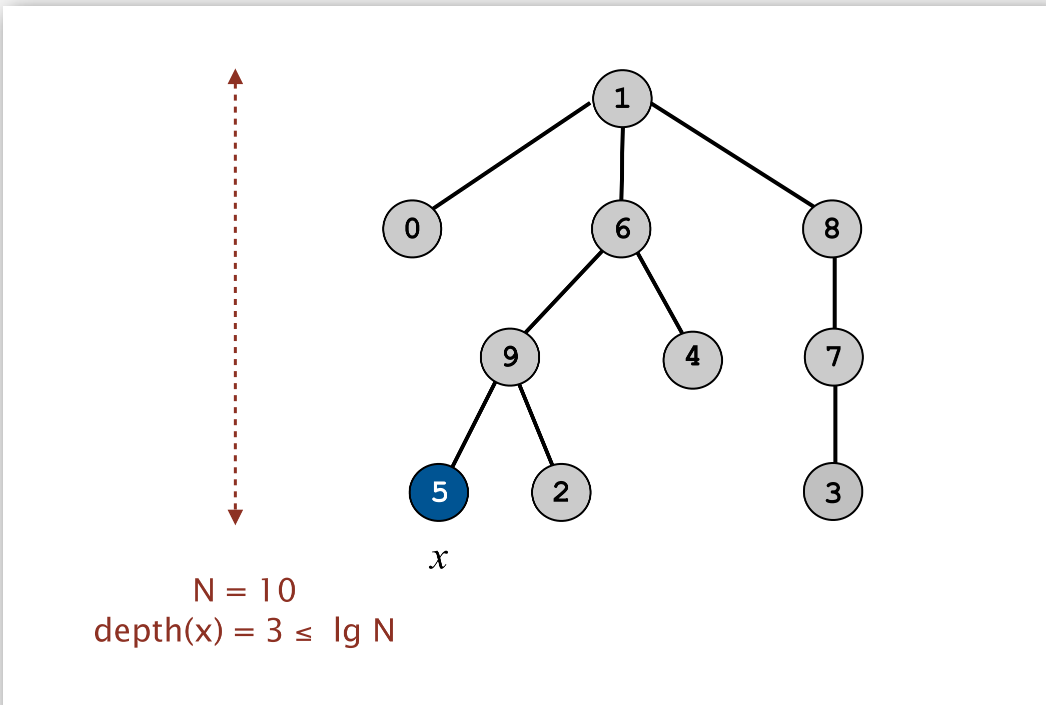
```
int i = root(p);
int j = root(q);
if  (sz[i] < sz[j]) { id[i] = j; sz[j] += sz[i]; }
else                { id[j] = i; sz[i] += sz[j]; }
```

Weighted quick-union analysis

Running time.

- Find: takes time proportional to depth of p and q.
- Union: takes constant time, given roots.

Proposition. Depth of any node x is at most $\lg N$.



Weighted quick-union analysis

Running time.

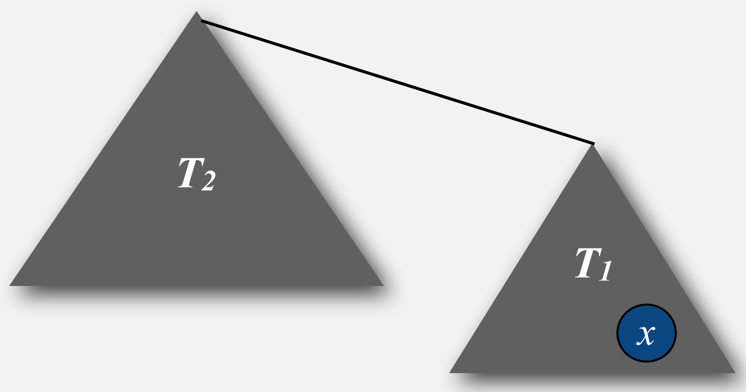
- Find: takes time proportional to depth of p and q.
- Union: takes constant time, given roots.

Proposition. Depth of any node x is at most $\lg N$.

Pf. When does depth of x increase?

Increases by 1 when tree T_1 containing x is merged into another tree T_2.

- The size of the tree containing x at least doubles since $|T_2| \geq |T_1|$.
- Size of tree containing x can double at most $\lg N$ times. Why?



Weighted quick-union analysis

Running time.

- Find: takes time proportional to depth of p and q.
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Proposition. Depth of any node x is at most $\lg N$.

algorithm	init	union	connected
quick-find	N	N	1
quick-union	N	N †	N
weighted QU	N	lg N †	lg N

