

2.3 QUICKSORT PARTITIONING DEMO



- ▶ Sedgwick 2-way
- ▶ Dijkstra 3-way
- ▶ Bentley-McIlroy 3-way

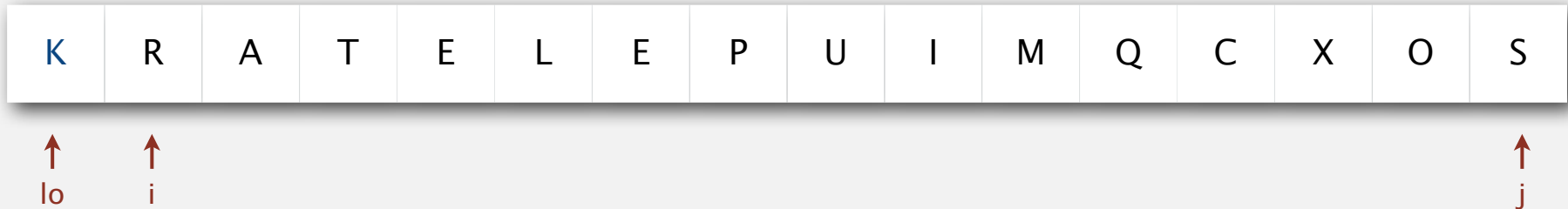
SEDGWICK 2-WAY PARTITIONING



Quicksort partitioning

Repeat until i and j pointers cross.

- Scan i from left to right so long as $a[i] < a[lo]$.
- Scan j from right to left so long as $a[j] > a[lo]$.
- Exchange $a[i]$ with $a[j]$.

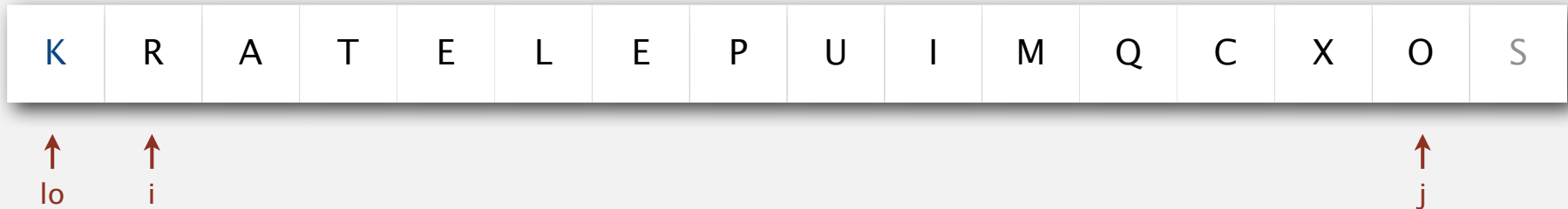


stop i scan because $a[i] \geq a[lo]$

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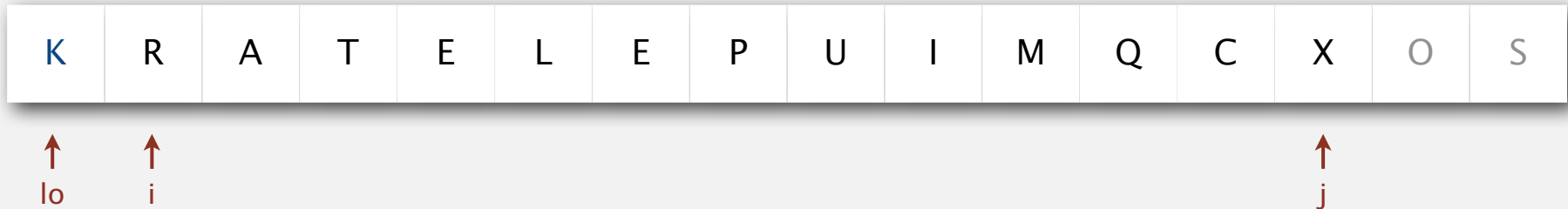
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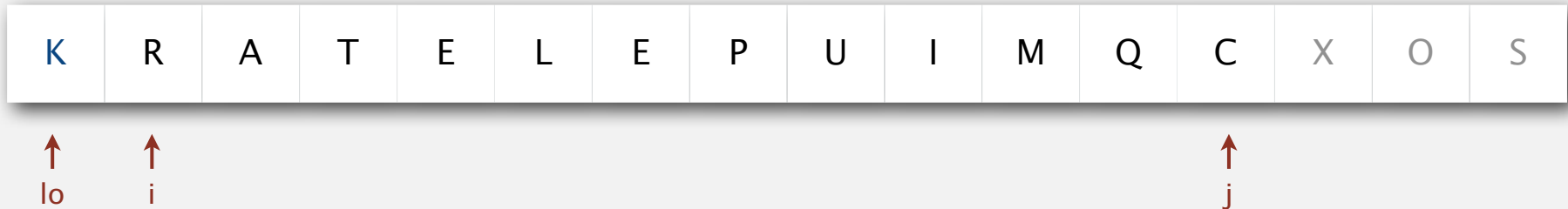
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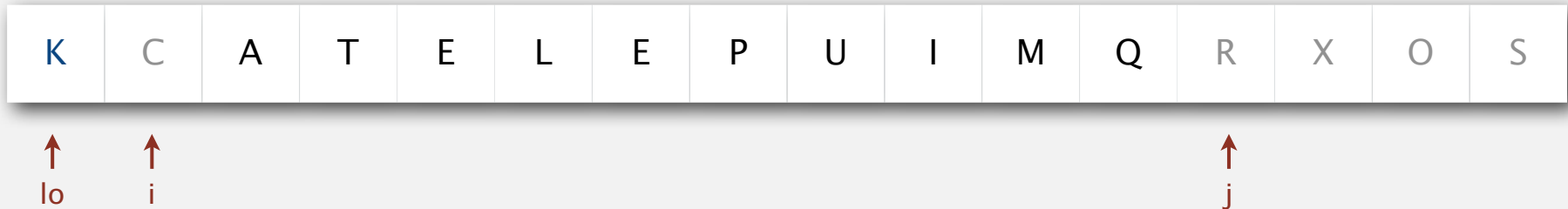


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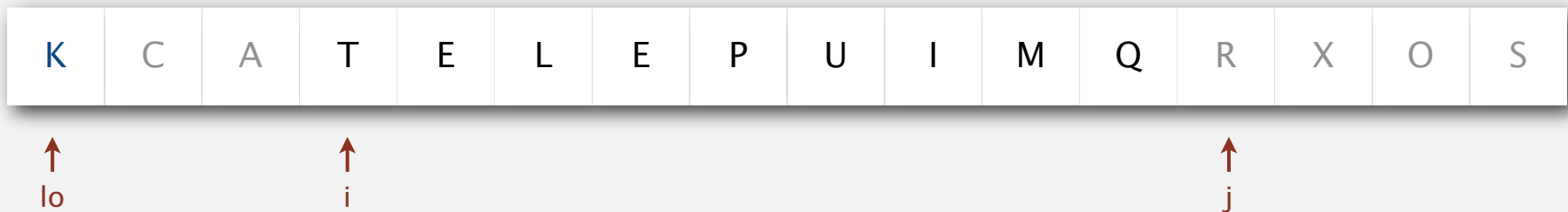
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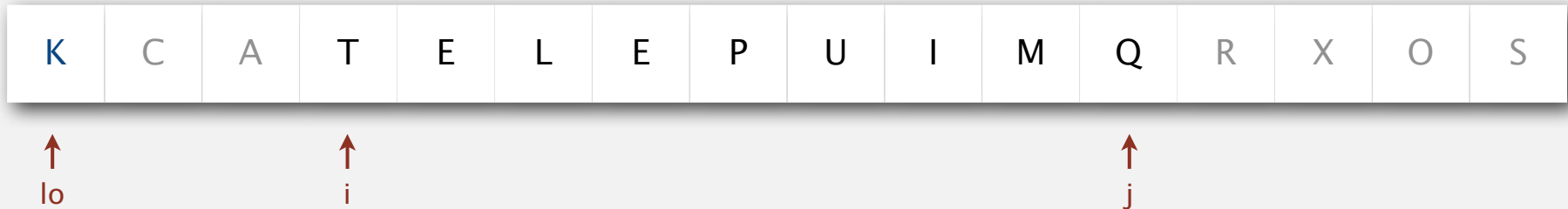


