

# Introduction to Shape Analysis

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## Motivation

Large repositories of 3D data are becoming available



Computer Graphics



Mechanical CAD



Molecular Biology



Medicine



Cultural Heritage



Computer Vision

## Lecture Outline

- Introduction
- Problems** ←
- Applications
- Simple example

## Shape Analysis Problems

Examples:

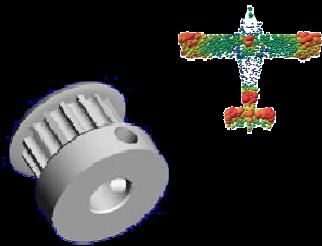
- Feature detection
- Segmentation
- Labeling
- Registration
- Matching
- Recognition
- Classification
- Clustering
- Retrieval

## Shape Analysis Problems

Examples:

### Feature detection

- Segmentation
- Labeling
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“How can we find significant geometric features robustly?”

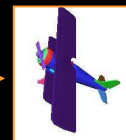
## Shape Analysis Problems

Examples:

- Feature detection
- Segmentation**
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Input Mesh



Part Decomposition

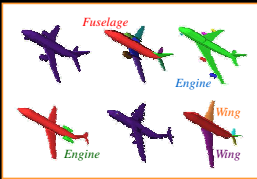
“How can we decompose a 3D model into its parts?”

## Shape Analysis Problems

Images courtesy of Ayellet Tal, Technion, Princeton University

Examples:

- Feature detection
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Semantic Labels  
(Golovinskiy, Lee, et al.)

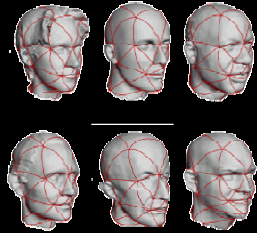
"How can we decompose a 3D model into its parts?"

## Shape Analysis Problems

Images courtesy of Emil Pratin

Examples:

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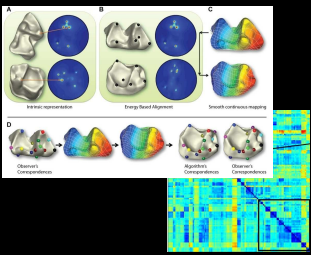
"How can we align features of 3D models?"

## Shape Analysis Problems

Images courtesy of [unintelligible]

Examples:

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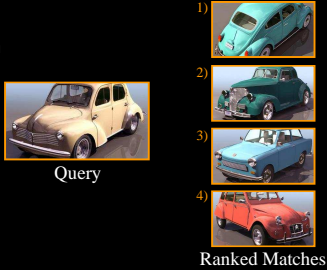
"How can we compute a measure of geometric similarity?"

## Shape Analysis Problems

Images courtesy of [unintelligible]

Examples:

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Query

Ranked Matches

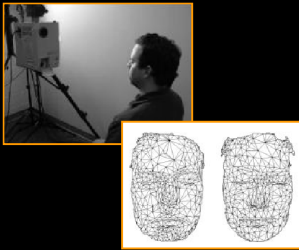
"How can we find 3D models best matching a query?"

## Shape Analysis Problems

Images courtesy of Florida State Univ.

Examples:

- Feature detection
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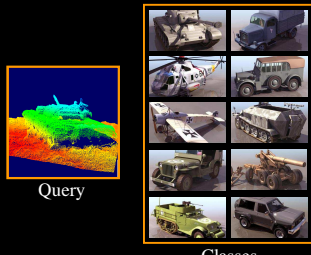
"How can we find a given 3D model in a large database?"

## Shape Analysis Problems

Images courtesy of Darpa E3D Project

Examples:

- Feature detection
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Query

Classes


"How can we determine the class of a 3D model?"

## Shape Analysis Problems

Images courtesy of Viewpoint

Examples:

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"How can we learn classes of 3D models automatically?"


## A Quick Diversion ...

Images courtesy of Georgia Tech and www.dreamhorse.com

Which is harder to analyze?