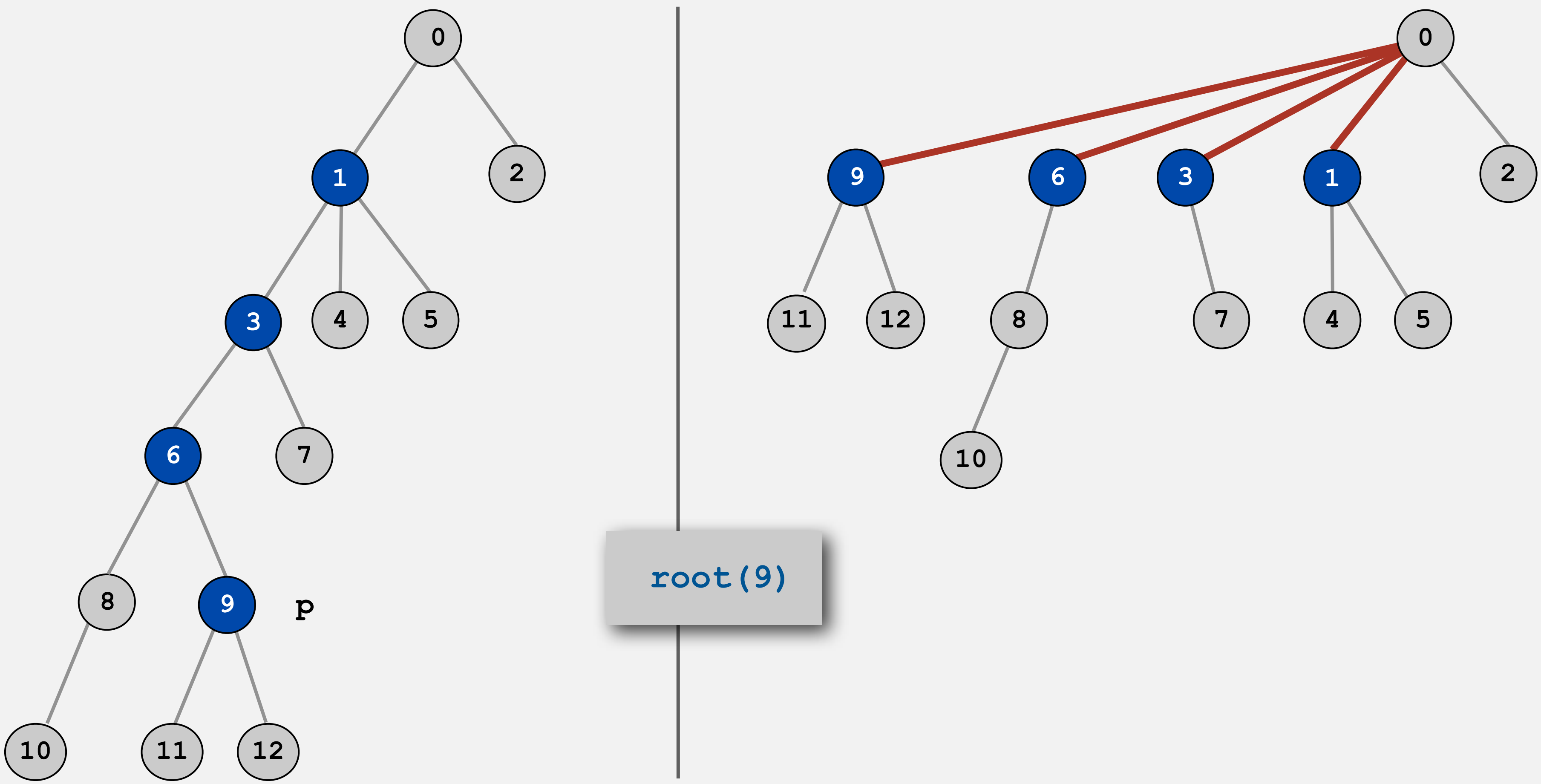
† includes cost of finding roots

Q. Stop at guaranteed acceptable performance?

A. No, easy to improve further.

Improvement 2: path compression

Quick union with path compression. Just after computing the root of p, set the id of each examined node to point to that root.



Path compression: Java implementation

Two-pass implementation: add second loop to `find()` to set the `id[]` of each examined node to the root.

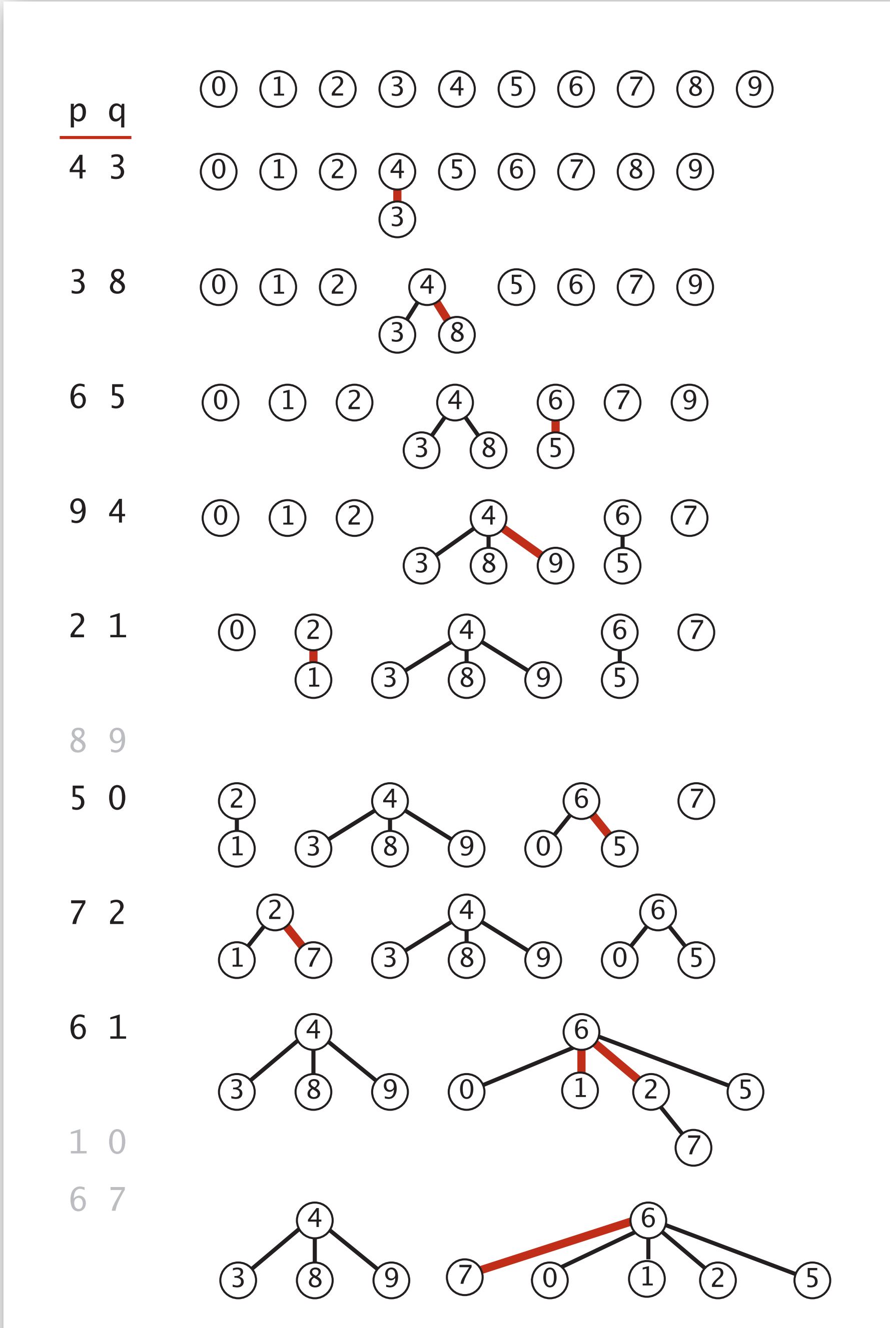
Simpler one-pass variant: Make every other node in path point to its grandparent (thereby halving path length).

```
private int root(int i)
{
   while (i != id[i])
   {
      id[i] = id[id[i]];
      i = id[i];
   }
   return i;
}
```

only one extra line of code !

In practice. No reason not to! Keeps tree almost completely flat.

Weighted quick-union with path compression example



1 linked to 6 because of path compression

7 linked to 6 because of path compression

Weighted quick-union with path compression: amortized analysis

Bob Tarjan
(Turing Award '86)

Proposition. Starting from an empty data structure, any sequence of M union–find operations on N objects makes at most proportional to $N + M \lg^* N$ array accesses.

