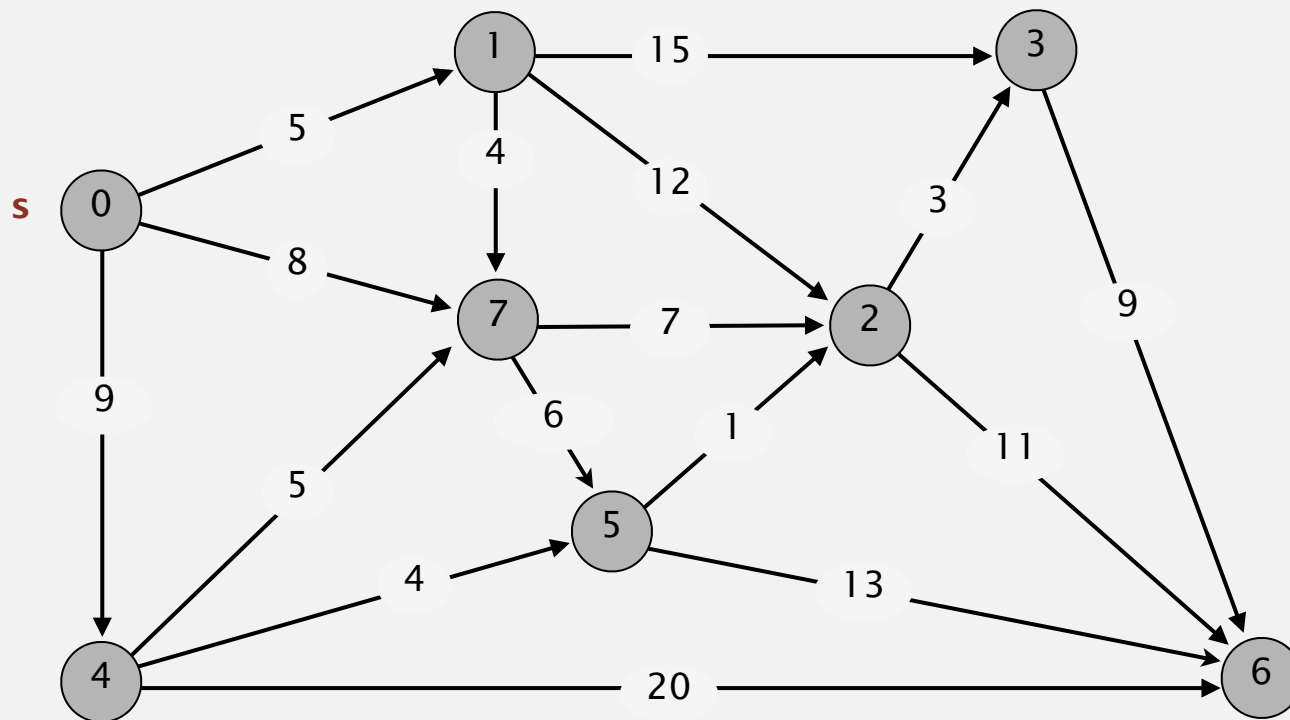


4.4 ACYCLIC SHORTEST PATHS DEMO



Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

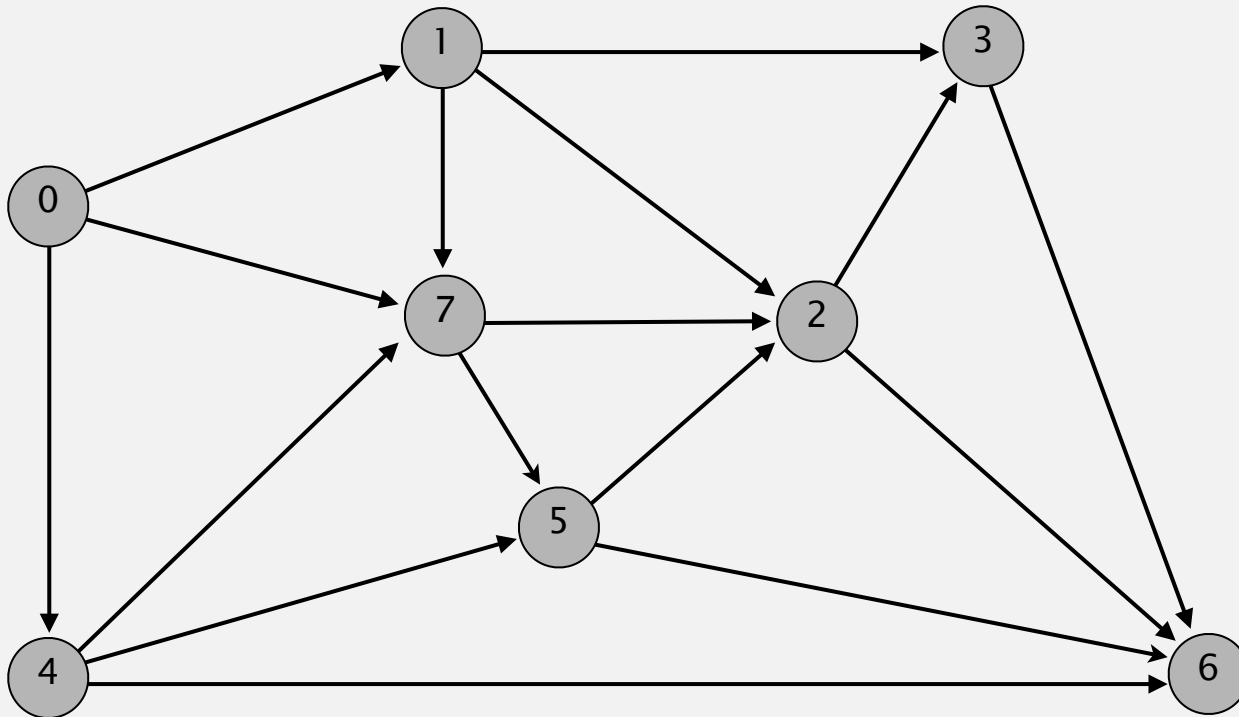


an edge-weighted DAG

| | |
|-----|------|
| 0→1 | 5.0 |
| 0→4 | 9.0 |
| 0→7 | 8.0 |
| 1→2 | 12.0 |
| 1→3 | 15.0 |
| 1→7 | 4.0 |
| 2→3 | 3.0 |
| 2→6 | 11.0 |
| 3→6 | 9.0 |
| 4→5 | 4.0 |
| 4→6 | 20.0 |
| 4→7 | 5.0 |
| 5→2 | 1.0 |
| 5→6 | 13.0 |
| 7→5 | 6.0 |
| 7→2 | 7.0 |

Topological sort algorithm

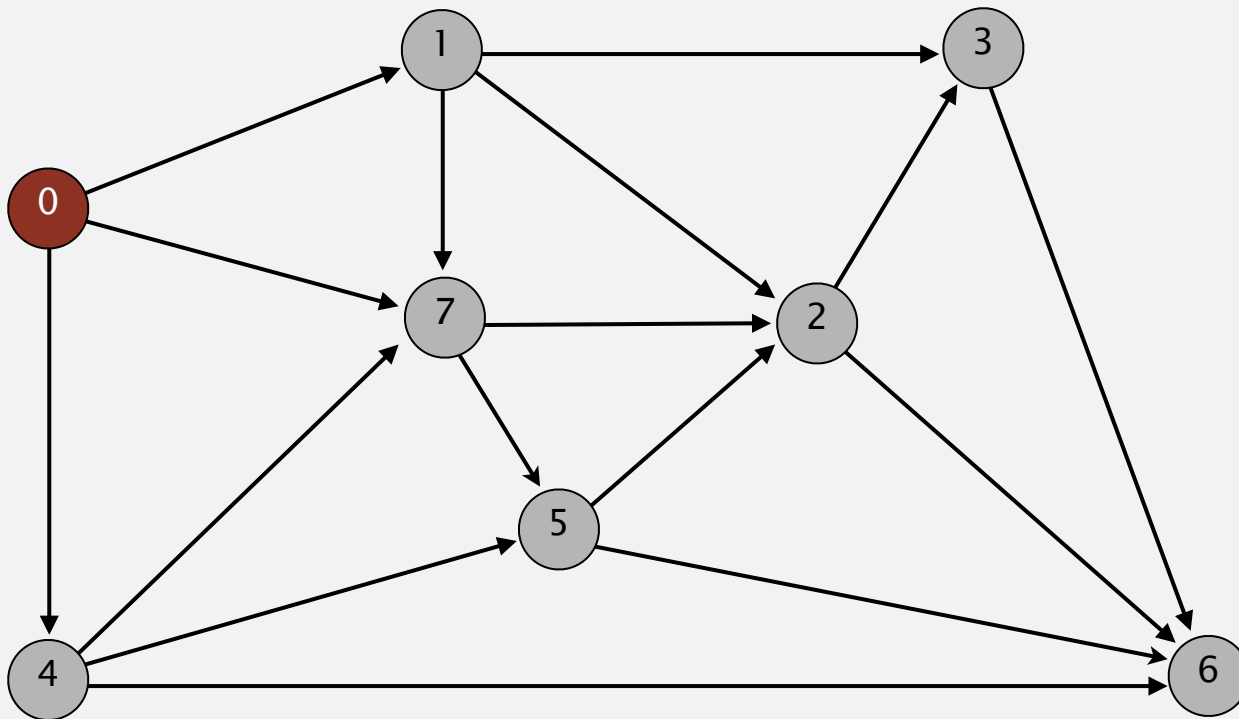
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



topological order: 0 1 4 7 5 2 3 6

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

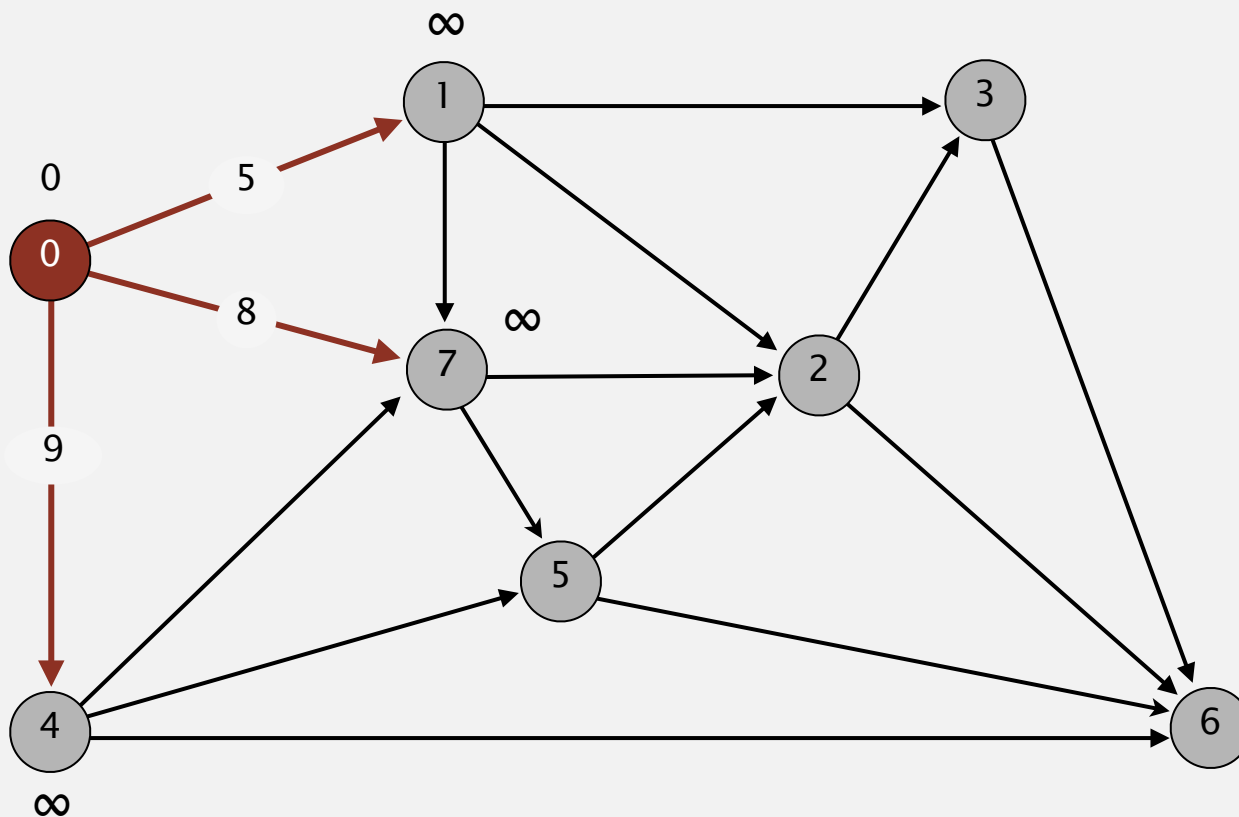


choose vertex 0

| v | distTo[] | edgeTo[] |
|-----|----------|----------|
| → 0 | 0.0 | - |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