3D Model



2D Image

## Lecture Outline

Introduction

Problems

Applications ←

Simple example

## Shape Analysis Applications

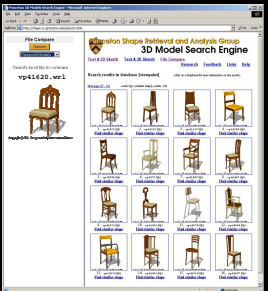
Examples:

- Computer graphics
- Geometric modeling
- Mechanical CAD
- Archaeology
- Virtual worlds
- Paleontology
- Molecular bio
- Medicine
- Forensics
- Art

## Shape Analysis Applications

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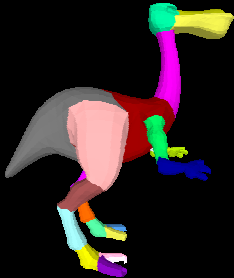


## Shape Analysis Applications

Image courtesy of Ayellet Tal, Technion and Princeton University

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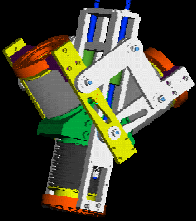


## Shape Analysis Applications

Images courtesy of Bill Roberg, Drexel University

Examples:

- Computer graphics
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- Art




## Shape Analysis Applications

Images courtesy of Weyrich, Brown, Rusinkiewicz, et al.

Examples:

- Computer graphics
- Geometric modeling
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- Art




Reconstructing Frescoes from Thera  
(Weyrich, Brown, Rusinkiewicz, et al.)

## Shape Analysis Applications

Images courtesy of Delson & Fraus

Examples:

- Computer graphics
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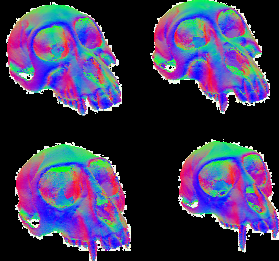


## Shape Analysis Applications

Images courtesy of Ilya Vakser, GRAM

Examples:

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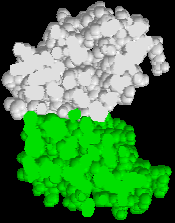


## Shape Analysis Applications

Image courtesy of Polina Golland, MIT

Examples:

- Computer graphics
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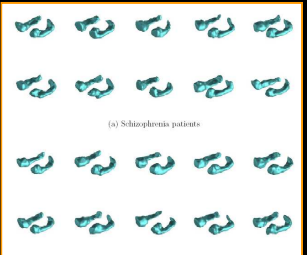


## Shape Analysis Applications

Image courtesy of Ilya Vakser, GRAM

Examples:

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Hippocampus-amygdala study in schizophrenia

## Shape Analysis Applications

Images courtesy of Boeing

Examples:

- Computer graphics
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- Mechanical CAD
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- Medicine
- **Forensics**
- Art




## Shape Analysis Applications

Images courtesy of Stanford University

Examples:

- Computer graphics
- Geometric modeling
- Mechanical CAD
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- Medicine
- Forensics
- **Art**



## Lecture Outline

Introduction


Problems

Applications





Simple example ←

## Simple Example

Shape-based retrieval:



Query

- 1) 
- 2) 
- 3) 
- 4) 

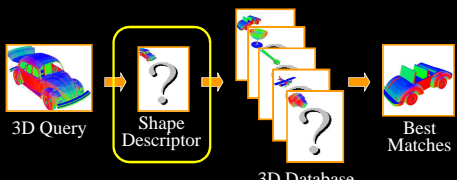
Ranked Matches

"How can we find 3D shapes best matching a query?"

## Shape Retrieval Challenges

Need shape descriptor & matching method that is:

- Concise to store
- Quick to compute
- Efficient to match
- Discriminating



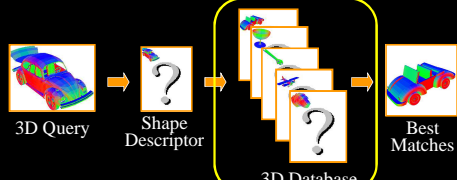
3D Query → Shape Descriptor → 3D Database → Best Matches

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3D Query → Shape Descriptor → 3D Database → Best Matches

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3D Query → Shape Descriptor → 3D Database → Best Matches

### Shape Retrieval Challenges

Need shape descriptor & matching method that is:

- Concise to store
- Quick to compute
- Efficient to match
- Discriminating
- **Invariant to transformations**
- Invariant to deformations
- Insensitive to noise
- Insensitive to topology
- Robust to degeneracies

Different Transformations (translation, scale, rotation, mirror)

### Shape Retrieval Challenges

Need shape descriptor & matching method that is:

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Different Articulated Poses

### Shape Retrieval Challenges

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Scanned Surface


Image courtesy of Ramamoorthi et al.