- Proof is very difficult.
- But the algorithm is still simple!
- Analysis can be improved to $N + M\,\alpha(M, N)$.

see COS 423

N	lg* N
1	0
2	1
4	2
16	3
65536	4
2^{65536}	5

lg* function

Linear-time algorithm for M union-find ops on N objects?

- Cost within constant factor of reading in the data.
- In theory, WQUPC is not quite linear.
- In practice, WQUPC is linear.

because $\lg^* N$ is a constant in this universe

Amazing fact. No linear-time algorithm exists.

in "cell-probe" model of computation

Summary

Bottom line. WQUPC makes it possible to solve problems that could not otherwise be addressed.

algorithm	worst-case time
quick-find	M N
quick-union	M N
weighted QU	N + M log N
QU + path compression	N + M log N
weighted QU + path compression	N + M lg* N

M union-find operations on a set of N objects

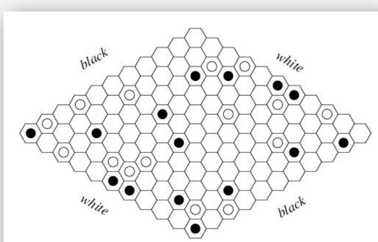
Ex. [10^9 unions and finds with 10^9 objects]

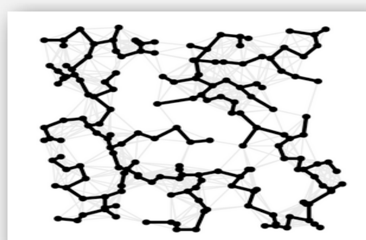
- WQUPC reduces time from 30 years to 6 seconds.
- Supercomputer won't help much; good algorithm enables solution.

- dynamic connectivity
- quick find
- quick union
- improvements
- **applications**

Union-find applications

- Percolation.
- Games (Go, Hex).
- ✓ Dynamic connectivity.
- Least common ancestor.
- Equivalence of finite state automata.
- Hoshen-Kopelman algorithm in physics.
- Hinley-Milner polymorphic type inference.
- Kruskal's minimum spanning tree algorithm.
- Compiling equivalence statements in Fortran.
- Morphological attribute openings and closings.
- Matlab's `bwlabel()` function in image processing.

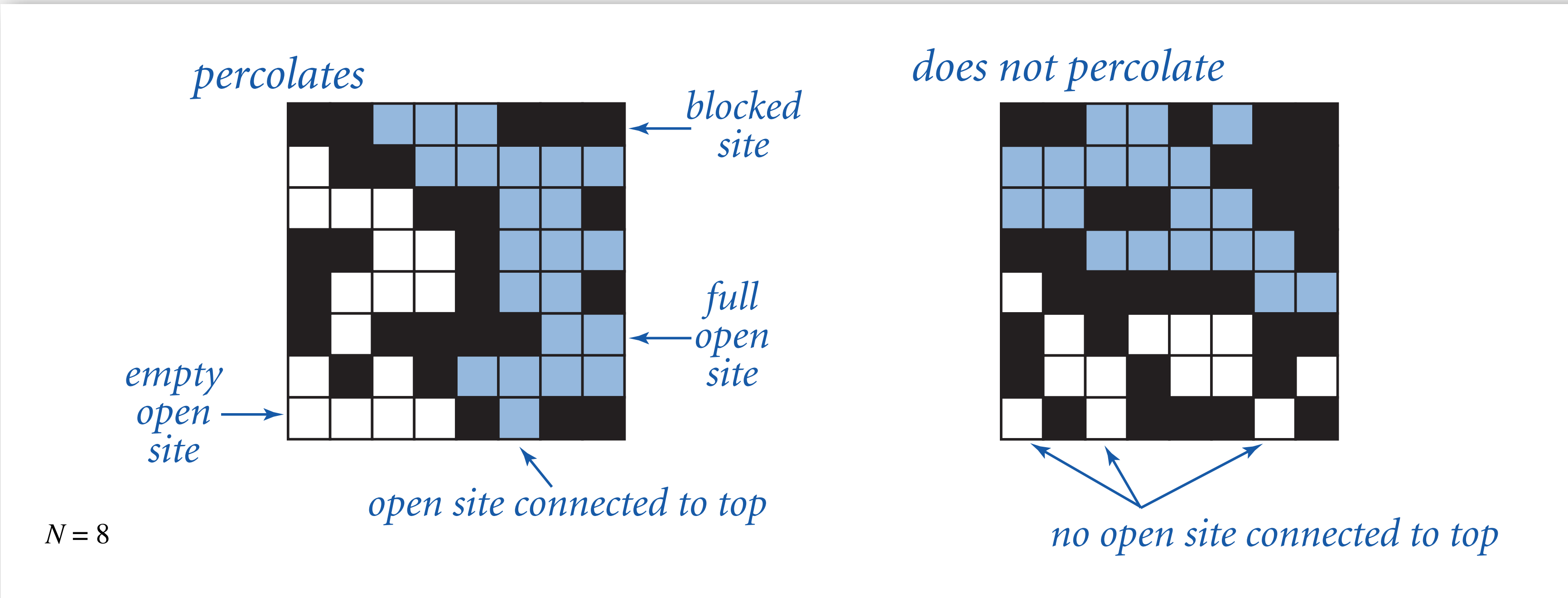




Percolation

A model for many physical systems:

- N-by-N grid of sites.
- Each site is open with probability p (or blocked with probability $1 - p$).
- System percolates iff top and bottom are connected by open sites.