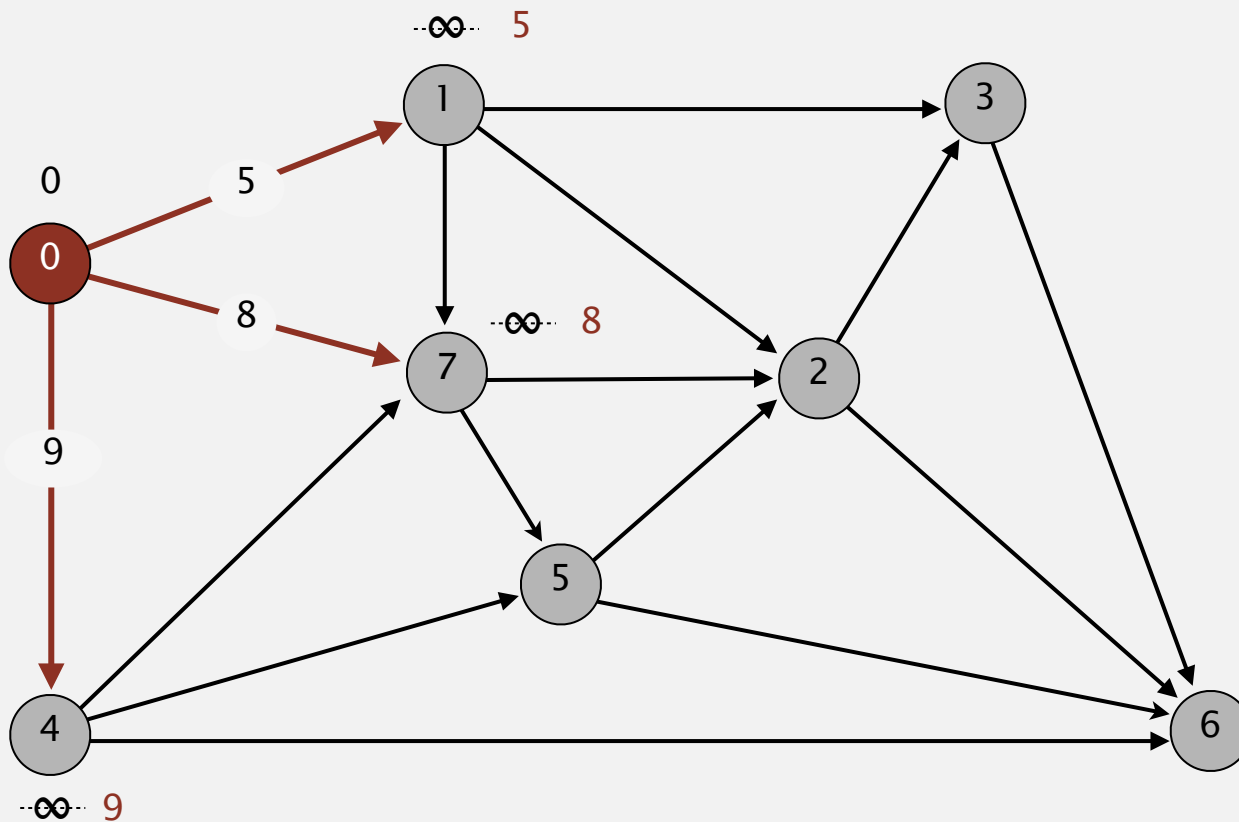


| v | distTo[] | edgeTo[] |
|-----|----------|----------|
| → 0 | 0.0 | - |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |

relax all edges incident from 0

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

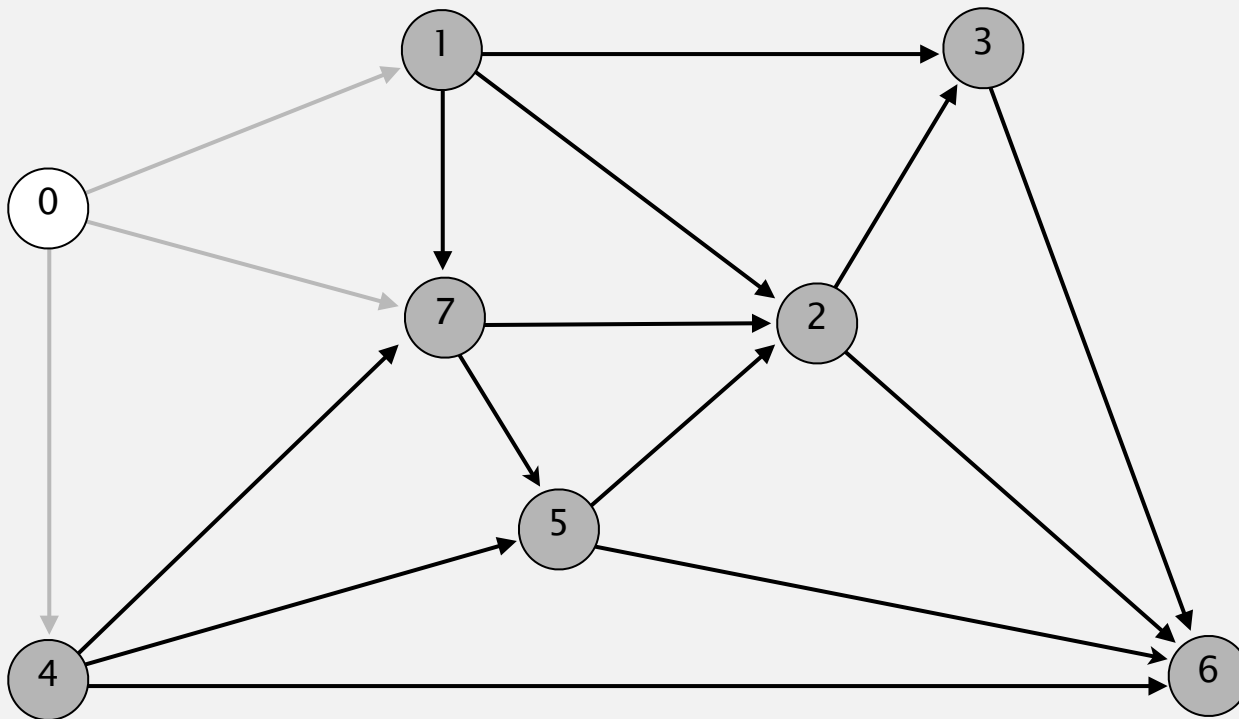


| v | distTo[] | edgeTo[] |
|---|----------|----------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | ∞ | - |
| 3 | ∞ | - |
| 4 | 9.0 | 0→4 |
| 5 | ∞ | - |
| 6 | ∞ | - |
| 7 | 8.0 | 0→7 |

relax all edges incident from 0

Topological sort algorithm

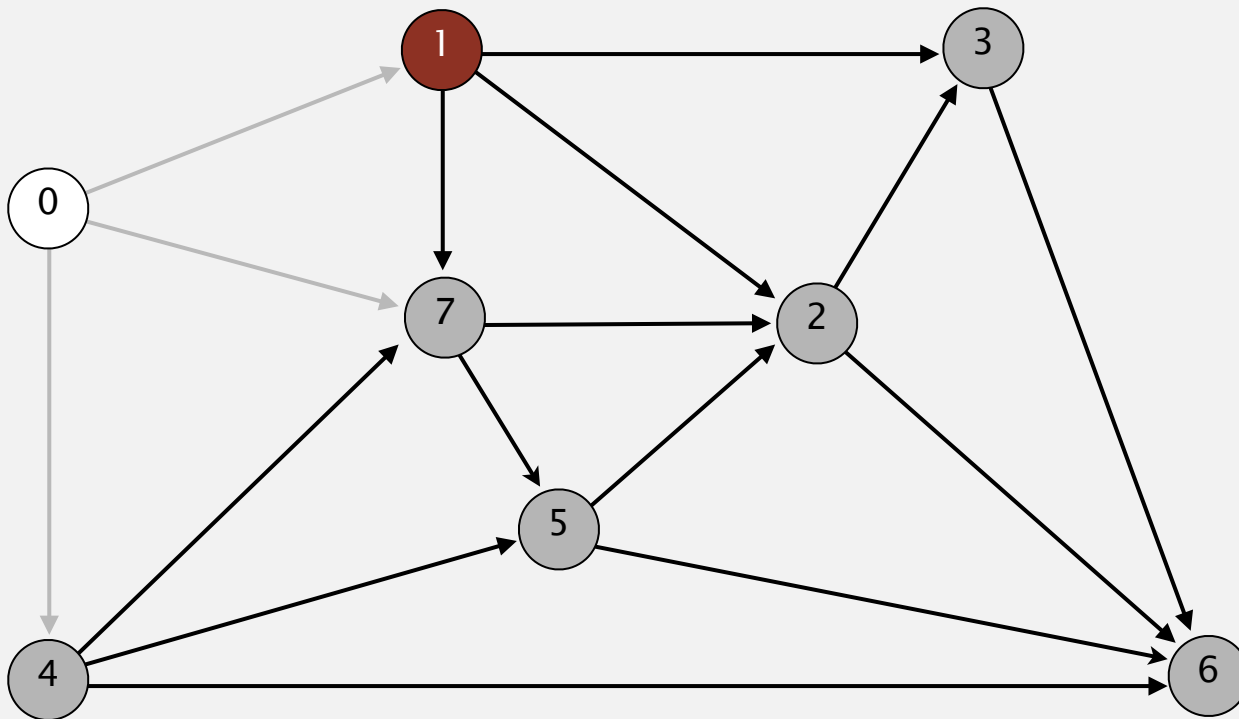
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | | |
| 3 | | |
| 4 | 9.0 | 0→4 |
| 5 | | |
| 6 | | |
| 7 | 8.0 | 0→7 |

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

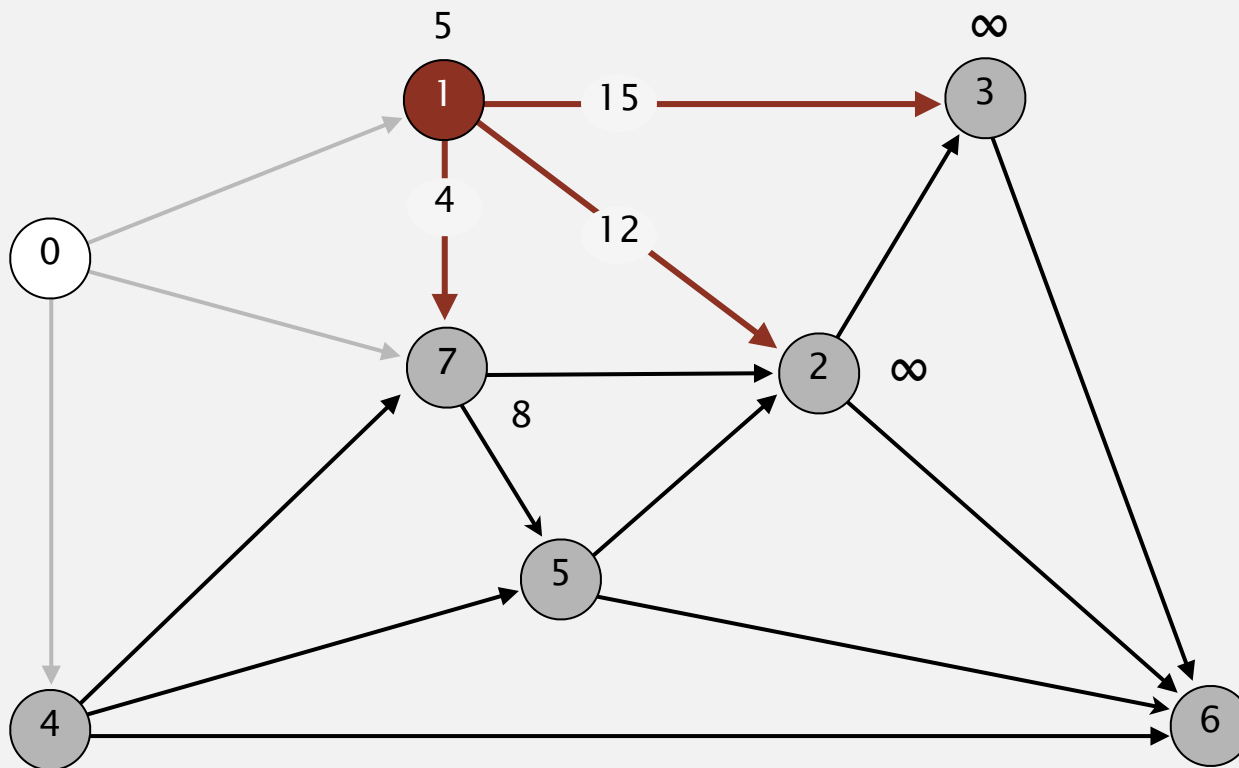


| v | distTo[] | edgeTo[] |
|-----|----------|----------|
| 0 | 0.0 | - |
| → 1 | 5.0 | 0→1 |
| 2 | | |
| 3 | | |
| 4 | 9.0 | 0→4 |
| 5 | | |
| 6 | | |
| 7 | 8.0 | 0→7 |