## Shape Retrieval Challenges


Images courtesy of Viewpoint & Stanford

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Different Genus




Different Tessellations

## Shape Retrieval Challenges


Images courtesy of Utah & De Espana

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No Bottom!




&\*Q?@#A%!

## Possible Shape Descriptors

Images courtesy of Amenta & Ossala

**Structural representations**

- Skeletons
- Part-based methods
- Feature-based methods



**Statistical representations**


- Attribute feature vectors
- Volumetric methods
- Surface-based methods
- View-based methods

## Possible Shape Descriptors

Images courtesy of Amenta & Ossala

**Structural representations**

- Skeletons
- Part-based methods
- Feature-based methods



**Statistical representations**

- Attribute feature vectors
- Volumetric methods
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- View-based methods

## Simple Method

Images courtesy of Amenta & Ossala

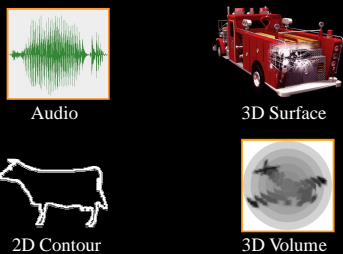
Shape distributions

- Shape representation: probability distributions
- Distance measure: difference between distributions

We are starting with discussion of a simple method to introduce the basic ideas

## Shape Distributions

Motivation: general approach to finding a common parameterization for matching



Audio      3D Surface

2D Contour      3D Volume

## Shape Distributions

Key idea: map 3D surfaces to common parameterization by randomly sampling shape function

3D Models → Randomly sample shape function → D2 Shape Distributions → Similarity Measure

## Which Shape Function?

Implementation: simple shape functions based on angles, distances, areas, and volumes

A3 (angle)    D1 (distance)    **D2 (distance)**    D3 (area)    D4 (volume)

[Ankerst 99]

## D2 Shape Distribution

Properties

- Concise to store?
- Quick to compute?
- Invariant to transforms?
- Efficient to match?
- Insensitive to noise?
- Insensitive to topology?
- Robust to degeneracies?
- Invariant to deformations?
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## D2 Shape Distribution

Properties

Concise to store?  
Quick to compute?

- Invariant to transforms?
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512 bytes (64 values)  
0.5 seconds ( $10^6$  samples)

## D2 Shape Distribution

Properties

Concise to store  
Quick to compute  
Invariant to transforms?

- Efficient to match?
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Translation  
Rotation  
Mirror  
Scale (w/ normalization)

Normalized Means

## D2 Shape Distribution

Properties

Concise to store  
Quick to compute  
Invariant to transforms  
Efficient to match?

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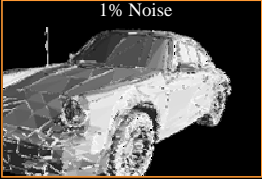


## D2 Shape Distribution

Properties

- Concise to store
- Quick to compute
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- Insensitive to noise?**
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- Robust to degeneracies?**

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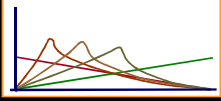


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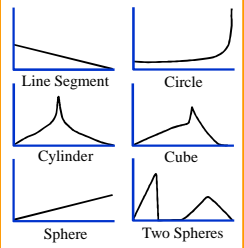


Ellipsoids with Different Eccentricities

## D2 Shape Distribution

Properties

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- ✗ Invariant to deformations**
- Discriminating?**



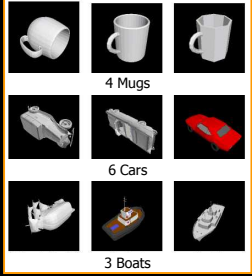
## D2 Shape Distribution Results

Question

- How discriminating are D2 shape distributions?