Percolation

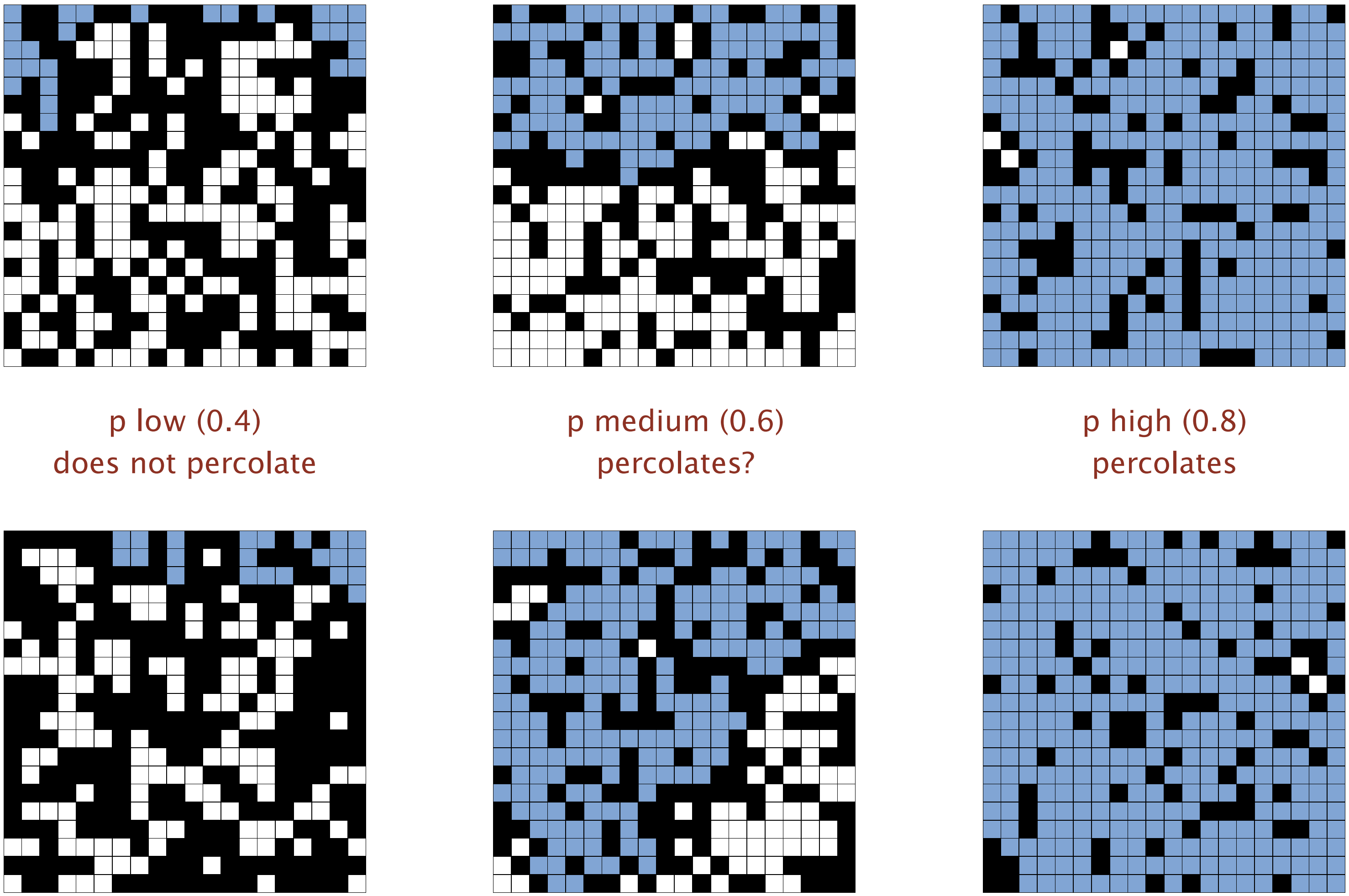
A model for many physical systems:

- N-by-N grid of sites.
- Each site is open with probability p (or blocked with probability $1 - p$).
- System percolates iff top and bottom are connected by open sites.

model	system	vacant site	occupied site	percolates
electricity	material	conductor	insulated	conducts
fluid flow	material	empty	blocked	porous
social interaction	population	person	empty	communicates

Likelihood of percolation

Depends on site vacancy probability p.



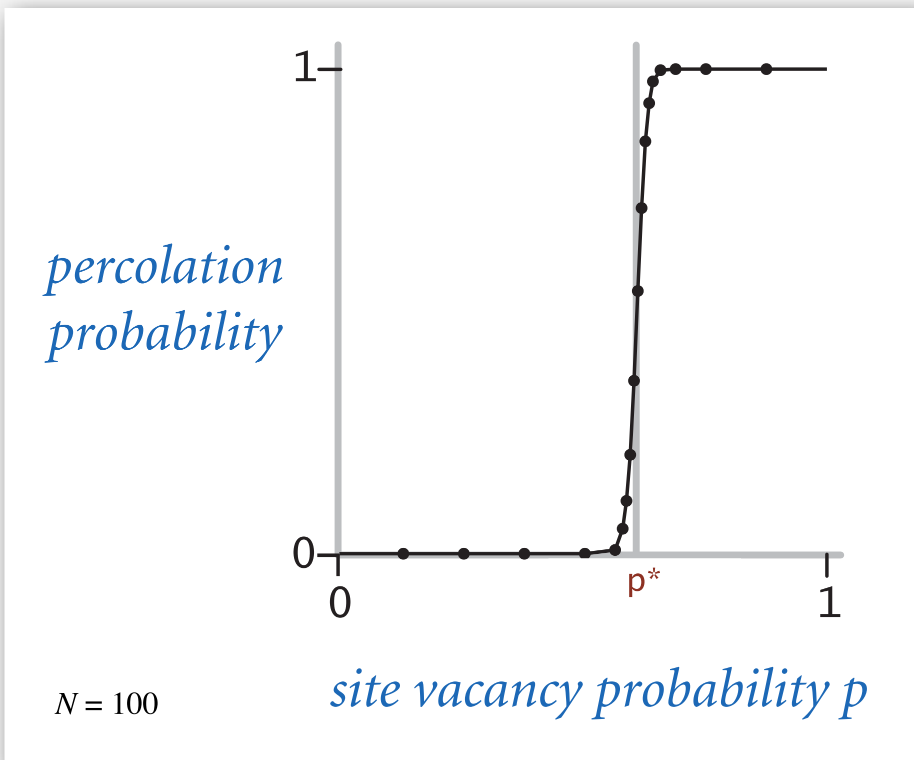
p low (0.4)	p medium (0.6)	p high (0.8)
does not percolate	percolates?	percolates

Percolation phase transition

When N is large, theory guarantees a sharp threshold p^*.