choose vertex 1

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

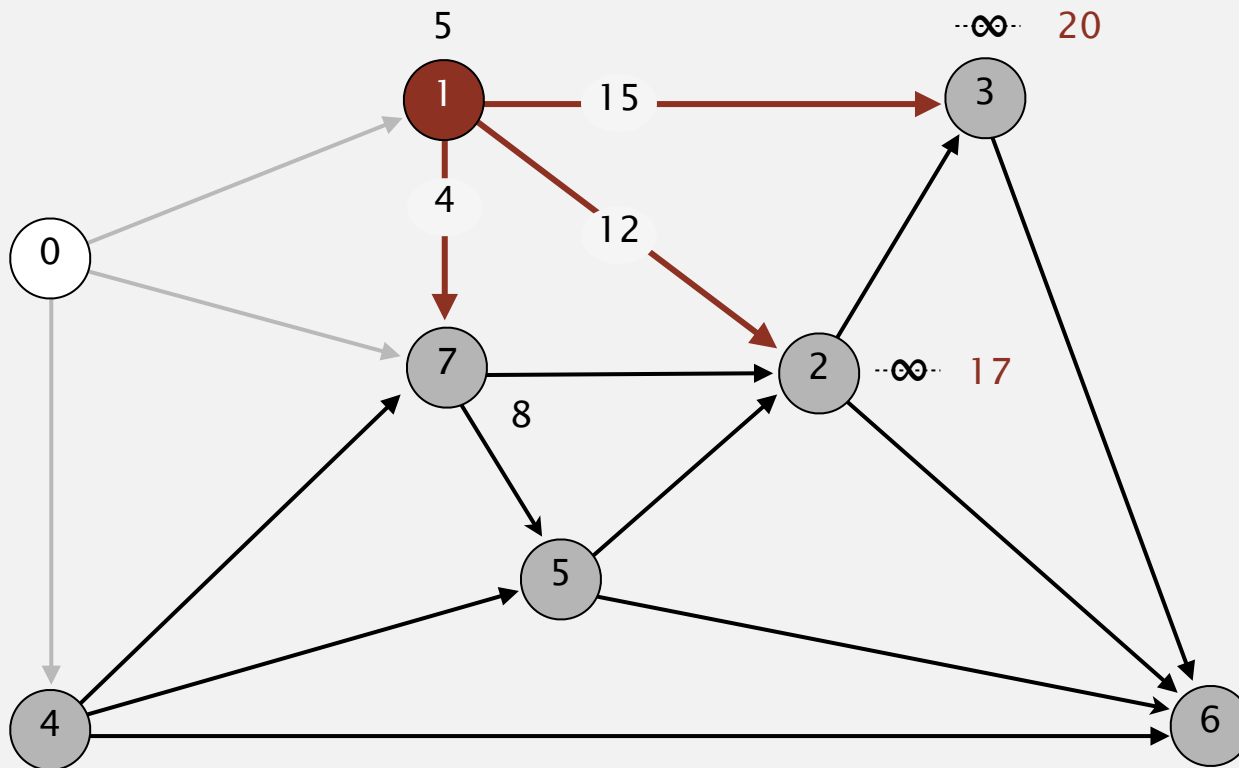


| v | $distTo[]$ | $edgeTo[]$ |
|-----|------------|------------|
| 0 | 0.0 | - |
| → 1 | 5.0 | 0→1 |
| 2 | | |
| 3 | | |
| 4 | 9.0 | 0→4 |
| 5 | | |
| 6 | | |
| 7 | 8.0 | 0→7 |

relax all edges incident from 1

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

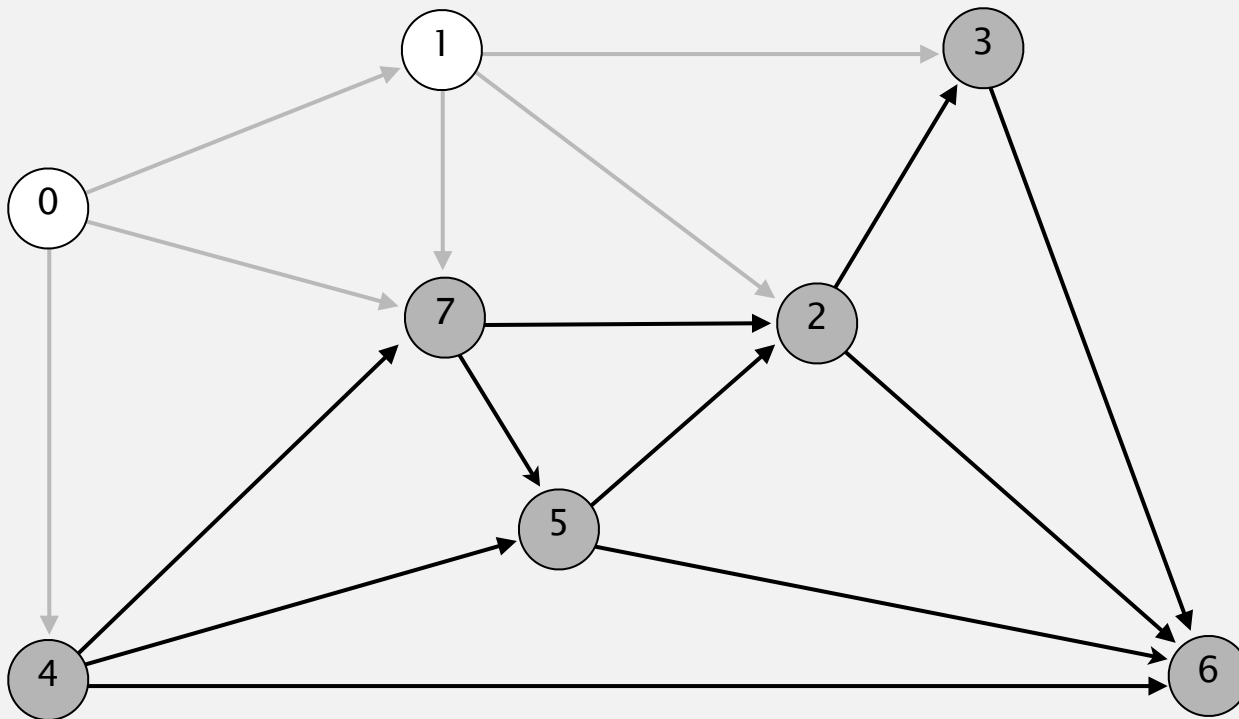


| v | distTo[] | edgeTo[] |
|---|----------|----------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 17.0 | 1→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| 5 | | |
| 6 | | |
| 7 | 8.0 ✓ | 0→7 |

relax all edges incident from 1

Topological sort algorithm

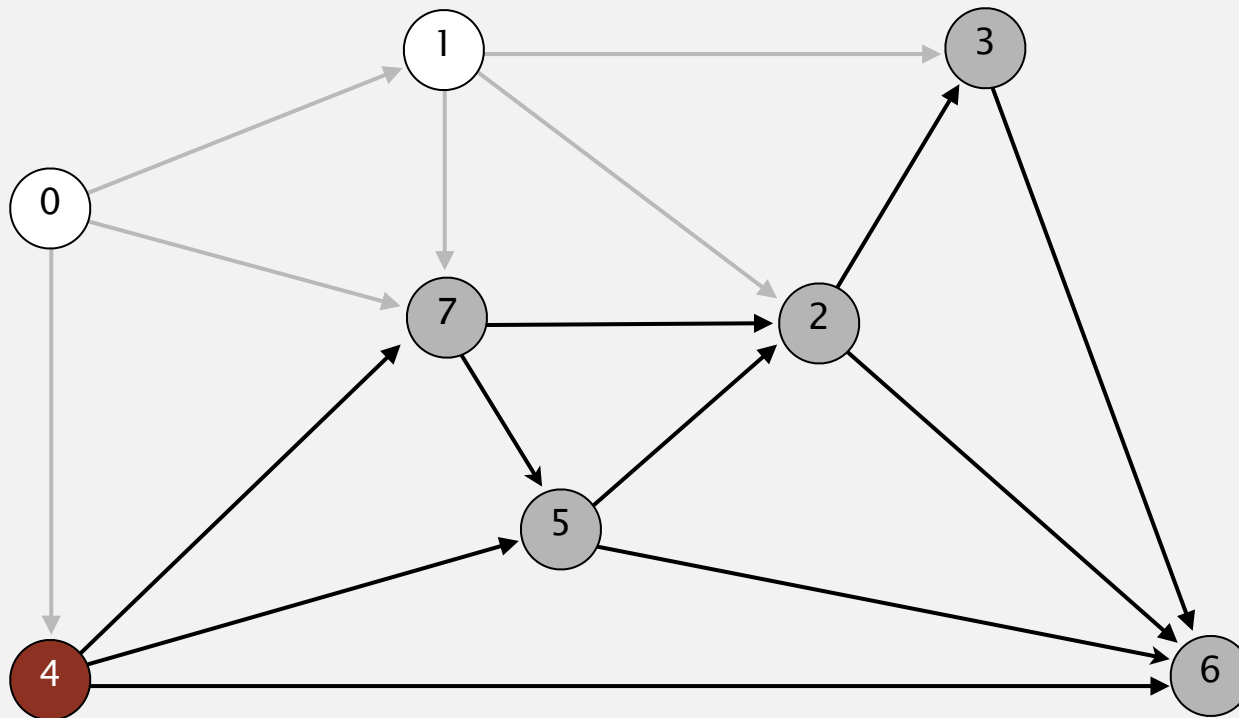
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 17.0 | 1→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| 5 | | |
| 6 | | |
| 7 | 8.0 | 0→7 |

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



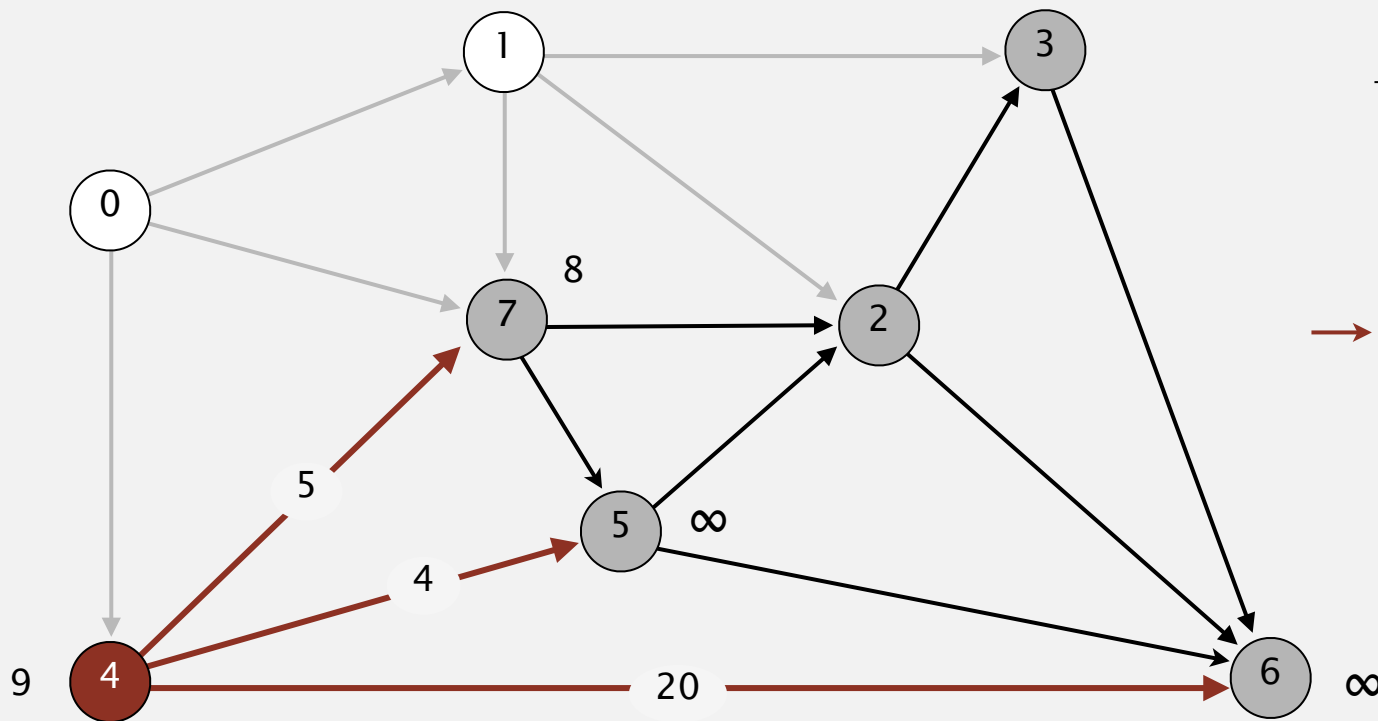
| v | distTo[] | edgeTo[] |
|-----|----------|----------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 17.0 | 1→2 |
| 3 | 20.0 | 1→3 |
| → 4 | 9.0 | 0→4 |
| 5 | | |
| 6 | | |
| 7 | 8.0 | 0→7 |

select vertex 4

(Dijkstra would have selected vertex 7)