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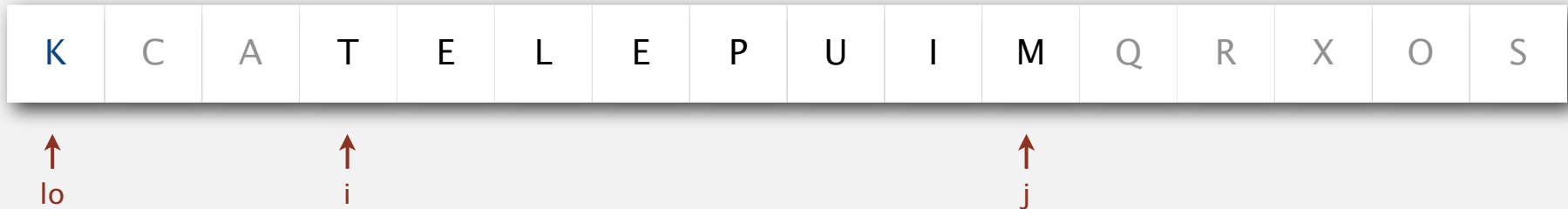
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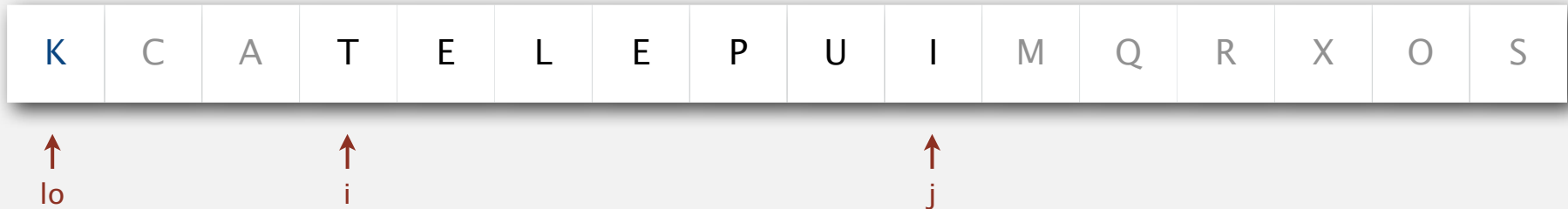
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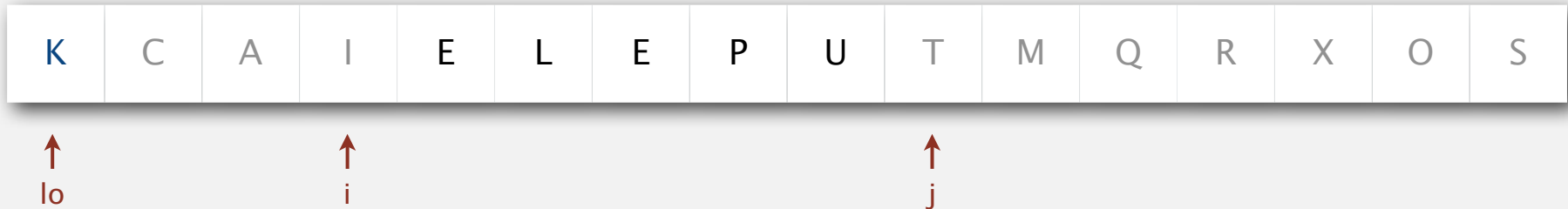


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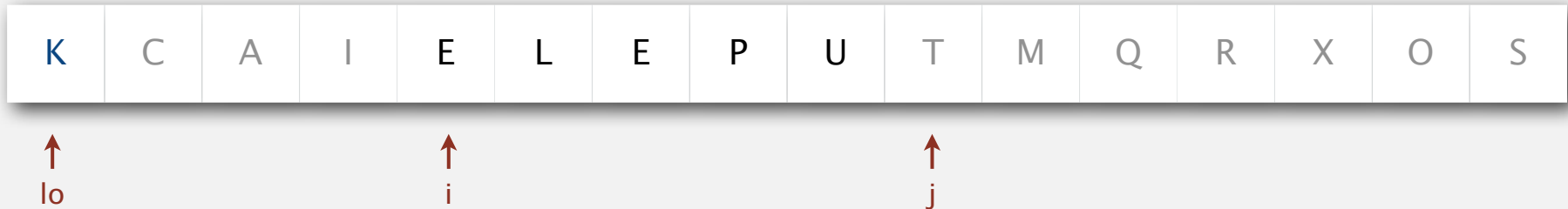
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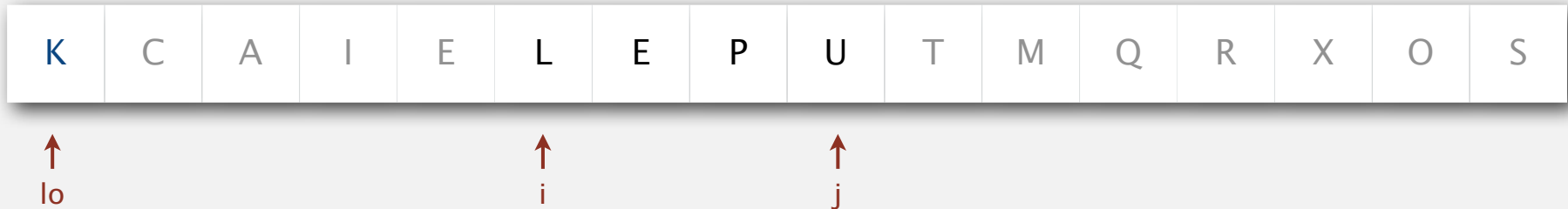


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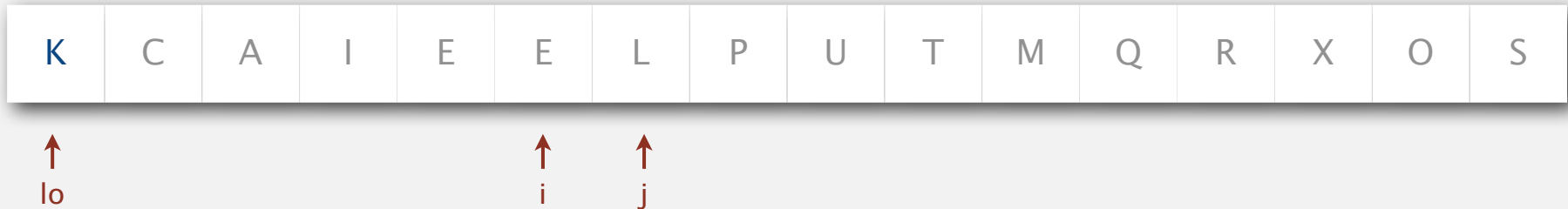


stop j scan and exchange $a[i]$ with $a[j]$

Quicksort partitioning

Repeat until i and j pointers cross.