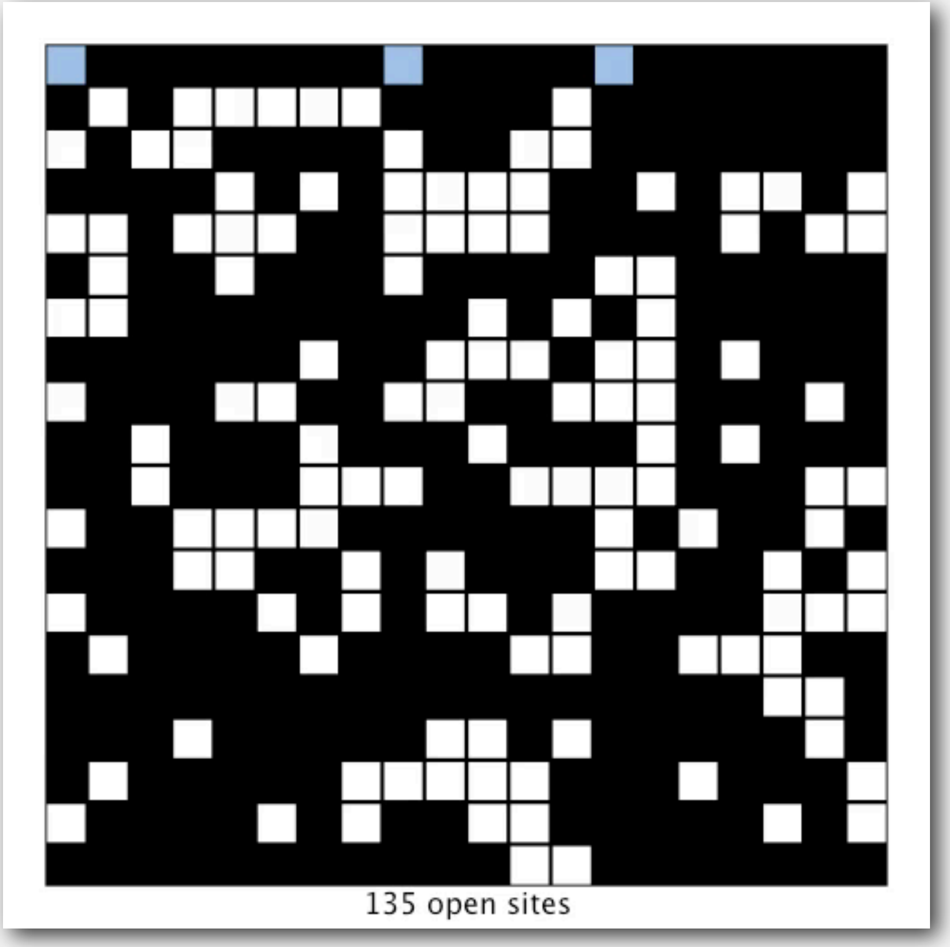
- $p > p^*$: almost certainly percolates.
- $p < p^*$: almost certainly does not percolate.

Q. What is the value of p^* ?



Monte Carlo simulation

- Initialize N-by-N whole grid to be blocked.
- Declare random sites open until top connected to bottom.
- Vacancy percentage estimates p^*.

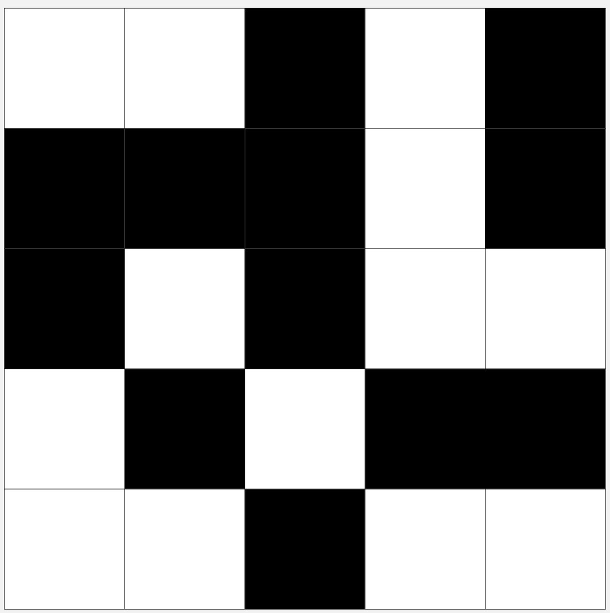


$N = 20$

Dynamic connectivity solution to estimate percolation threshold

Q. How to check whether an N-by-N system percolates?

$N = 5$



open site

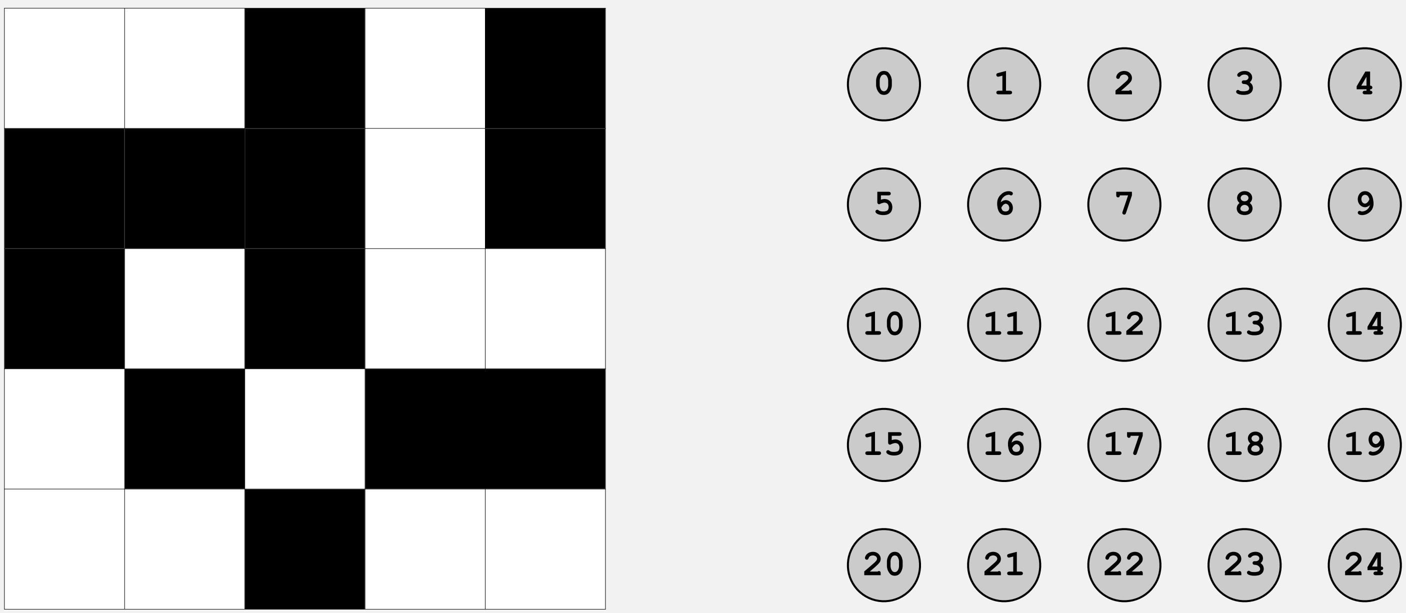
blocked site

Dynamic connectivity solution to estimate percolation threshold

Q. How to check whether an N-by-N system percolates?

- Create an object for each site and name them 0 to $N^2 - 1$.