Topological sort algorithm

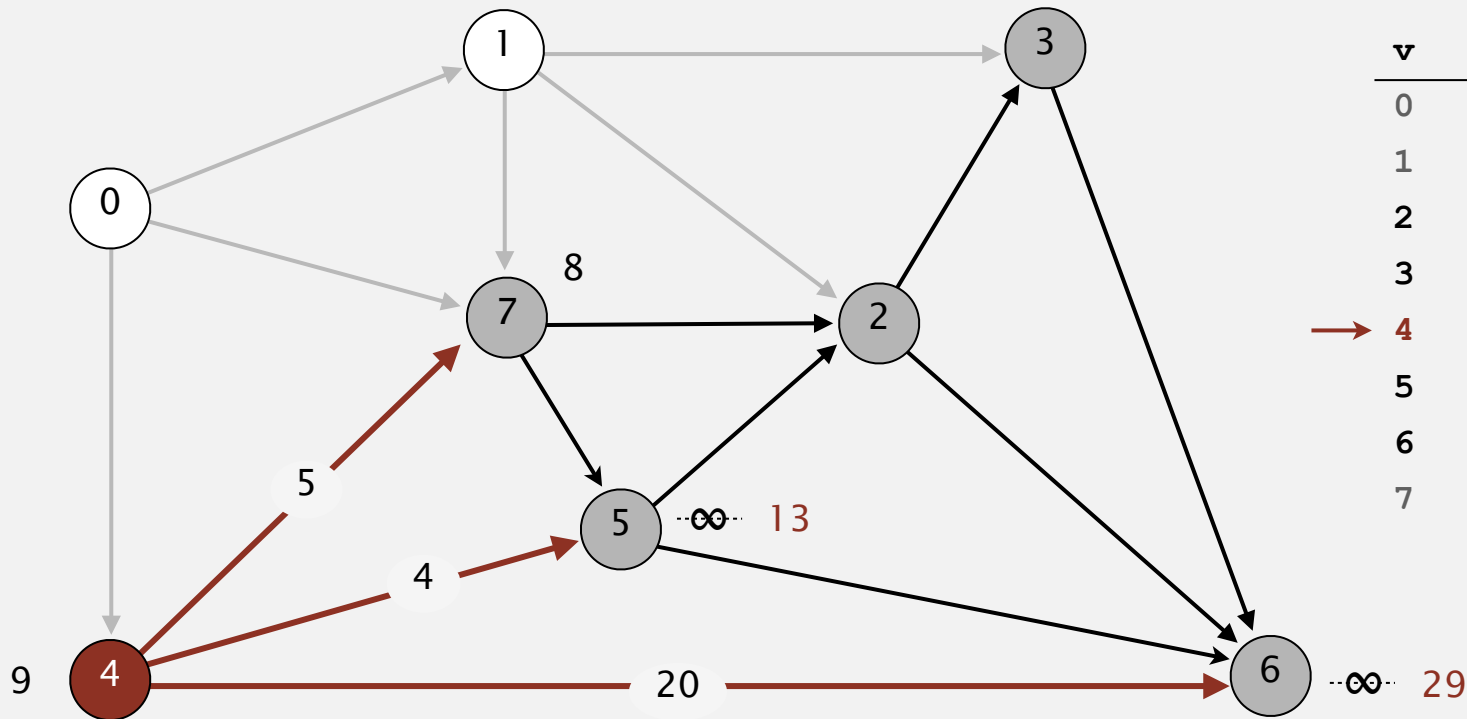
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



relax all edges incident from 4

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

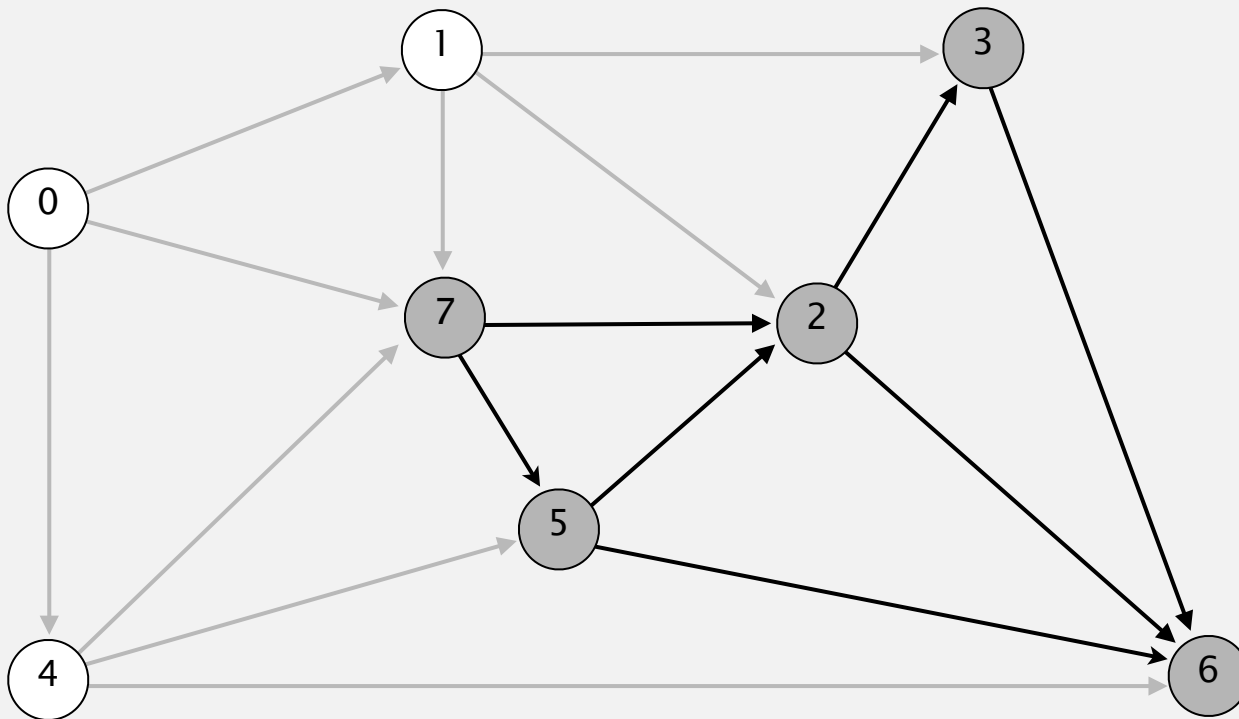


| v | distTo[] | edgeTo[] |
|-----|----------|----------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 17.0 | 1→2 |
| 3 | 20.0 | 1→3 |
| → 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 29.0 | 4→6 |
| 7 | 8.0 ✓ | 0→7 |

relax all edges incident from 4

Topological sort algorithm

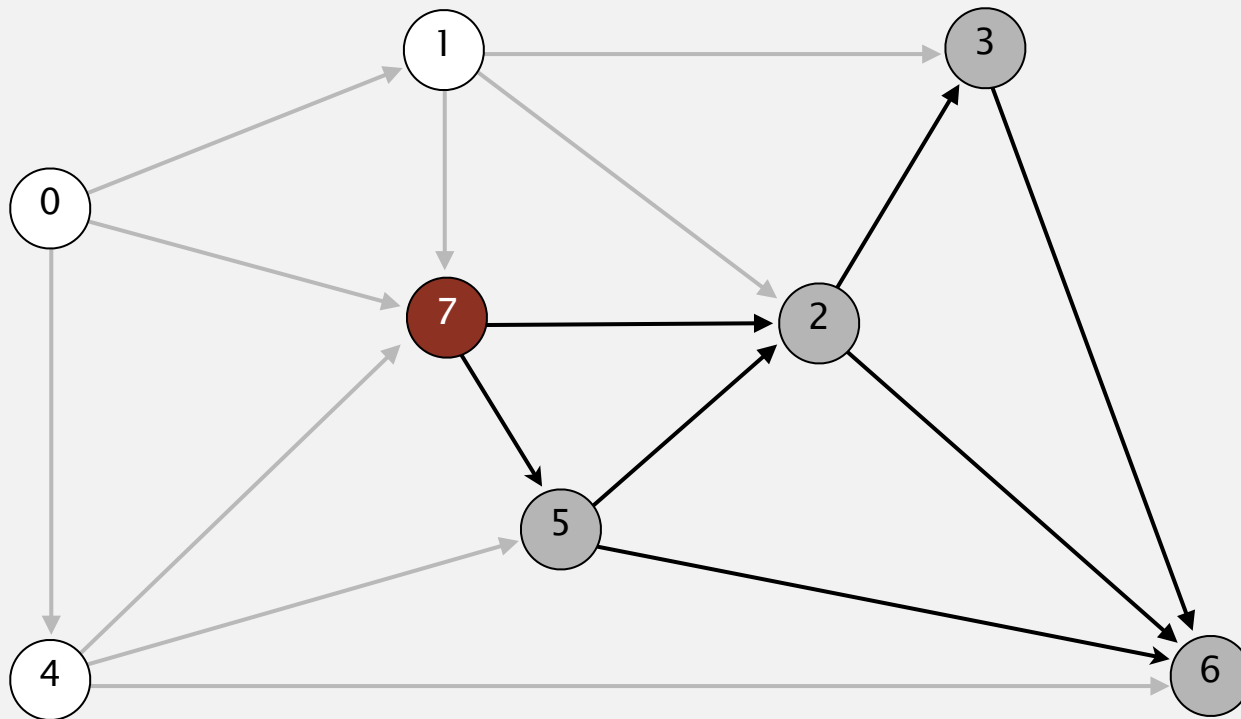
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 17.0 | 1→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 29.0 | 4→6 |
| 7 | 8.0 | 0→7 |

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

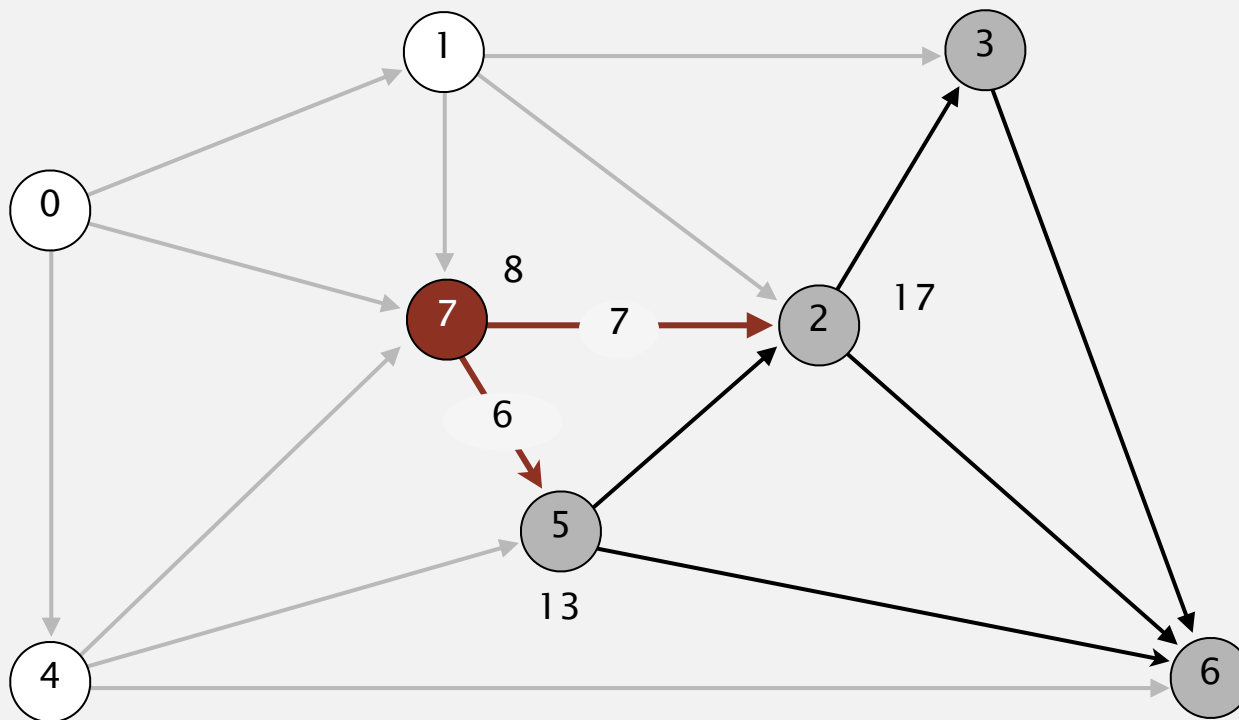


| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 17.0 | 1→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 29.0 | 4→6 |
| → 7 | 8.0 | 0→7 |

choose vertex 7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

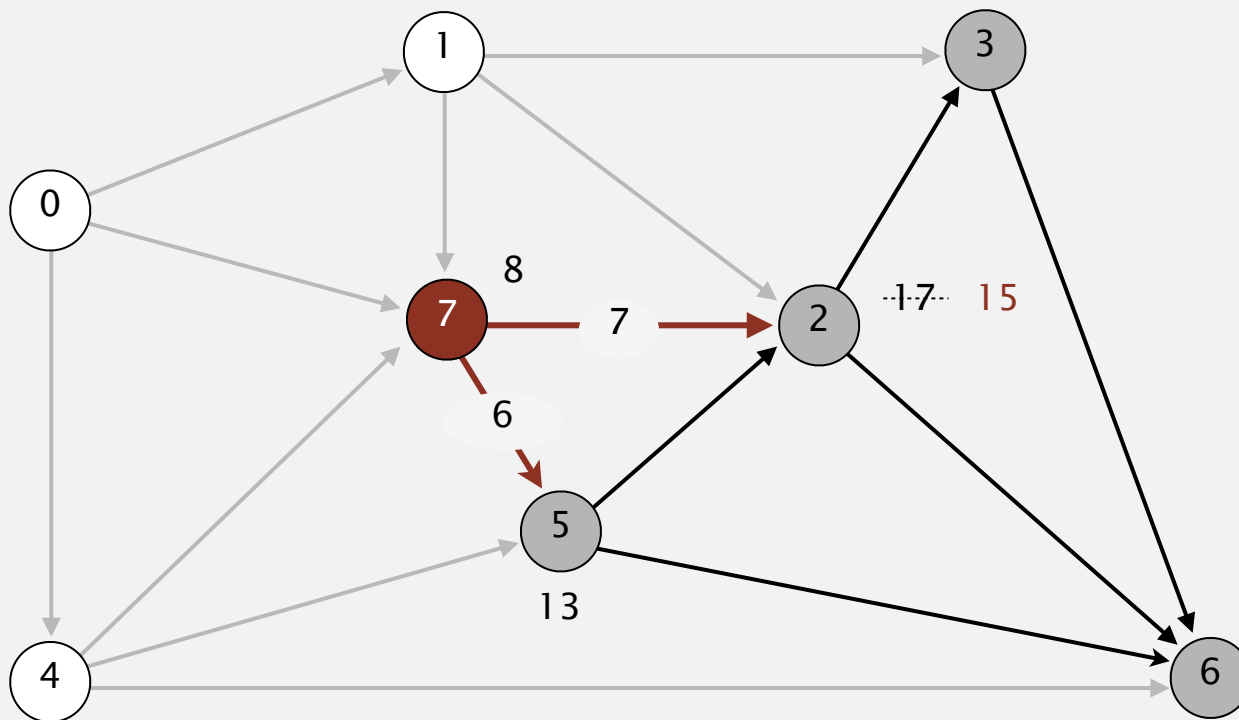


| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 17.0 | 1→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 29.0 | 4→6 |
| → 7 | 8.0 | 0→7 |

relax all edges incident from 7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