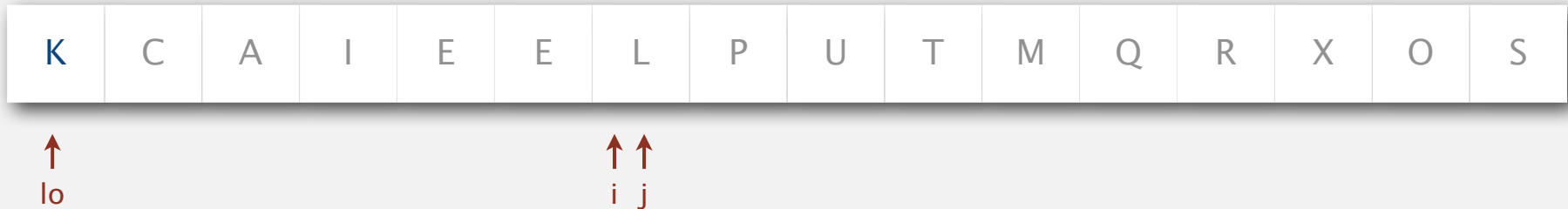
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stop i scan because $a[i] \geq a[lo]$

Quicksort partitioning

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- Exchange $a[i]$ with $a[j]$.



stop j scan because $a[j] \leq a[lo]$

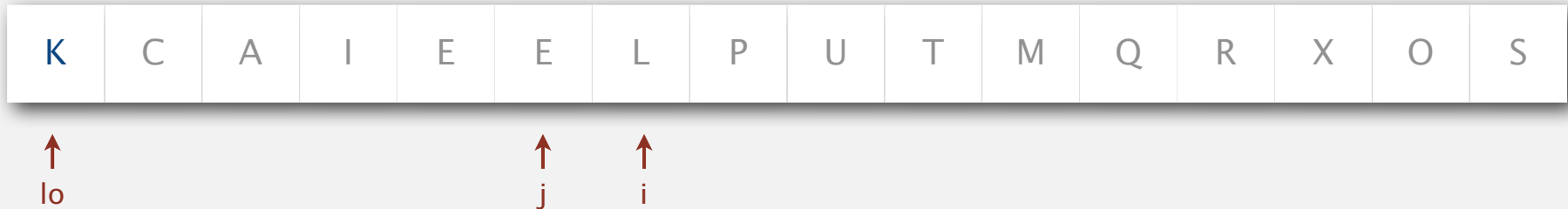
Quicksort partitioning

Repeat until i and j pointers cross.

- Scan i from left to right so long as $a[i] < a[lo]$.
- Scan j from right to left so long as $a[j] > a[lo]$.
- Exchange $a[i]$ with $a[j]$.

When pointers cross.

- Exchange $a[lo]$ with $a[j]$.



pointers cross: exchange $a[lo]$ with $a[j]$

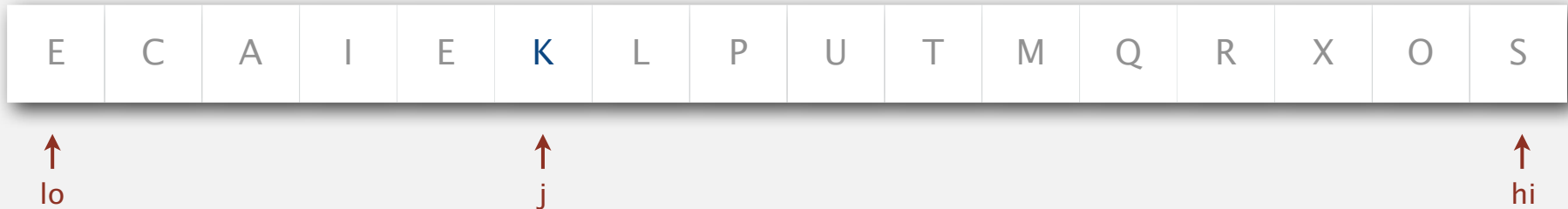
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When pointers cross.

- Exchange $a[lo]$ with $a[j]$.



partitioned!

DIJKSTRA 3-WAY PARTITIONING

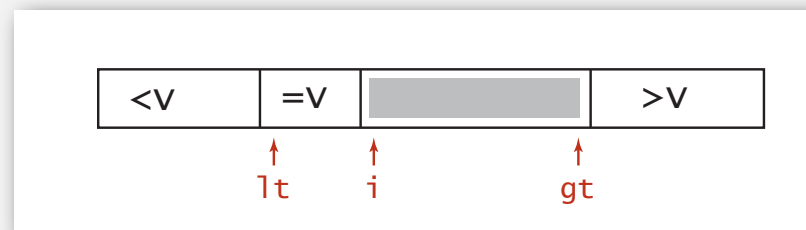


Dijkstra 3-way partitioning

- Let v be partitioning item $a[lo]$.
- Scan i from left to right.
 - ($a[i] < v$): exchange $a[lt]$ with $a[i]$ and increment both lt and i
 - ($a[i] > v$): exchange $a[gt]$ with $a[i]$ and decrement gt
 - ($a[i] == v$): increment i

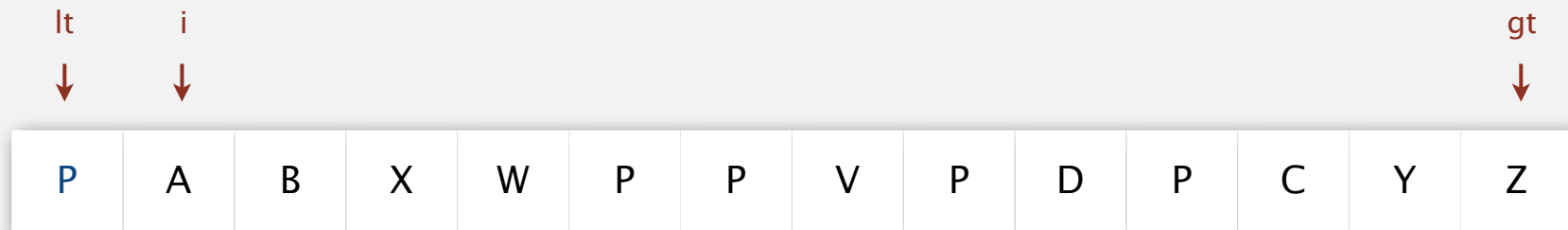


invariant

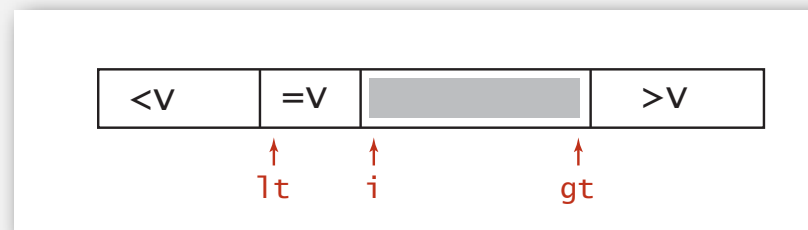


Dijkstra 3-way partitioning

- Let v be partitioning item $a[l_0]$.
- Scan i from left to right.
 - ($a[i] < v$): exchange $a[l_t]$ with $a[i]$ and increment both l_t and i
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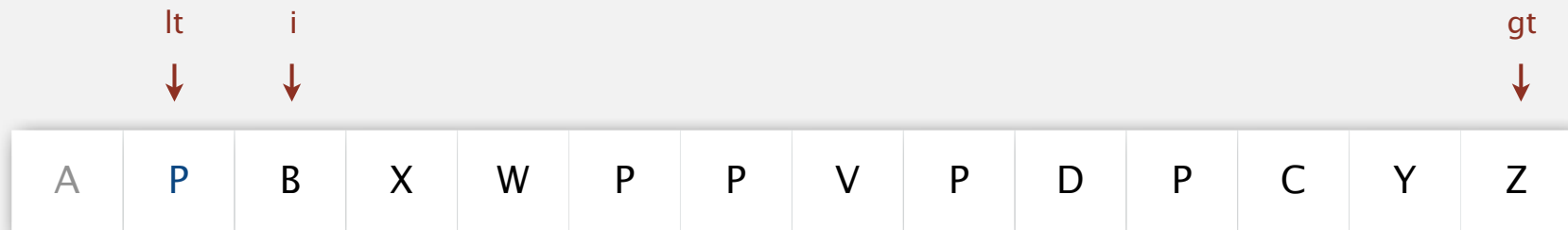