$N = 5$



open site

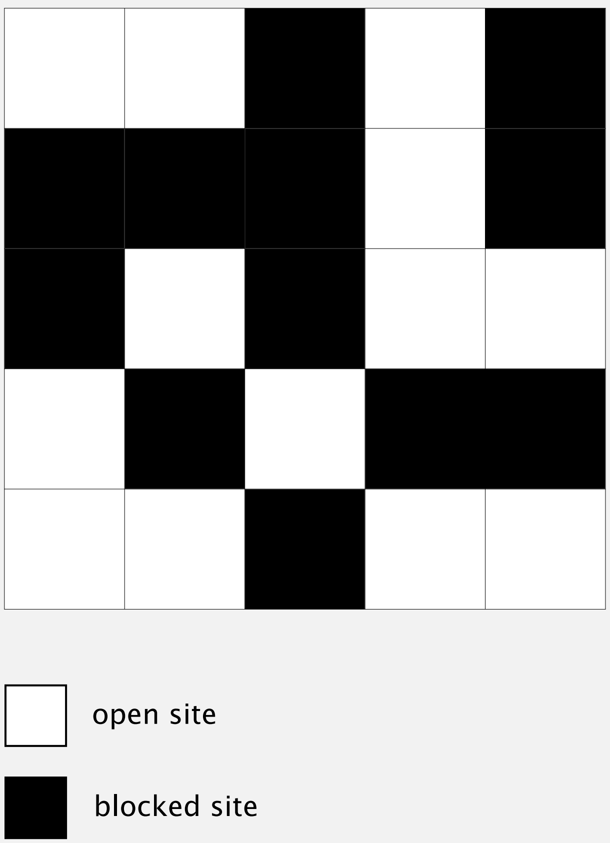
blocked site

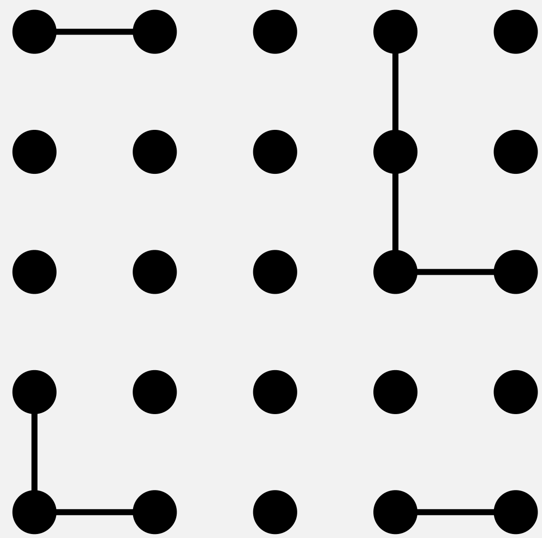
Dynamic connectivity solution to estimate percolation threshold

Q. How to check whether an N-by-N system percolates?

- Create an object for each site and name them 0 to $N^2 - 1$.
- Sites are in same component if connected by open sites.

$N = 5$





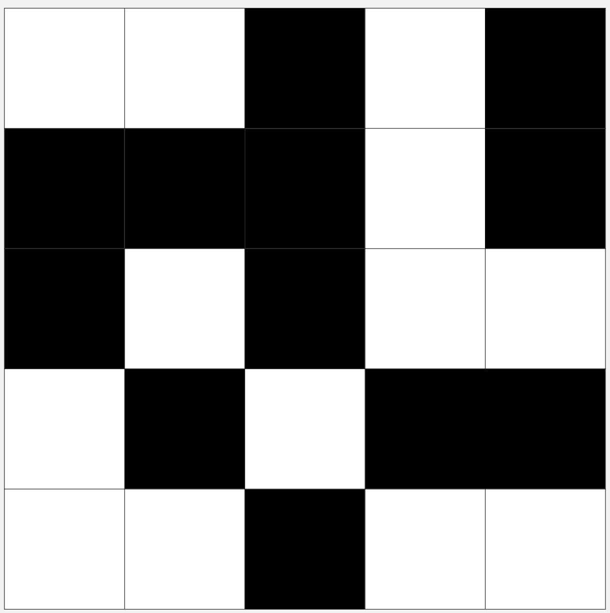
Dynamic connectivity solution to estimate percolation threshold

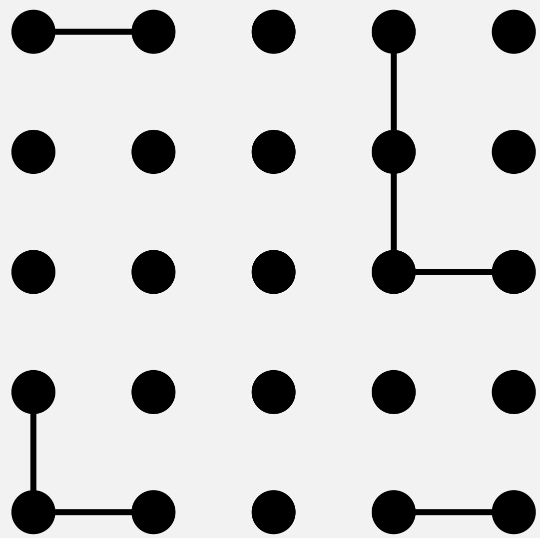
Q. How to check whether an N-by-N system percolates?

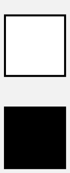
- Create an object for each site and name them 0 to $N^2 - 1$.
- Sites are in same component if connected by open sites.
- Percolates iff any site on bottom row is connected to site on top row.

brute-force algorithm: N^2 calls to connected()

$N = 5$







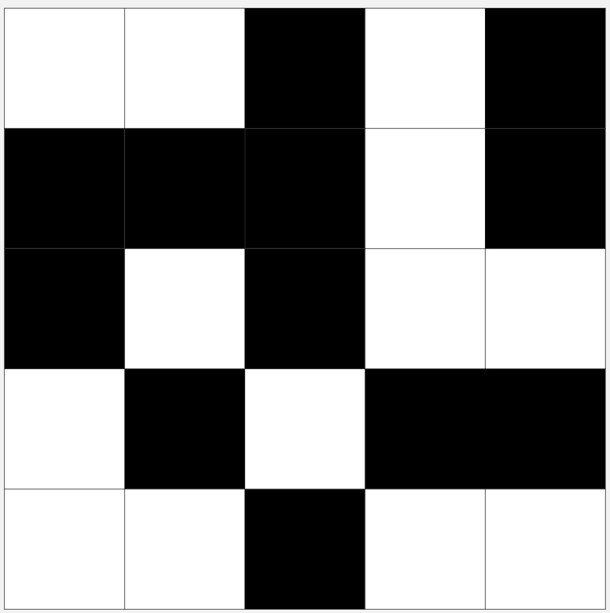
Dynamic connectivity solution to estimate percolation threshold

Clever trick. Introduce two virtual sites (and connections to top and bottom).

- Percolates iff virtual top site is connected to virtual bottom site.