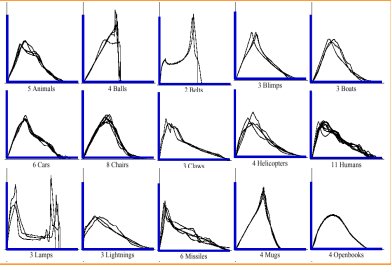
Test database

- 133 polygonal models
- 25 classes



## D2 Shape Distribution Results

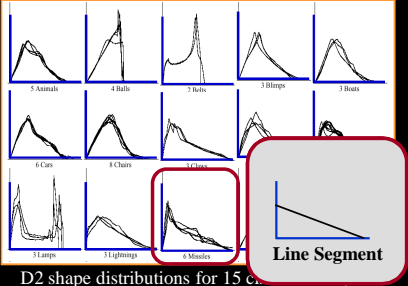
D2 distributions are different across classes



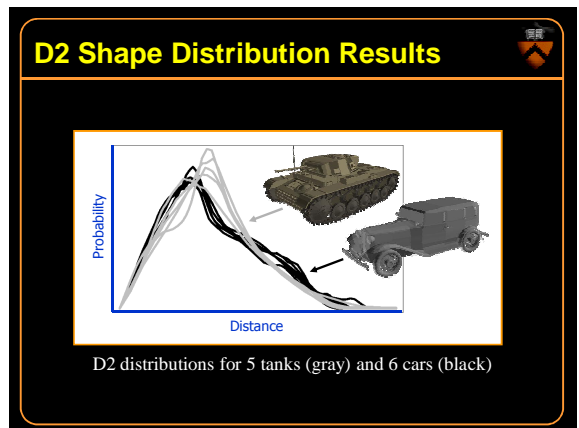
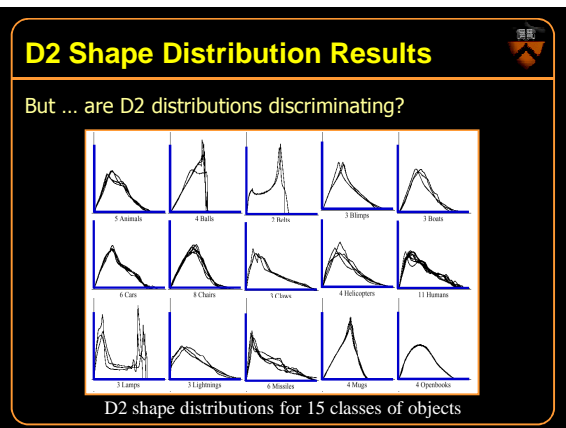
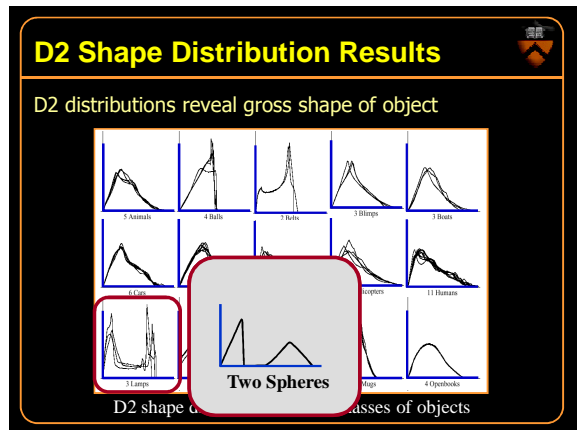
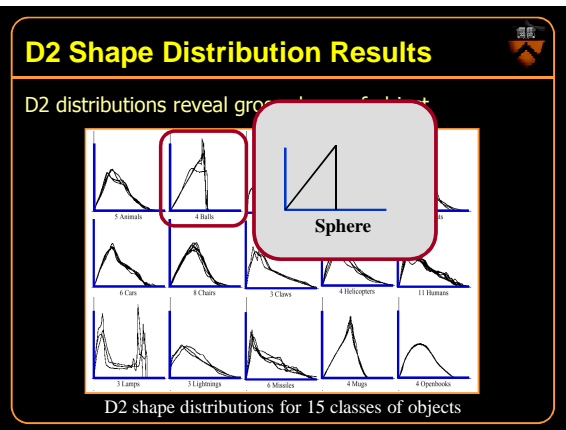
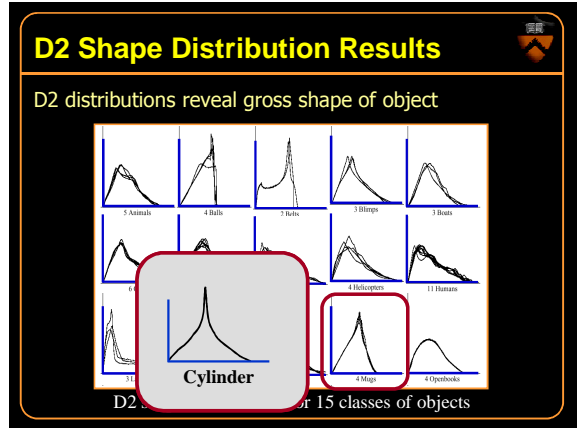
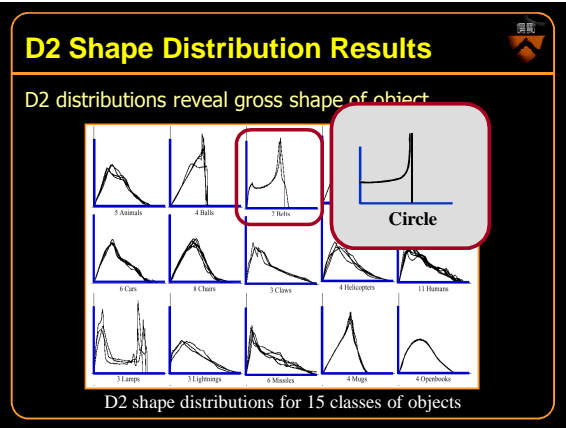
D2 shape distributions for 15 classes of objects