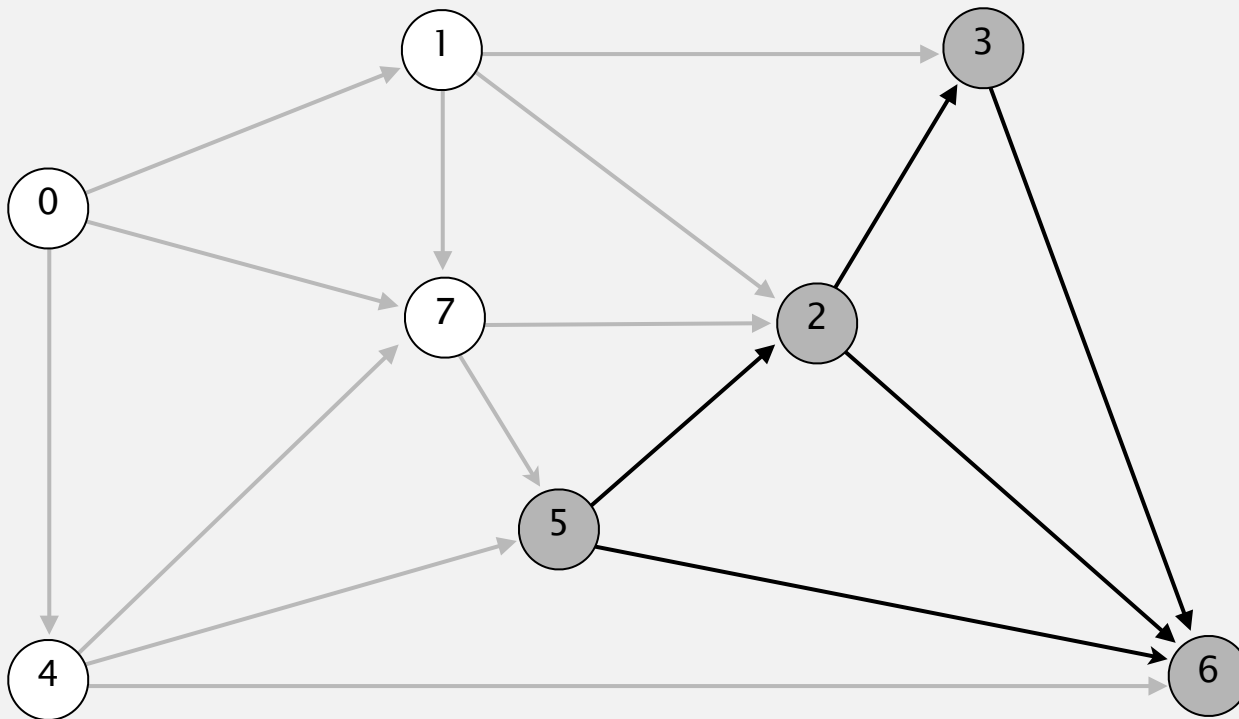


| v | distTo[] | edgeTo[] |
|-----|----------|----------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 15.0 | 7→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 29.0 | 4→6 |
| → 7 | 8.0 | 0→7 |

relax all edges incident from 7

Topological sort algorithm

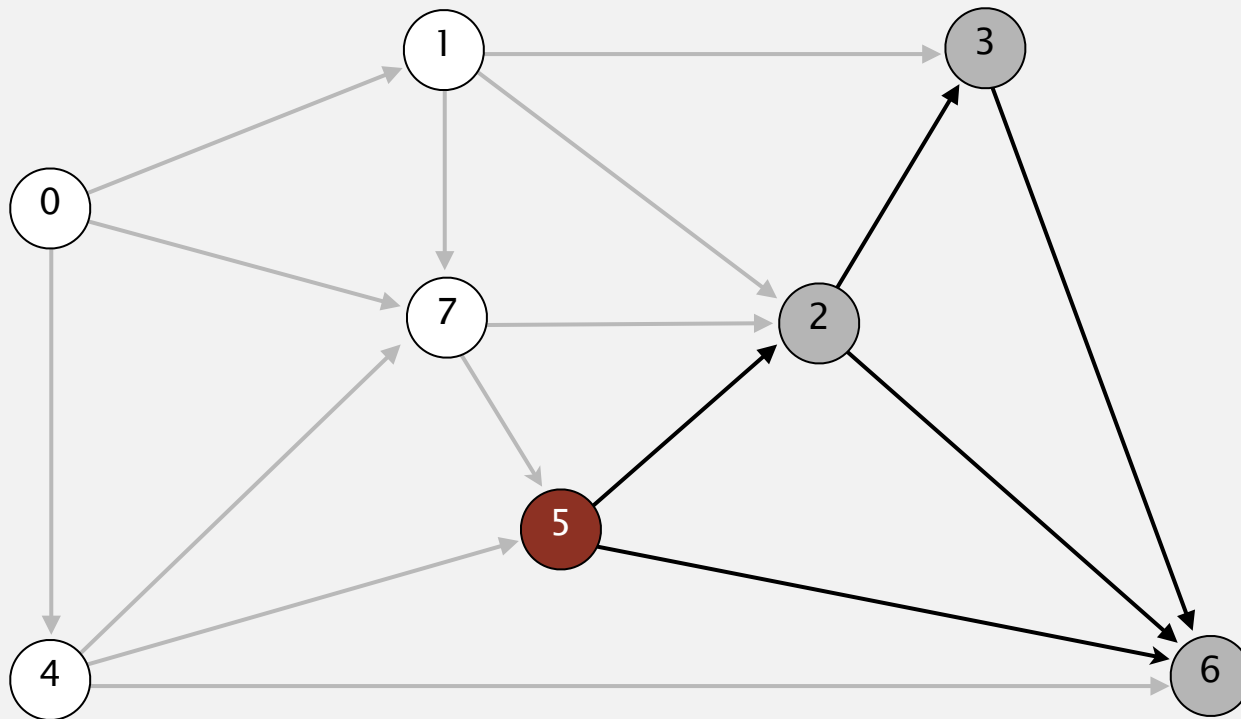
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 15.0 | 7→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 29.0 | 4→6 |
| 7 | 8.0 | 0→7 |

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

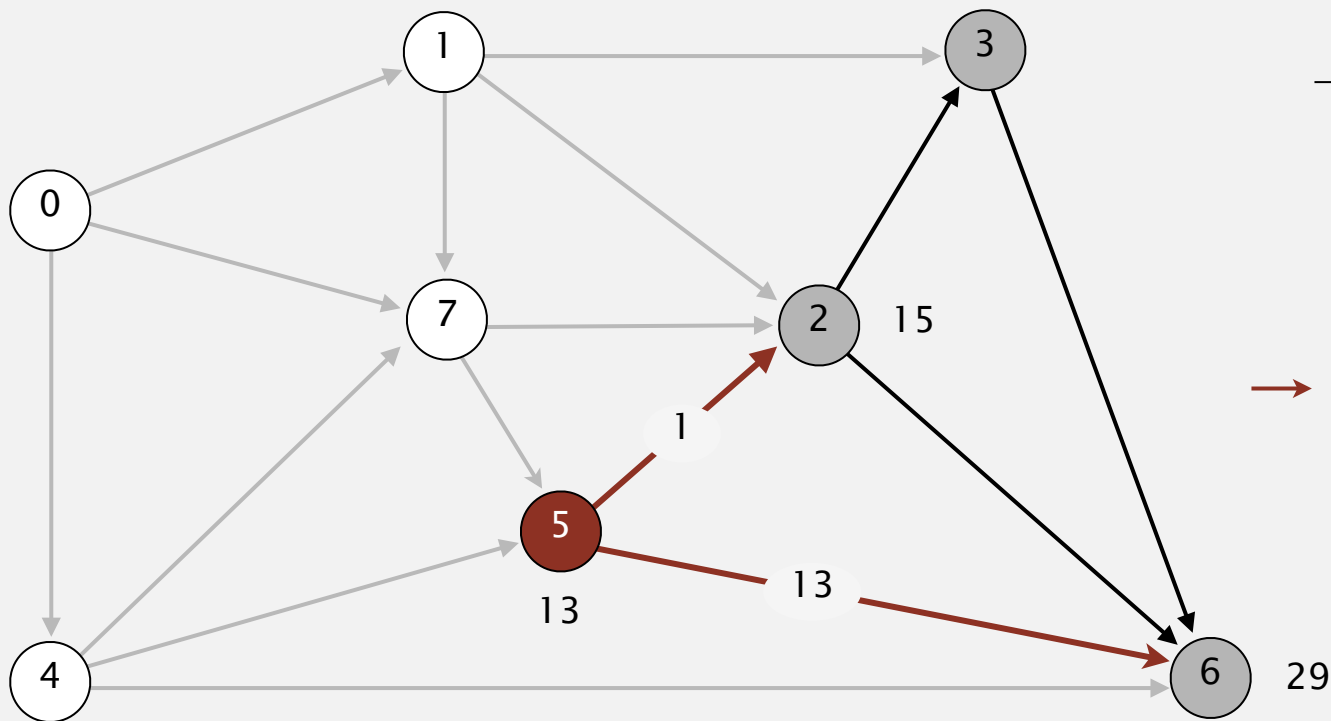


| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 15.0 | 7→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| → 5 | 13.0 | 4→5 |
| 6 | 29.0 | 4→6 |
| 7 | 8.0 | 0→7 |

select vertex 5

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

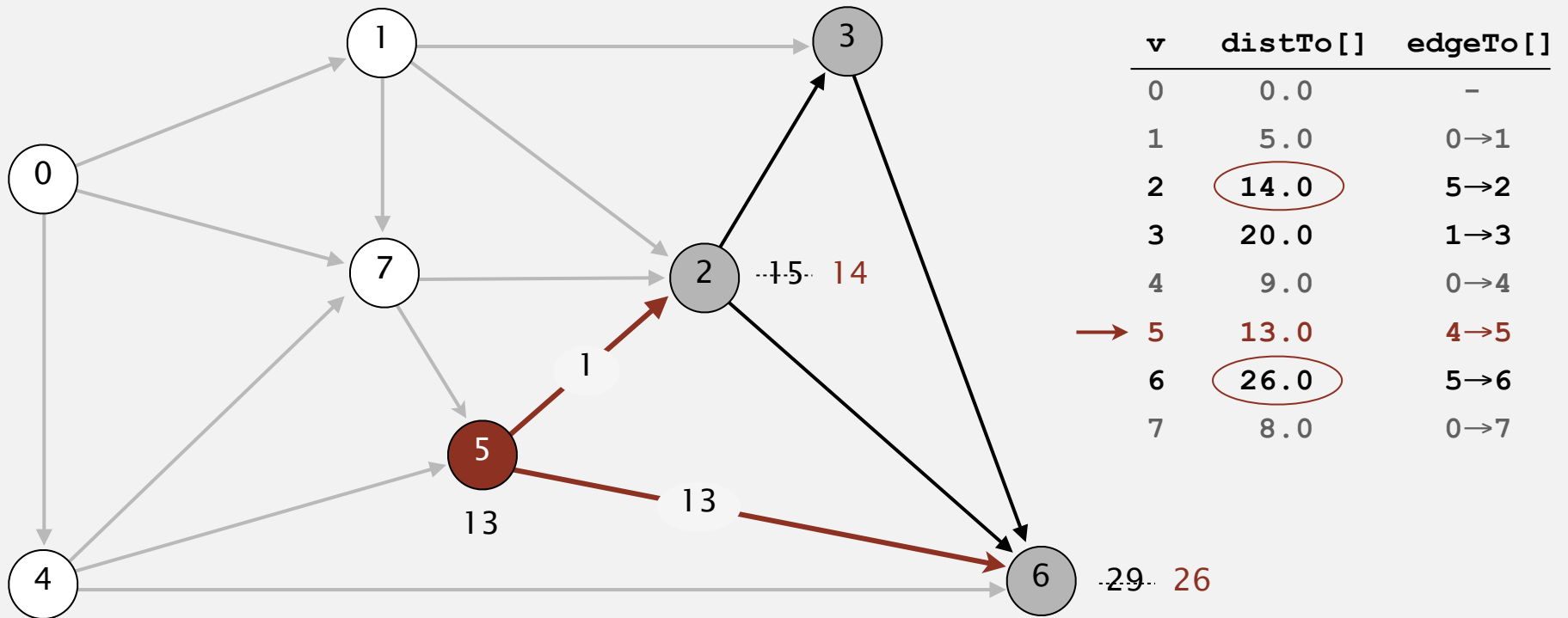


| v | distTo[] | edgeTo[] |
|-----|----------|----------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 15.0 | 7→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| → 5 | 13.0 | 4→5 |
| 6 | 29.0 | 4→6 |
| 7 | 8.0 | 0→7 |