invariant

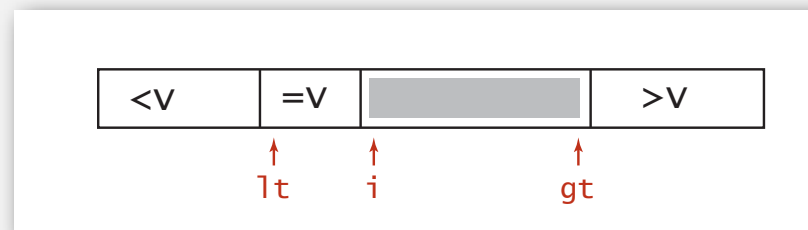


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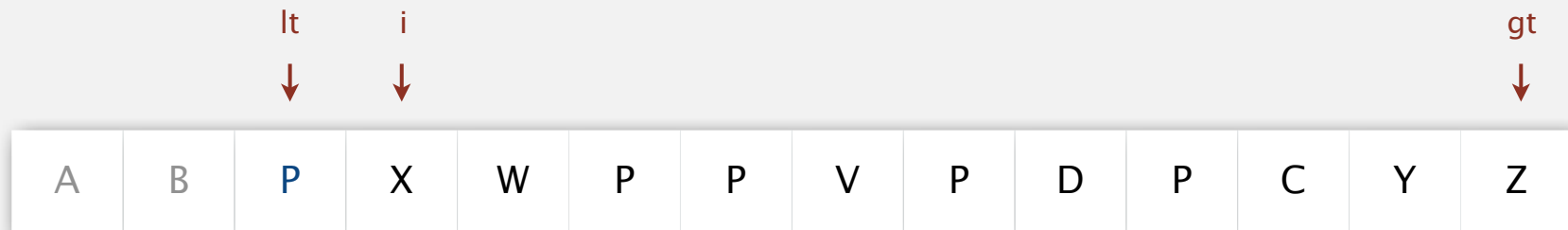


invariant

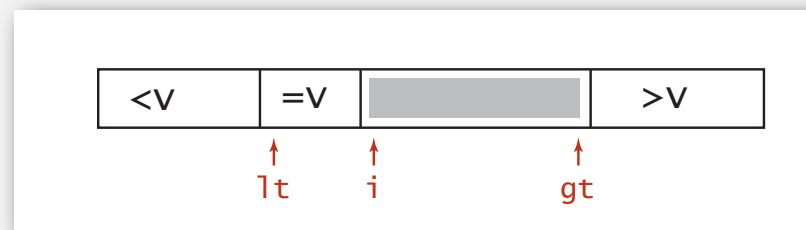


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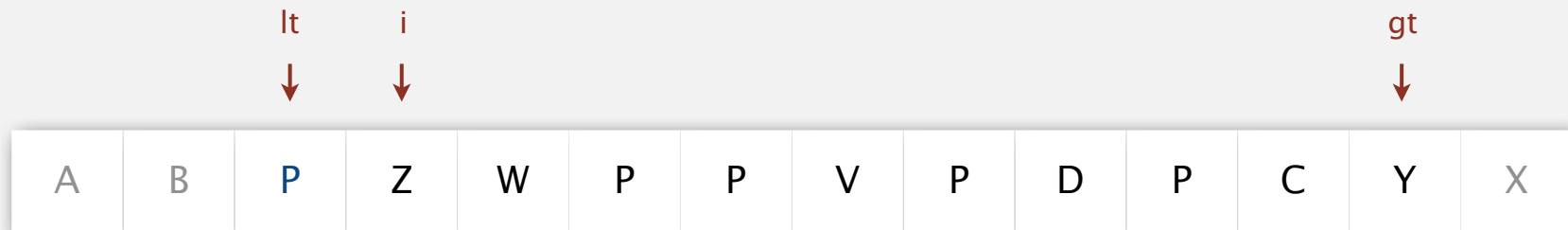


invariant

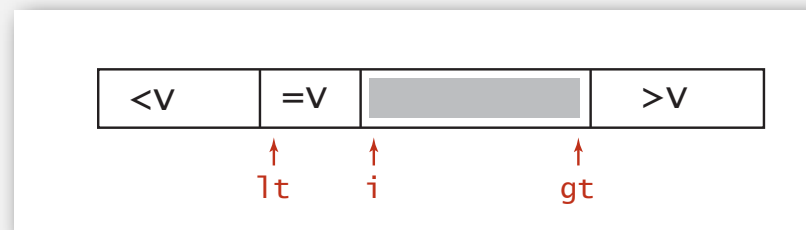


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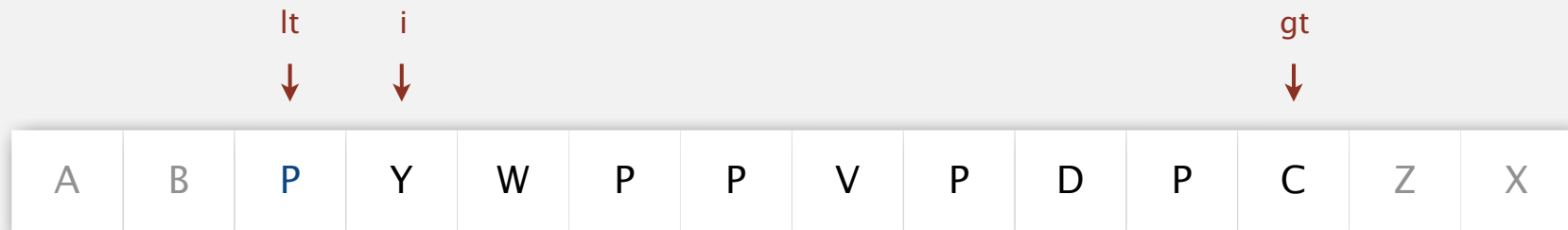


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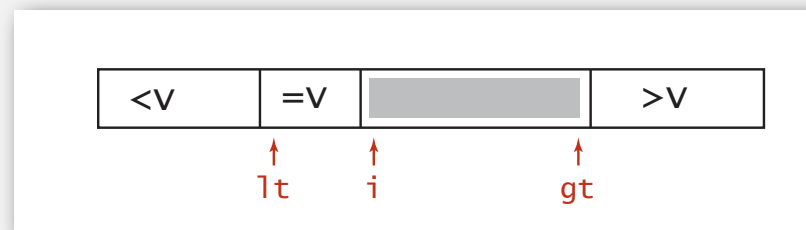