## D2 Shape Distribution Results

D2 distributions reveal gross shape of object



D2 shape distributions for 15 classes of objects



### D2 Shape Distribution Results

Similarity Matrix

- Darkness represents similarity

Blocks

- Tanks, cars
- Airplanes
- Humans
- Helicopters

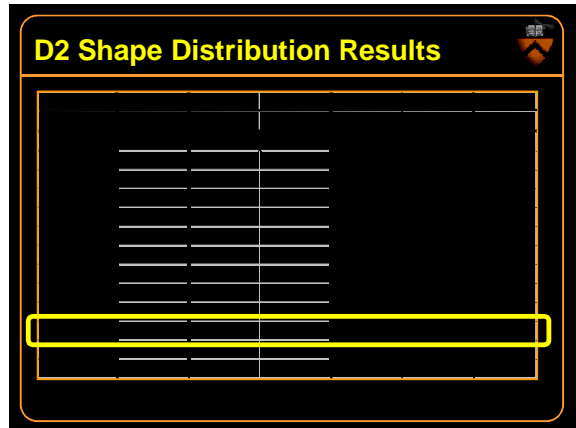
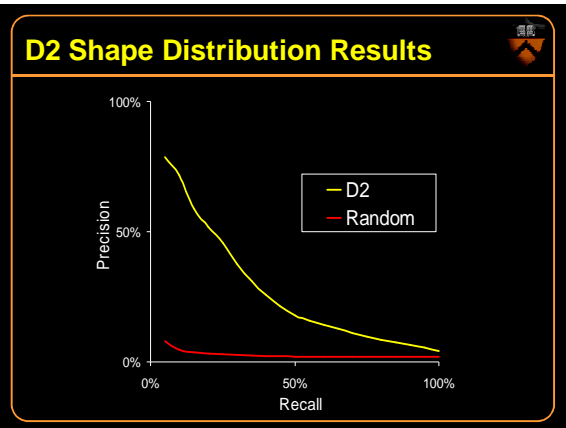
### D2 Shape Distribution Results

Princeton Shape Benchmark  
1814 classified models, 161 classes

51 potted plants    33 faces    15 desk chairs    22 dining chairs

100 humans    28 biplanes    14 flying birds    11 ships

<http://shape.cs.princeton.edu/benchmark>



### Next Lectures ...

Better shape representations  
More shape analysis methods