relax all edges incident from 5

Topological sort algorithm

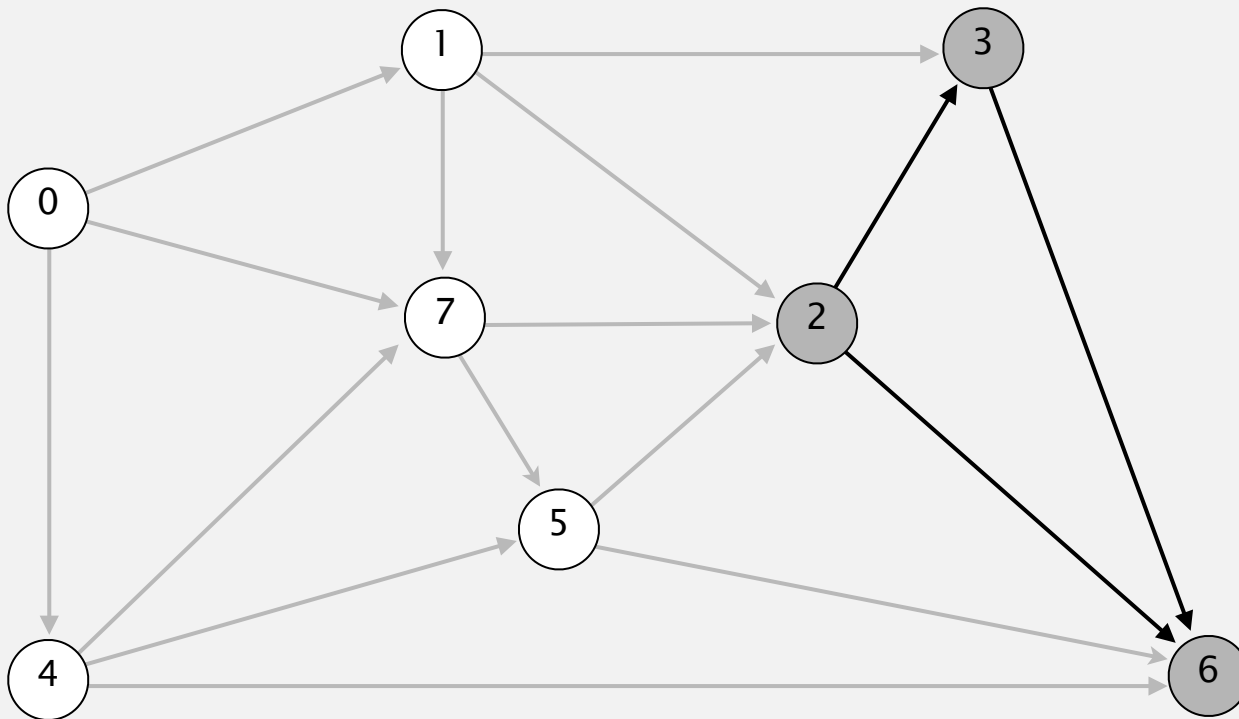
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



relax all edges incident from 5

Topological sort algorithm

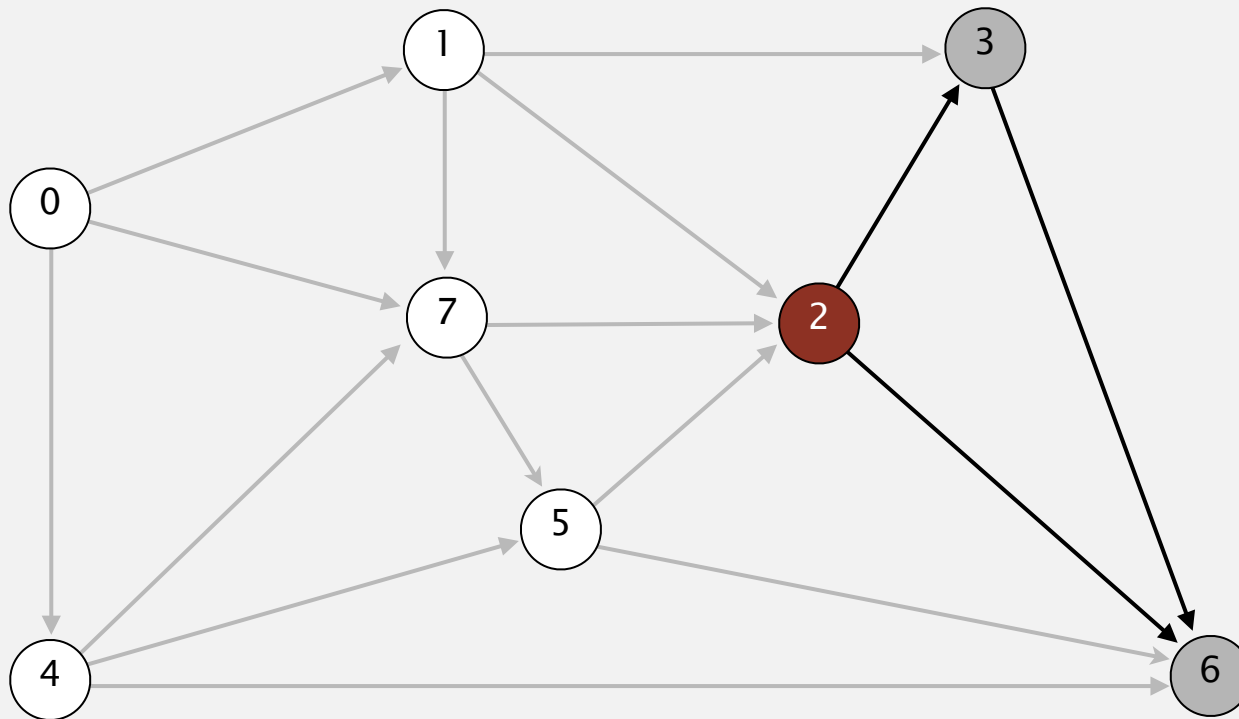
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 14.0 | 5→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 26.0 | 5→6 |
| 7 | 8.0 | 0→7 |

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

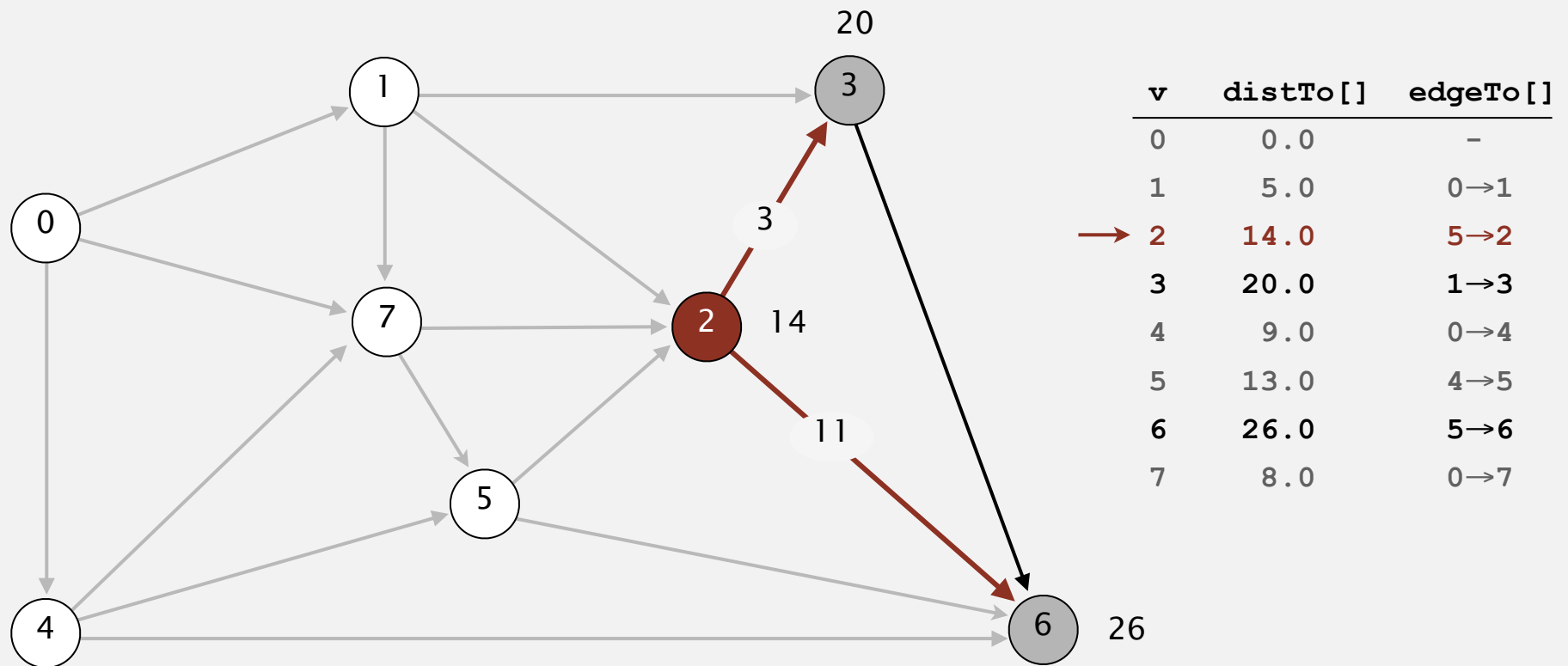


| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| → 2 | 14.0 | 5→2 |
| 3 | 20.0 | 1→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 26.0 | 5→6 |
| 7 | 8.0 | 0→7 |

select vertex 2

Topological sort algorithm

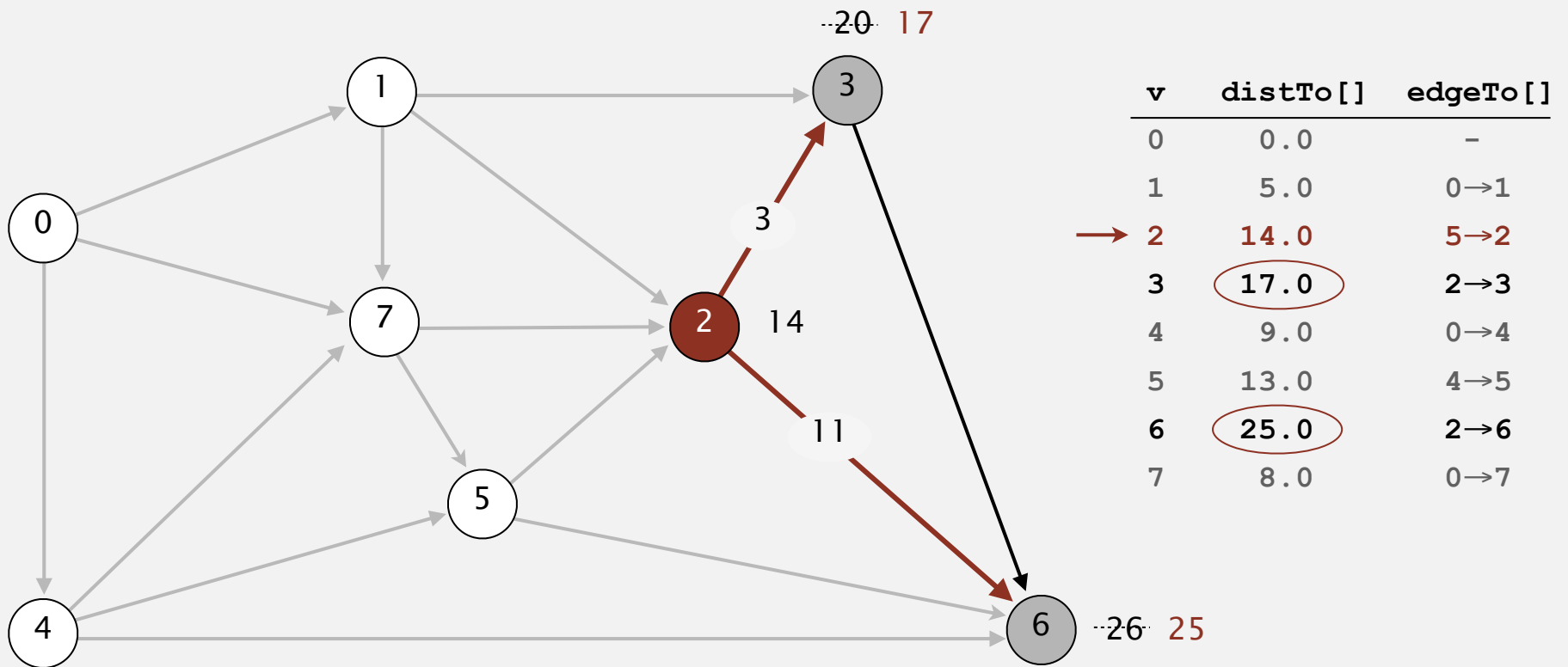
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