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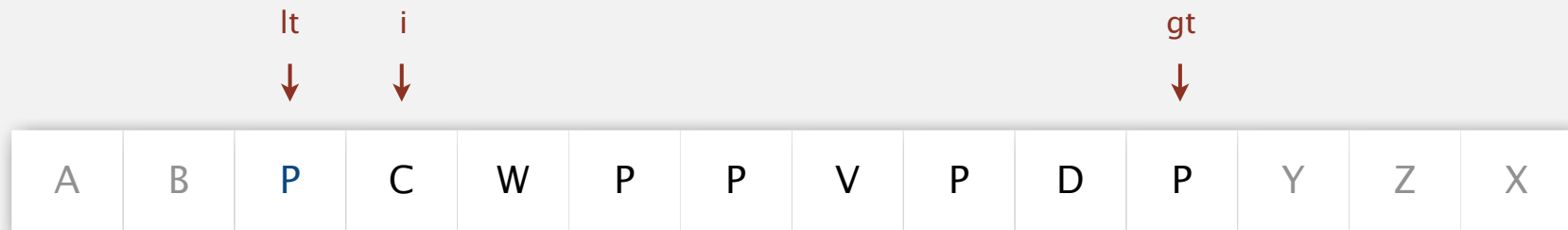


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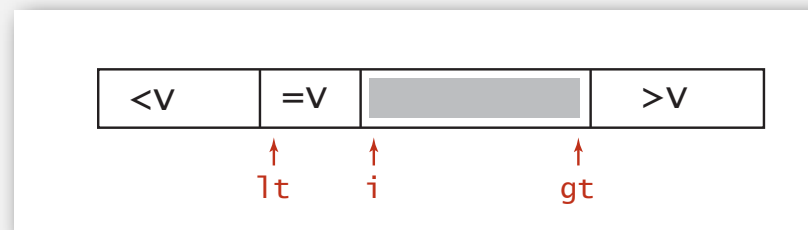


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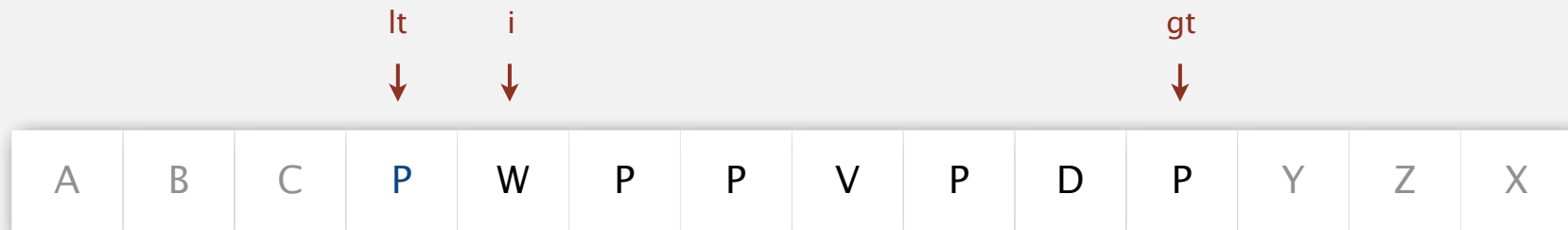


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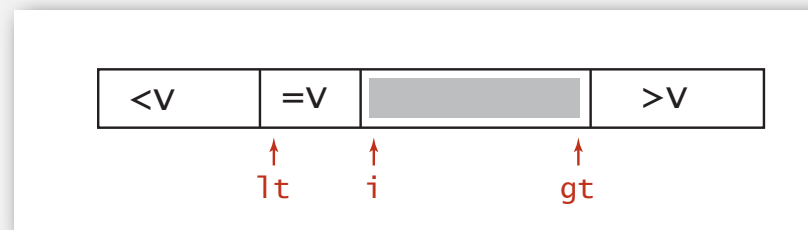


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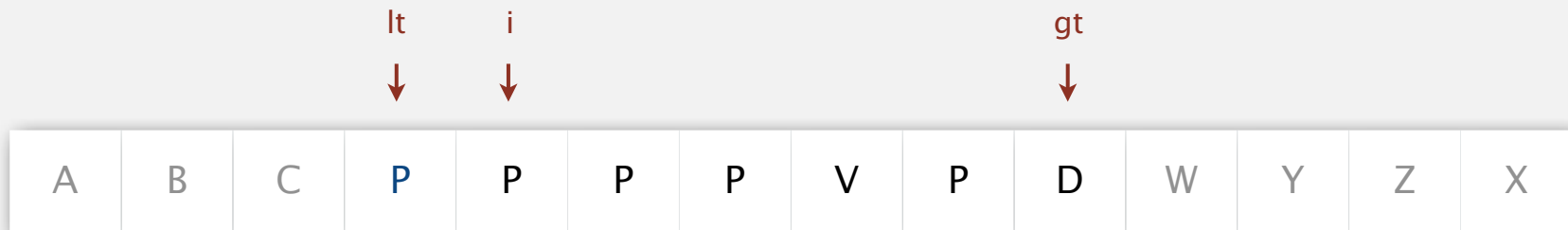


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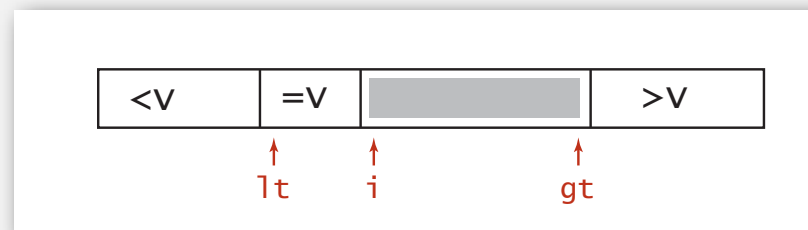


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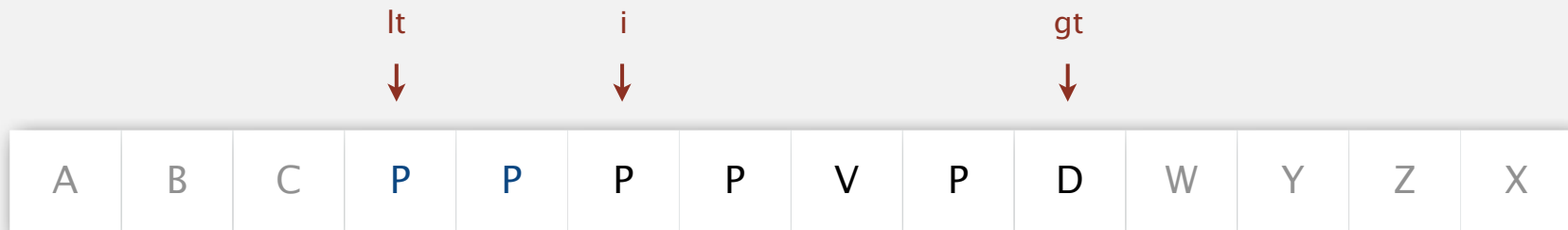


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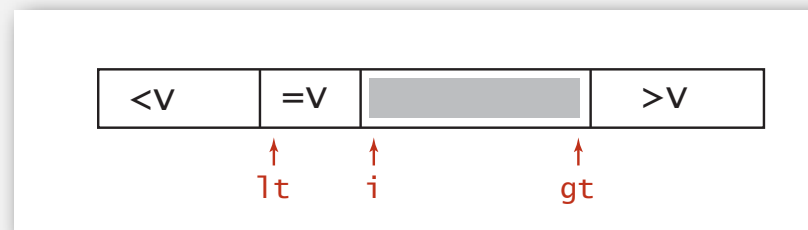


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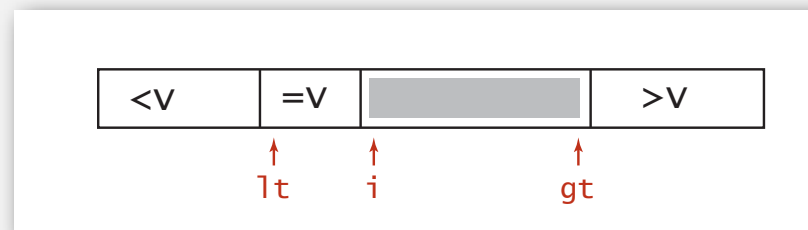