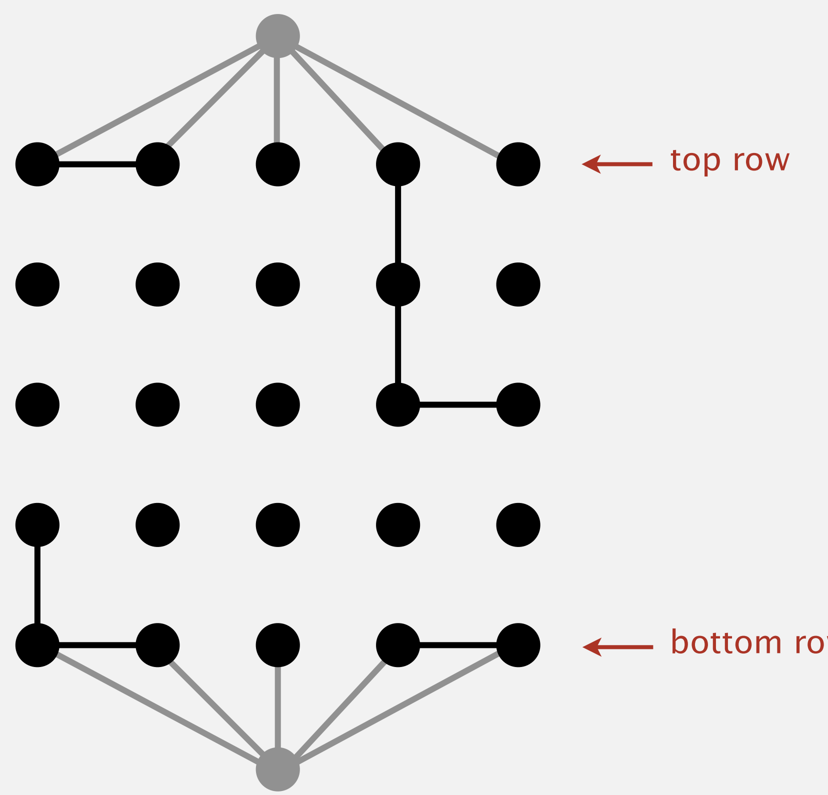
efficient algorithm: only 1 call to connected()