relax all edges incident from 2

Topological sort algorithm

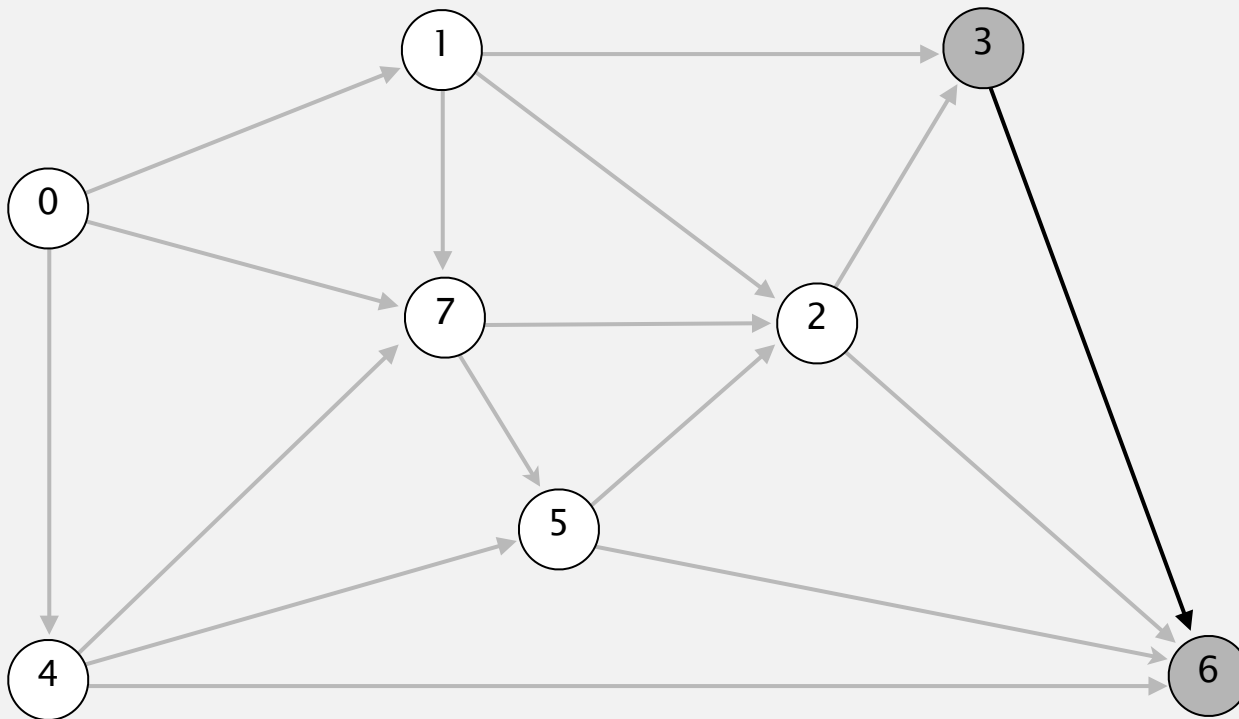
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



relax all edges incident from 2

Topological sort algorithm

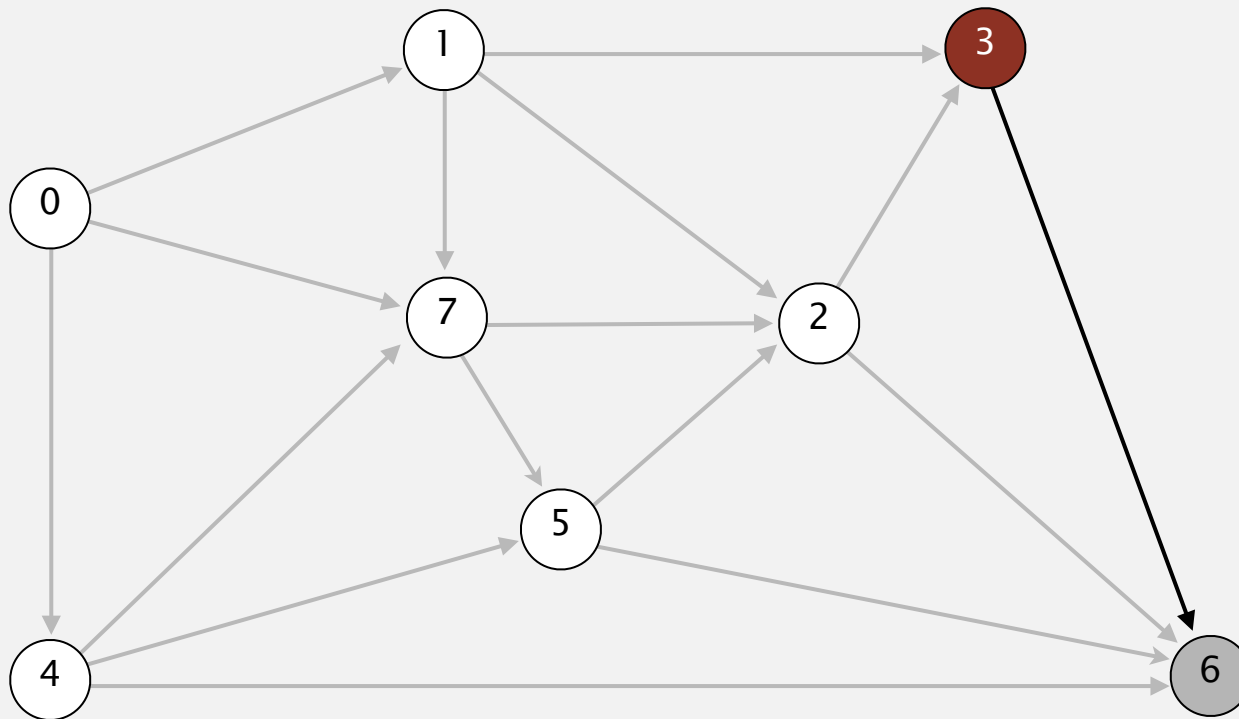
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 14.0 | 5→2 |
| 3 | 17.0 | 2→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 25.0 | 2→6 |
| 7 | 8.0 | 0→7 |

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

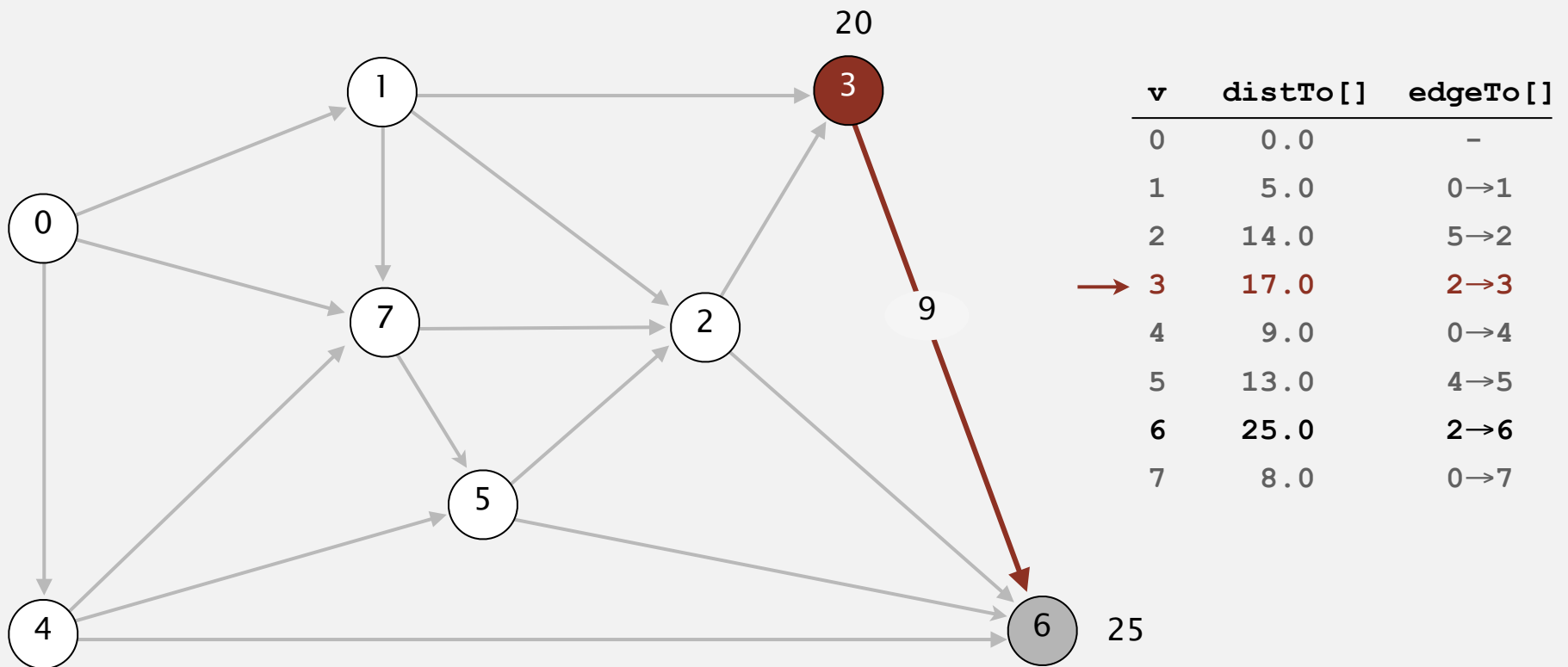


| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 14.0 | 5→2 |
| → 3 | 17.0 | 2→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 25.0 | 2→6 |
| 7 | 8.0 | 0→7 |

select vertex 3

Topological sort algorithm

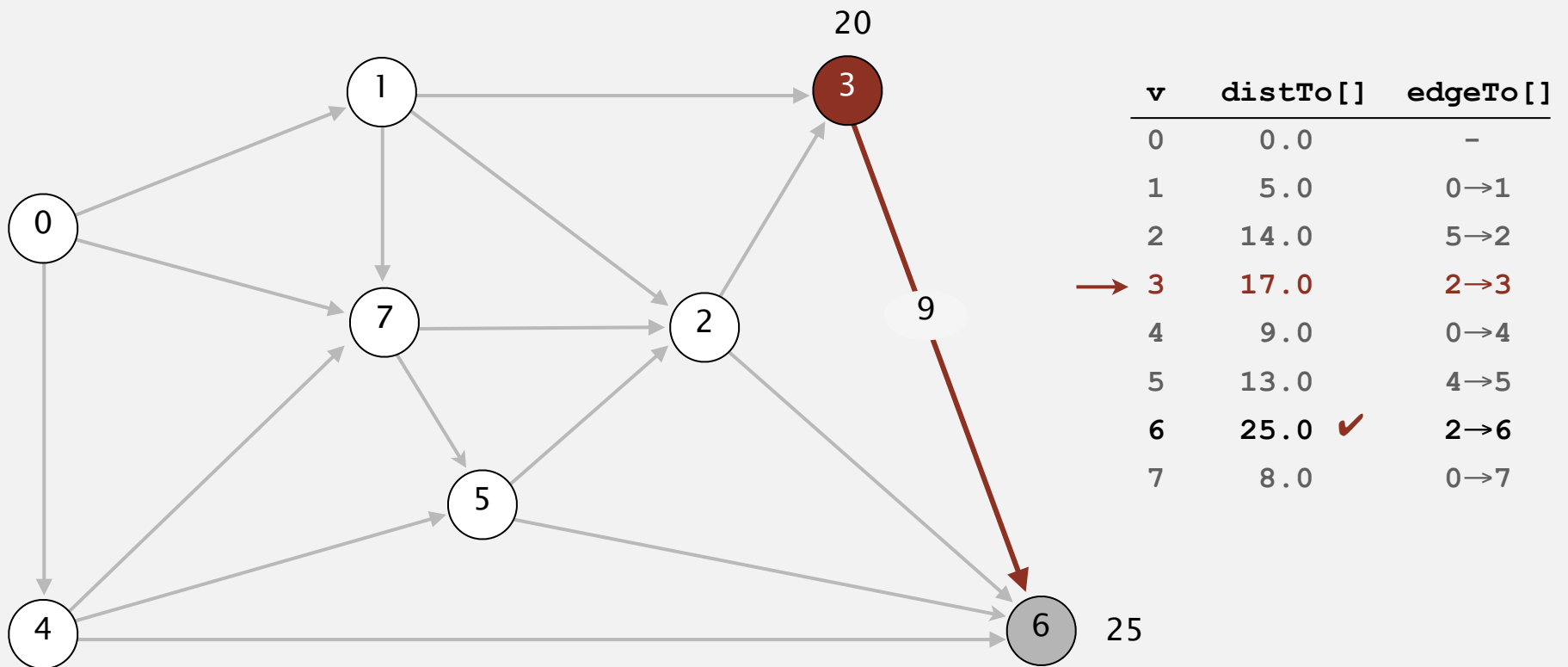
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



relax all edges incident from 3

Topological sort algorithm

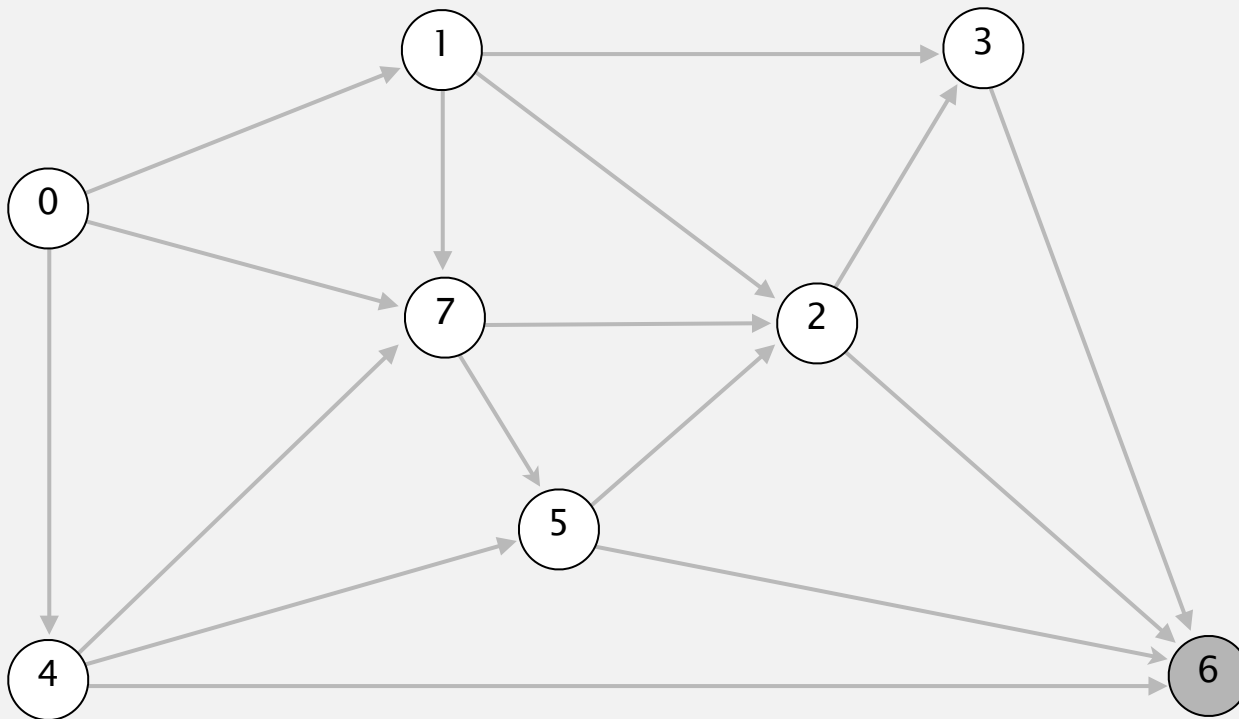
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