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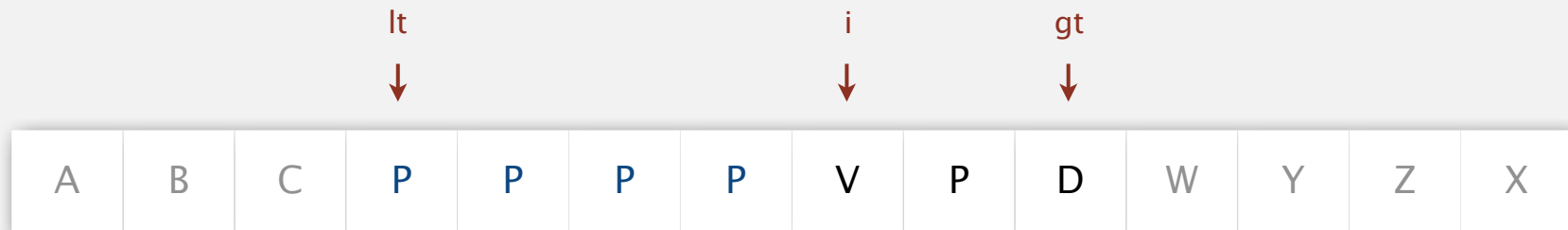


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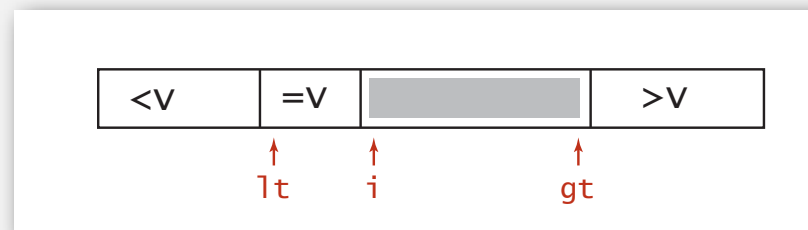


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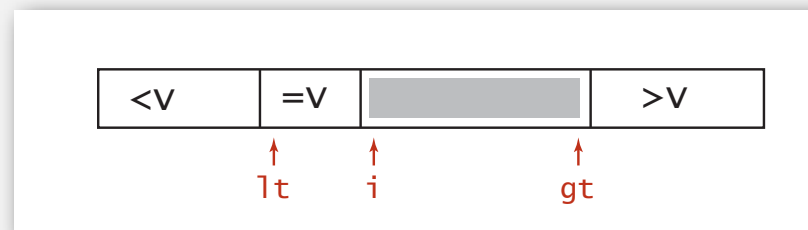


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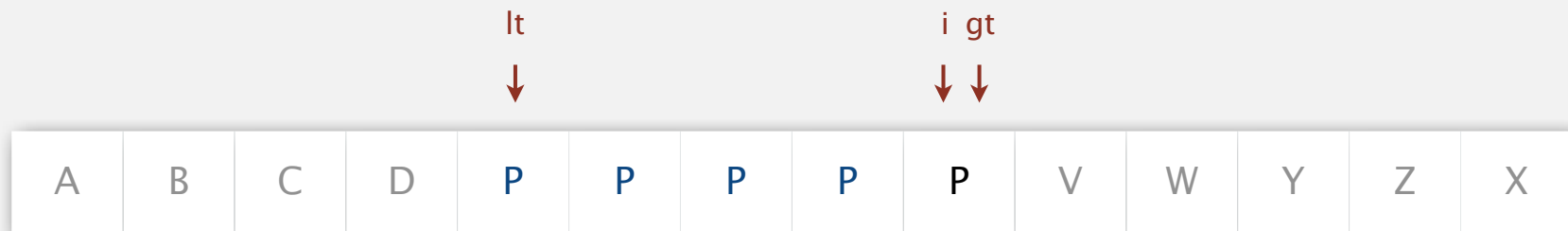


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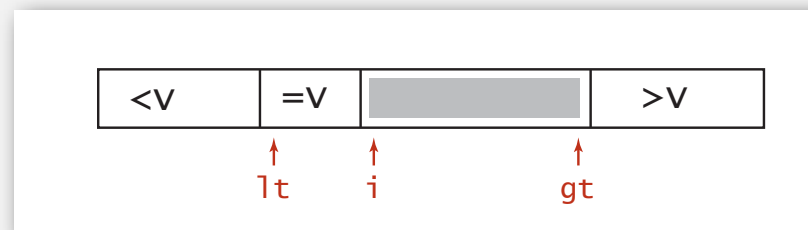


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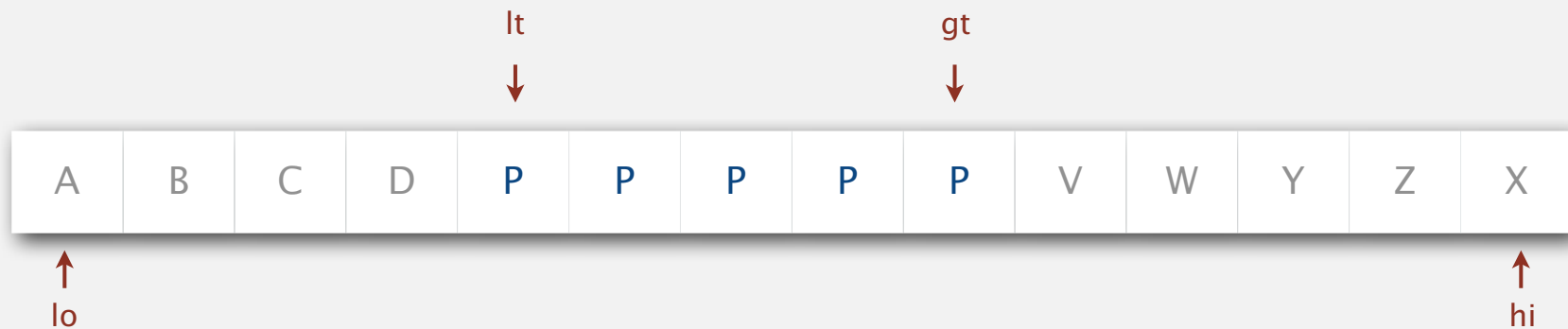


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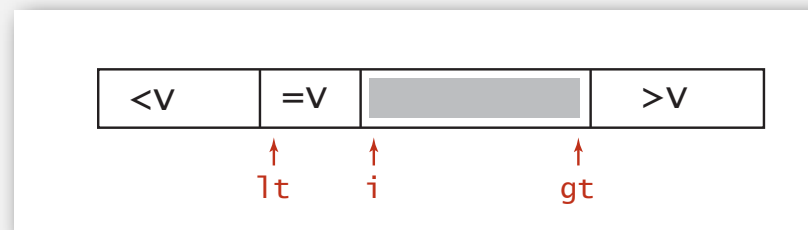


Dijkstra 3-way partitioning

- Let v be partitioning item $a[lo]$.
- Scan i from left to right.
 - ($a[i] < v$): exchange $a[l_t]$ with $a[i]$ and increment both l_t and i
 - ($a[i] > v$): exchange $a[g_t]$ with $a[i]$ and decrement g_t
 - ($a[i] == v$): increment i



invariant



BENTLEY-MCILROY 3-WAY PARTITIONING



Bentley-McIlroy 3-way partitioning

Repeat until i and j pointers cross.