$N = 5$



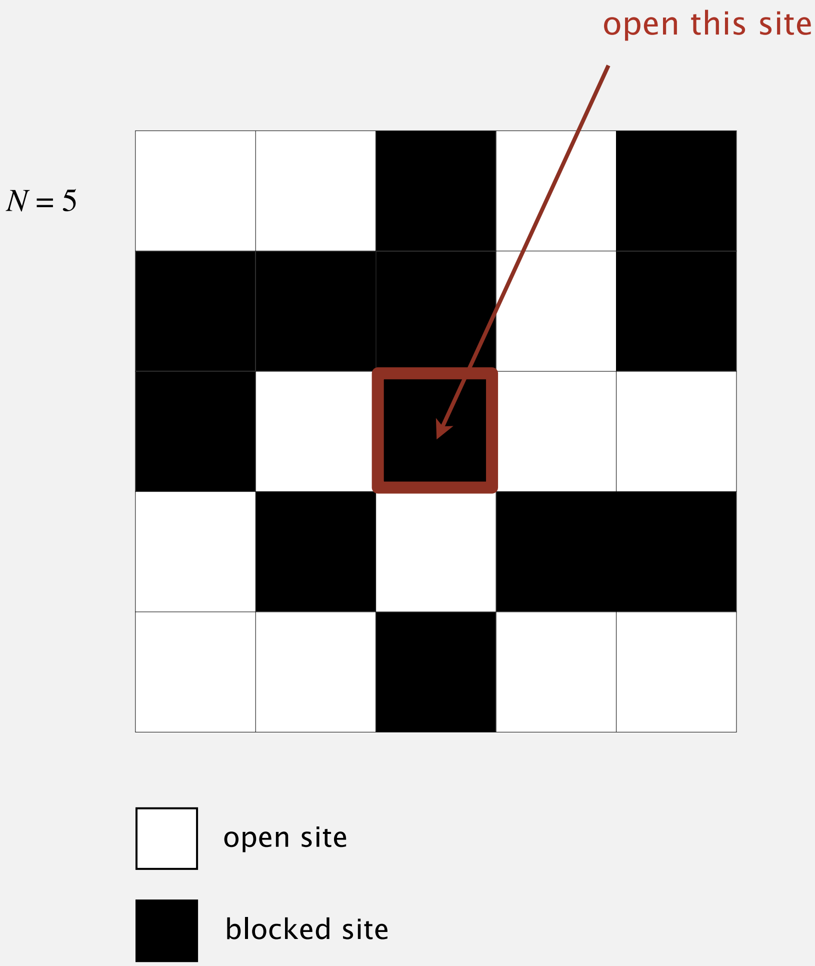
open site

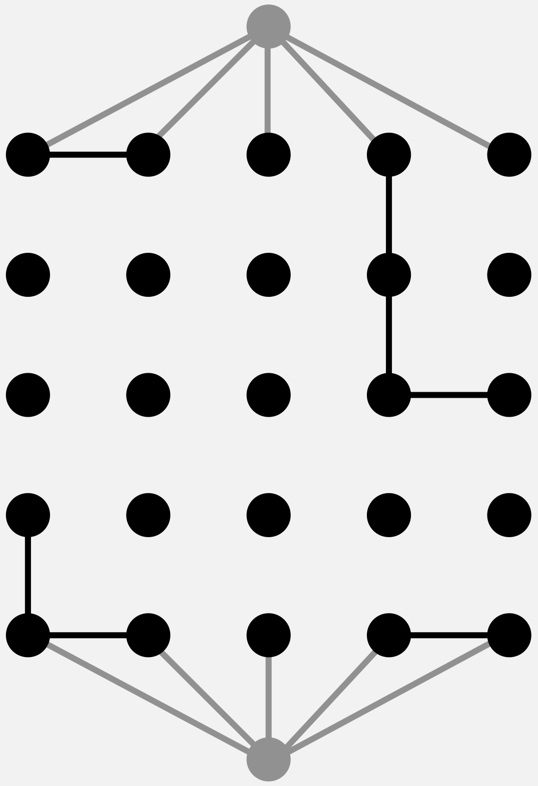
blocked site



Dynamic connectivity solution to estimate percolation threshold

Q. How to model as dynamic connectivity problem when opening a new site?



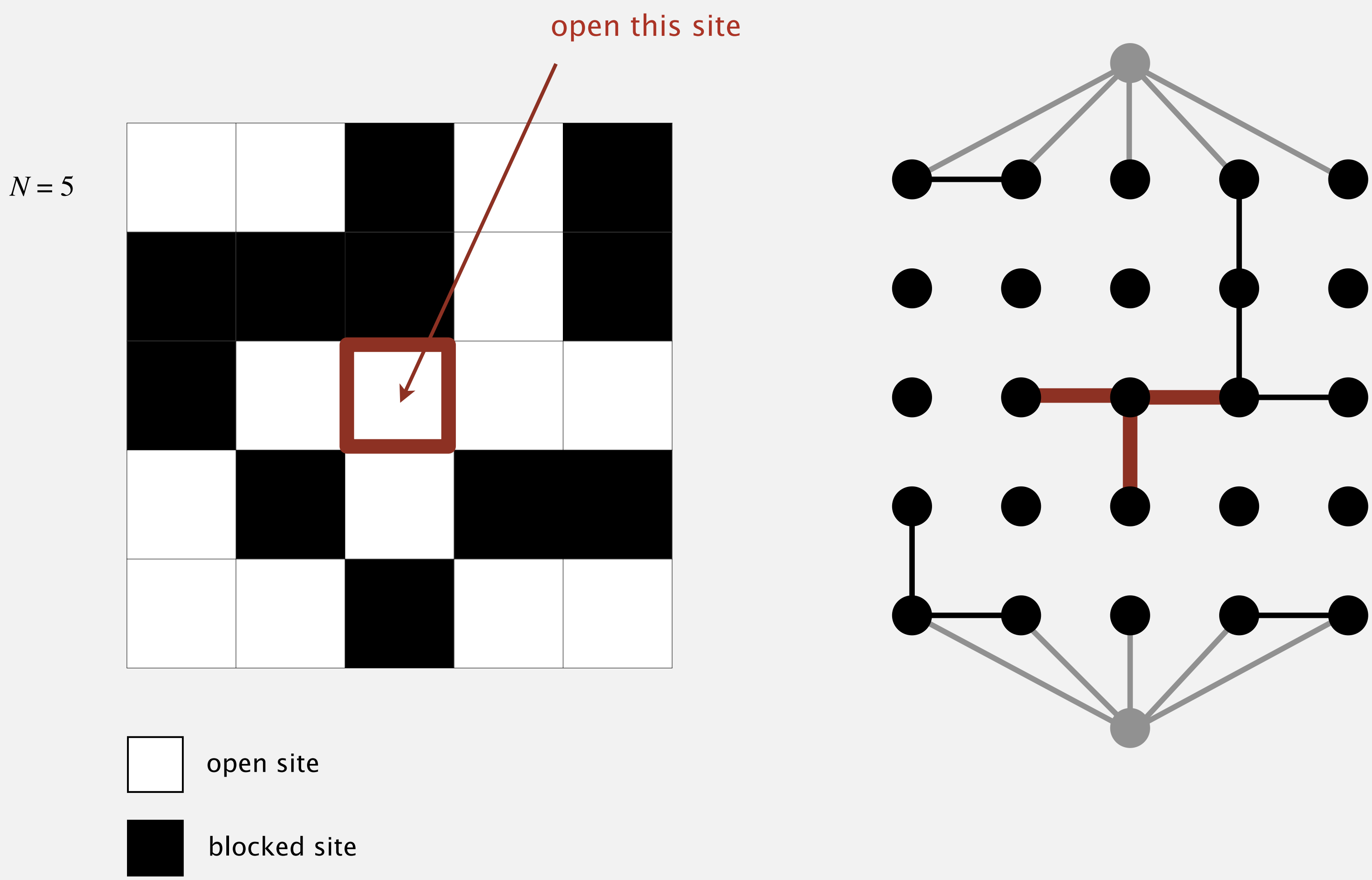


Dynamic connectivity solution to estimate percolation threshold

Q. How to model as dynamic connectivity problem when opening a new site?

A. Connect newly opened site to all of its adjacent open sites.

up to 4 calls to union()

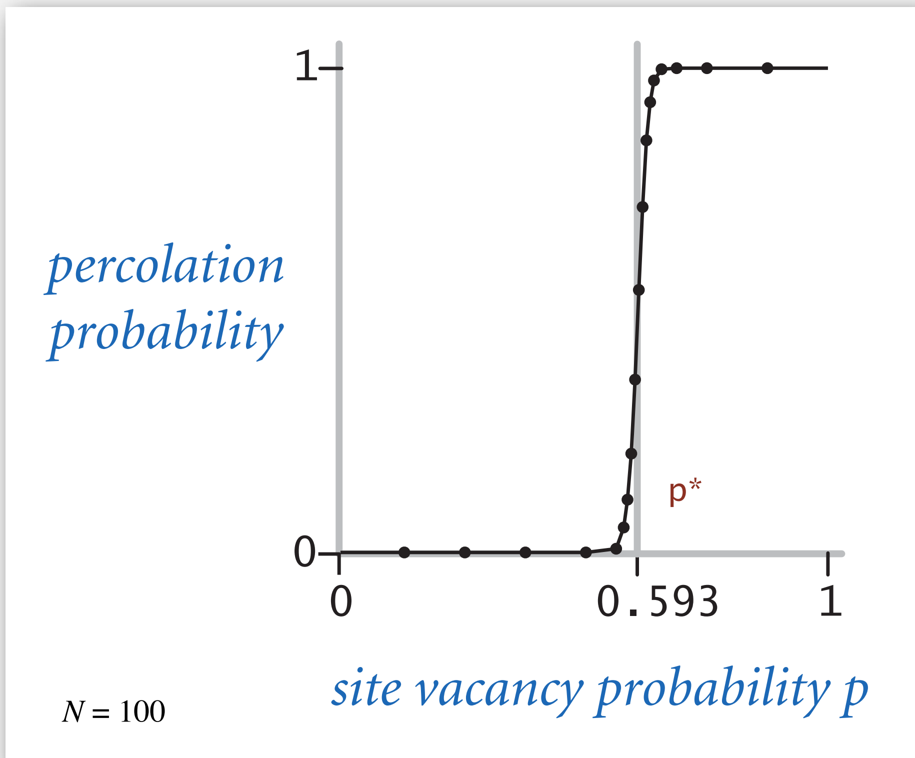


Percolation threshold

Q. What is percolation threshold p^* ?

A. About 0.592746 for large square lattices.

constant know only via simulation



Fast algorithm enables accurate answer to scientific question.

Subtext of today's lecture (and this course)

Steps to developing a usable algorithm.

- Model the problem.
- Find an algorithm to solve it.
- Fast enough? Fits in memory?
- If not, figure out why.
- Find a way to address the problem.
- Iterate until satisfied.

The scientific method.

Mathematical analysis.