relax all edges incident from 3

Topological sort algorithm

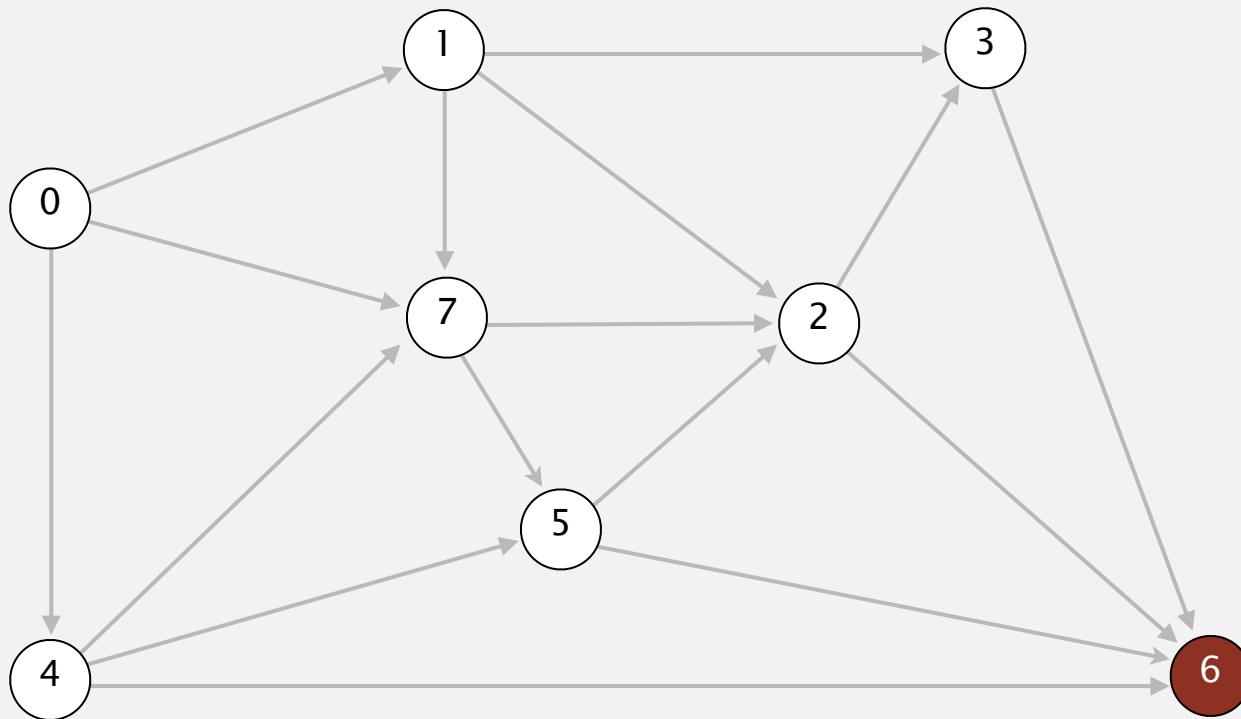
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 14.0 | 5→2 |
| 3 | 17.0 | 2→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 25.0 | 2→6 |
| 7 | 8.0 | 0→7 |

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

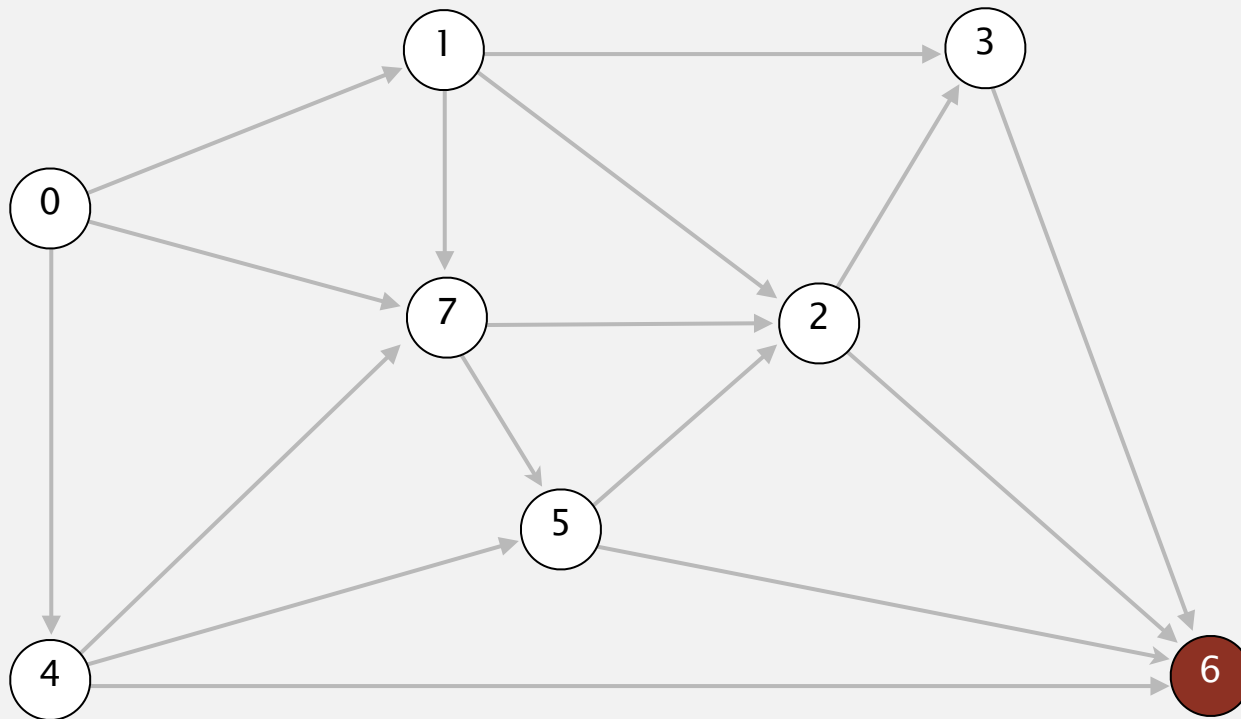


| <u>v</u> | <u>distTo[]</u> | <u>edgeTo[]</u> |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 14.0 | 5→2 |
| 3 | 17.0 | 2→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| → 6 | 25.0 | 2→6 |
| 7 | 8.0 | 0→7 |

select vertex 6

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

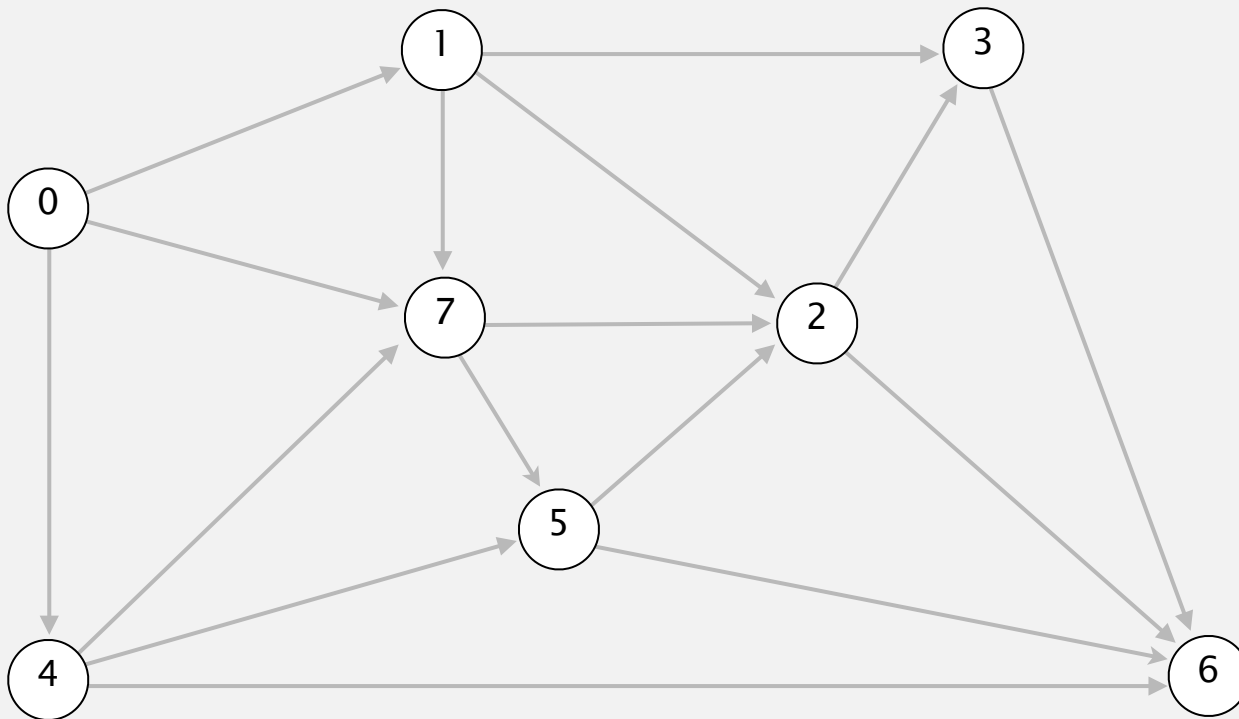


| <u>v</u> | <u>distTo[]</u> | <u>edgeTo[]</u> |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 14.0 | 5→2 |
| 3 | 17.0 | 2→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| → 6 | 25.0 | 2→6 |
| 7 | 8.0 | 0→7 |

relax all edges incident from 6

Topological sort algorithm

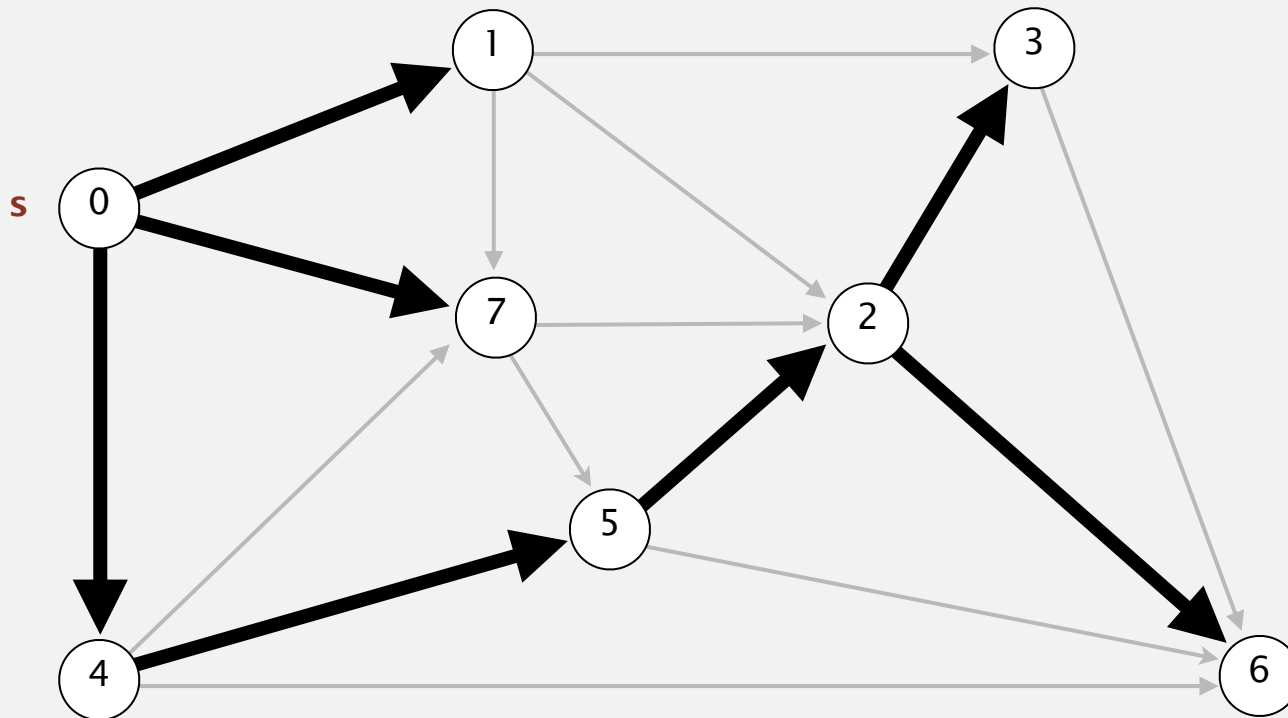
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 14.0 | 5→2 |
| 3 | 17.0 | 2→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 25.0 | 2→6 |
| 7 | 8.0 | 0→7 |

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



| v | distTo[] | edgeTo[] |
|----------|-----------------|-----------------|
| 0 | 0.0 | - |
| 1 | 5.0 | 0→1 |
| 2 | 14.0 | 5→2 |
| 3 | 17.0 | 2→3 |
| 4 | 9.0 | 0→4 |
| 5 | 13.0 | 4→5 |
| 6 | 25.0 | 2→6 |
| 7 | 8.0 | 0→7 |

shortest-paths tree from vertex s