- Scan i from left to right so long as $a[i] < a[lo]$.
- Scan j from right to left so long as $a[j] > a[lo]$.
- Exchange $a[i]$ with $a[j]$.
- If $a[i] == a[lo]$, exchange $a[i]$ with $a[p]$ and increment p .
- If $a[j] == a[lo]$, exchange $a[j]$ with $a[q]$ and decrement q .



Bentley-McIlroy 3-way partitioning

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Bentley-McIlroy 3-way partitioning

Repeat until i and j pointers cross.

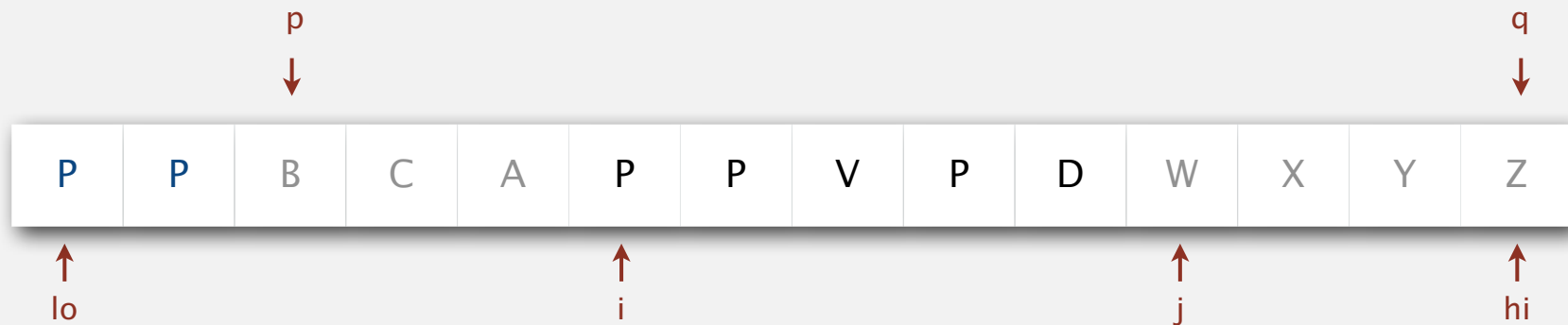
- Scan i from left to right so long as $a[i] < a[lo]$.
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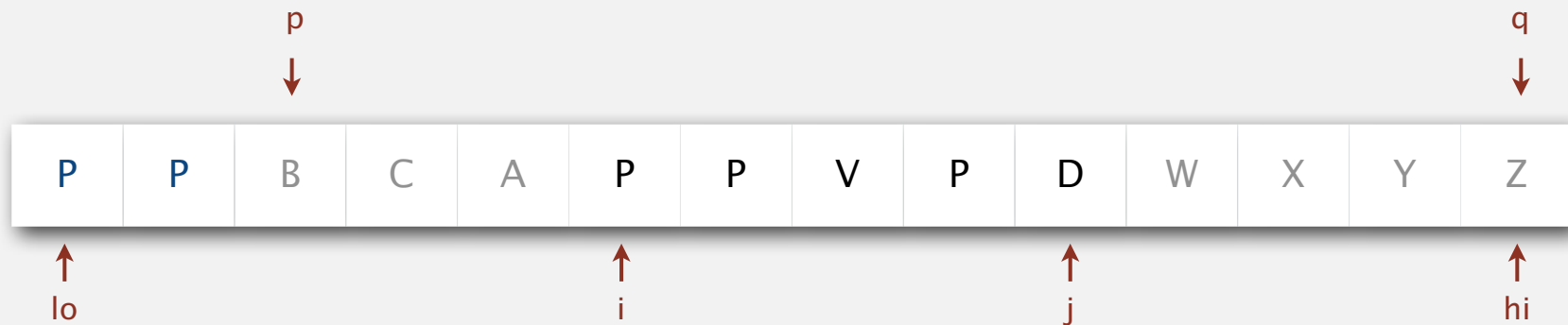
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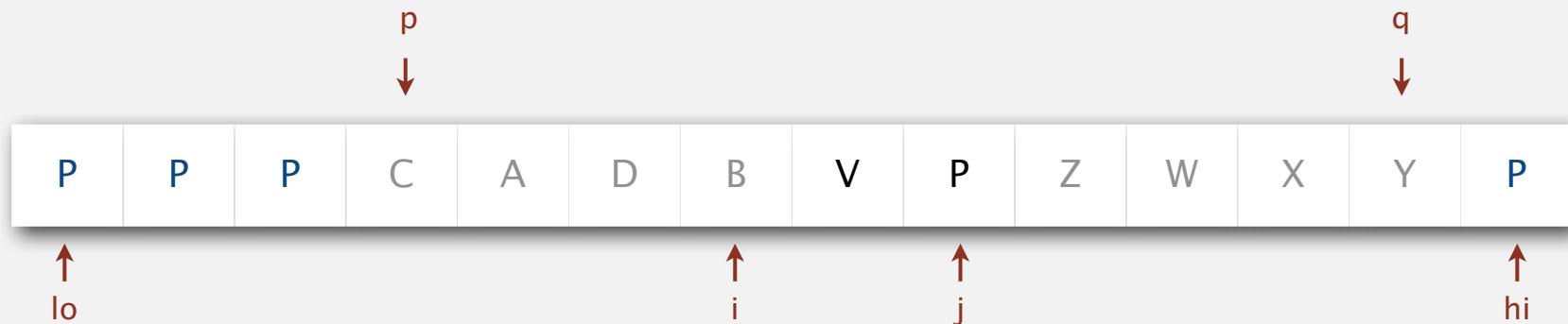


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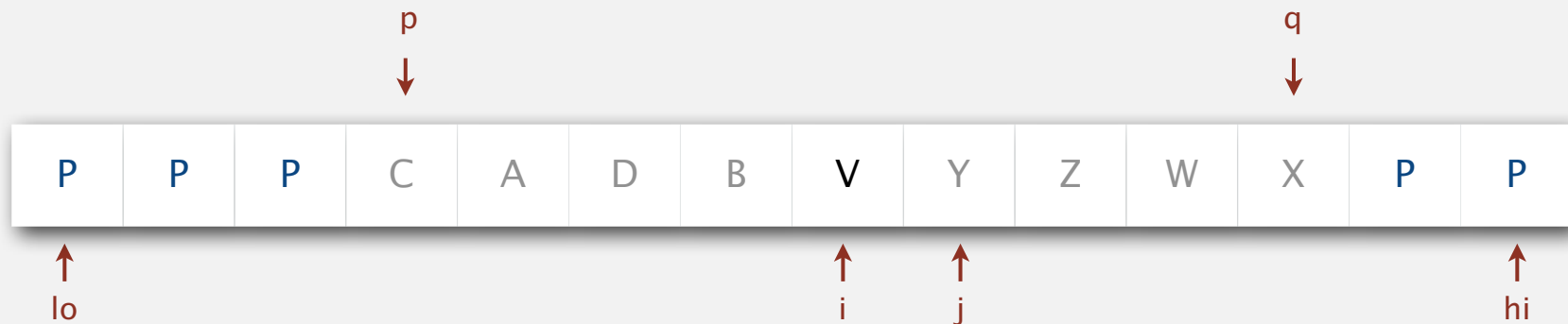
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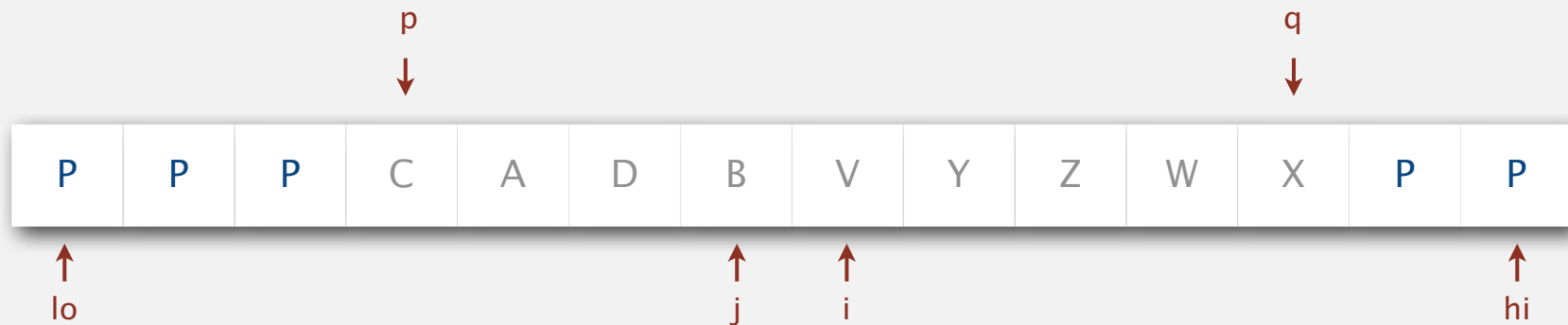
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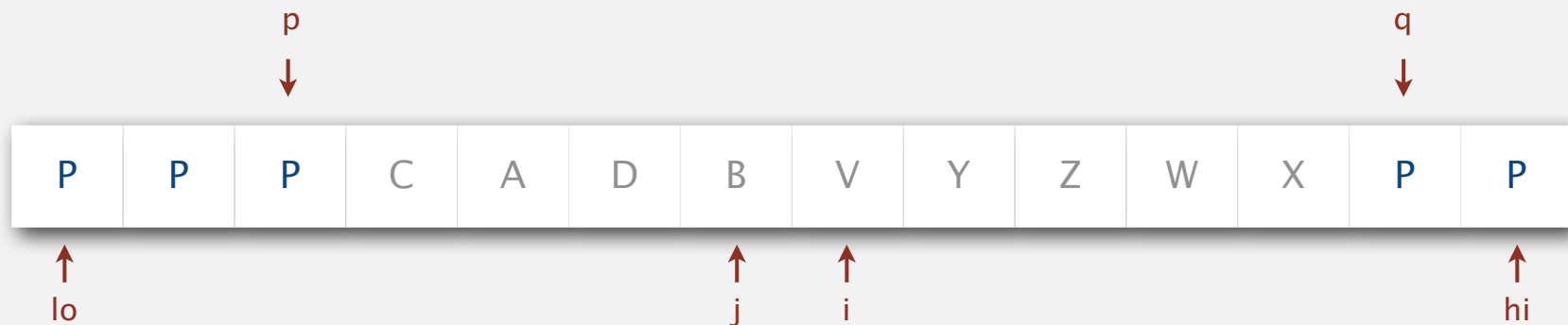


pointers cross

Bentley-McIlroy 3-way partitioning

Afterwards, swap equal keys to the center.

- Scan j and p from right to left and exchange $a[j]$ with $a[p]$.
- Scan i and q from left to right and exchange $a[i]$ with $a[q]$.

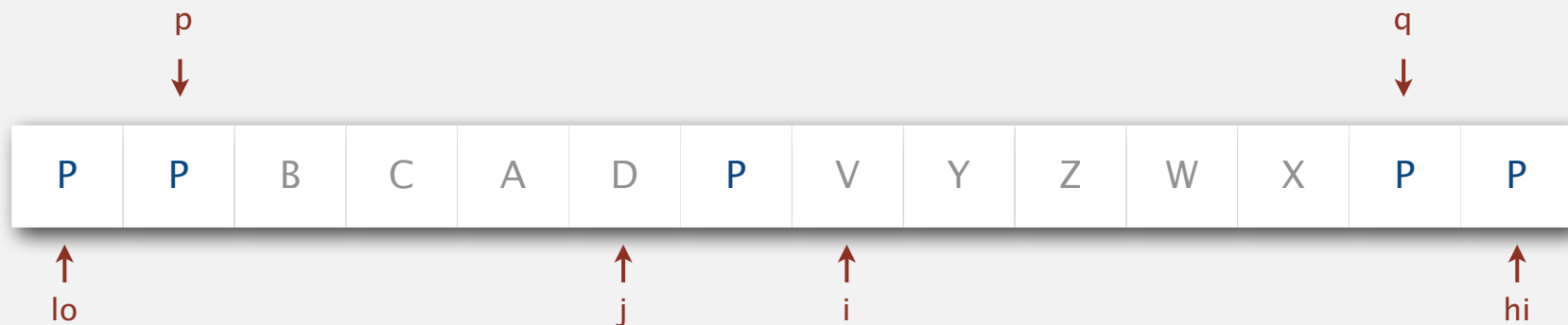


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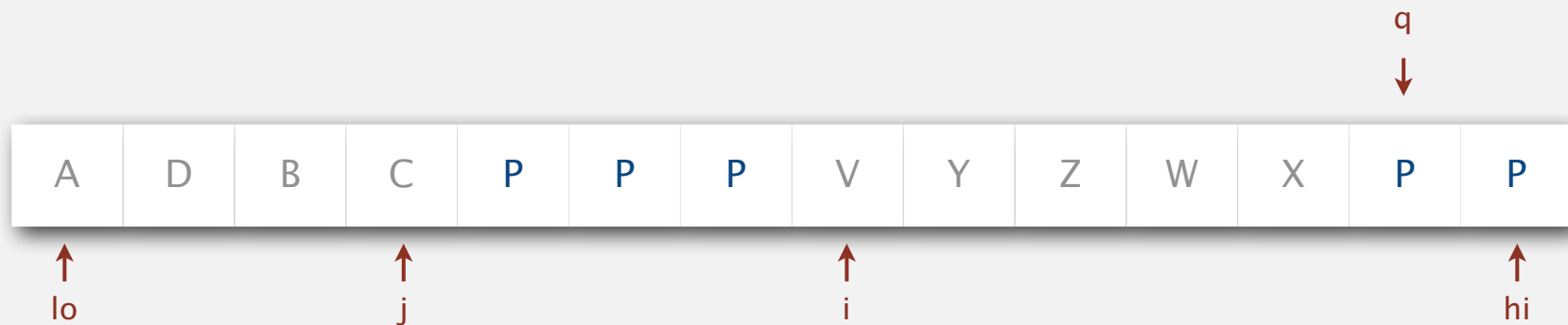


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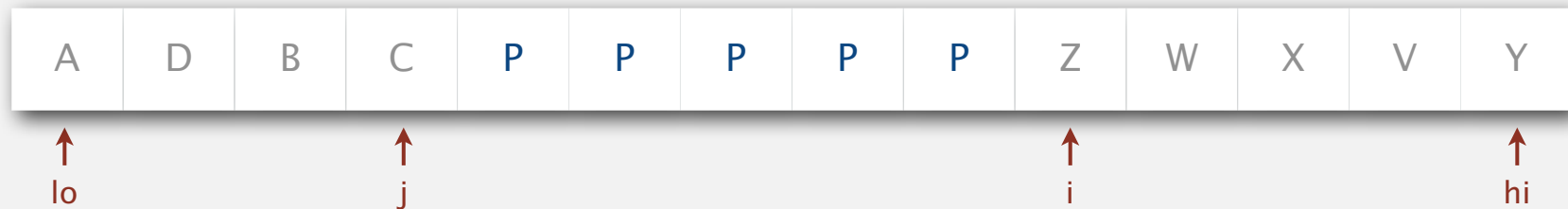


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3-way partitioned