COS 487: Theory of Computation **Fall 2008**

Mid-Term Exam

Due: Tuesday, November 4 *Sanjeev Arora*

Instructions: Give yourself 5 hours to do the test. Write answers legibly in the space provided. If you need extra space for an answer, you may attach extra sheets, *which the instructor will read at his discretion.* You can consult any notes/handouts from this class as well as the text. You cannot consult another person or source in any way. You can however Feel free to quote, without proof, any results from class or the text.

DO NOT READ THE TEST BEFORE YOU ARE READY TO WORK ON IT.

Your Name:

Write and sign the honor code pledge[1] here:

Prof. Arora will hold office hours on Mon Nov 3 from 3 to 4pm and on Tues. Nov 4, from 2pm to 3pm. If you have questions during the exam you can try to reach him between 10am and 10pm at 609-356-9076 or Aditya Bhaskara at ..... Both will also check email and answer it as quickly as possible.

[1]The pledge is "I pledge my honor that I have not violated the honor code during this exam."

Question 1 (25 points) This question lists five languages over $\{0, 1\}$. You have to classify each in to one of the following classes: *Class 1:* Regular. *Class 2:* Context-free but not regular. *Class 3:* Decidable but not context-free. *Class 4:* Recognizable but not Decidable. *Class 5:* Not recognizable.

If x is an integer, let $[x]_2$ denote its binary representation. Classify each of the languages as described above. **Give brief explanations**.

(i) (5 points) $\{[2^p]_2 \ : \ p \text{ is a prime number}\}$.

(ii) (5 points) $L = \{0^p \ : \ p \text{ is a prime}\}$.

(iii) (5 points) L^* where L was defined in question (ii).

(iv) (5 points) $\{[n^2]_2 \ : \ n \geq 0\}$ (mild hand-waving allowed).

(iv) (5 points) Let M be a Turing machine. The language $\{[n]_2 : \ M$ accepts at least one string of length $\geq n\}$.

Question 2 (30 points) Say whether the following statements are true or false, and justify your answer in a few lines. No credit will be given if the justification is wrong.

ADITYA: I SUGGEST REMOVING PART (I) AND JUST MAKE THIS QUESTION ABOUT THE JUMBLE OPERATOR.

(i) (5 points) If L is any context-free language, then the following language is context-free: (recall that w^R is just w written backwards)

$$L^R = \left\{ w^R : w \in L \right\}.$$

(ii) (5 points) For a language L, define

$$\mathsf{Jumble}(L) = \{x_1 x_3 \dots x_{2n-1} x_{2n} x_{2n-2} \dots x_2 \mid x_1 x_2 \dots x_{2n} \in L\}$$

If L is any regular language, then $\mathsf{Jumble}(L)$ is regular.

(iii) (10 points) If L is regular, $\mathsf{Jumble}(L)$ is context-free.

(iv) (10 points) If L is context-free, $\mathsf{Jumble}(L)$ is context-free.

Question 3 (20 points) Let $L = \{\langle M \rangle \mid M \text{ halts on every input}\}$. Show that L is not recursively enumerable. *(Hint):* Suppose there is a machine that enumerates L and try using a diagonalization argument to construct a machine M that does not occur in the enumeration.

Question 4 (25 points) For a language L, let $L_{\frac{1}{2}-}$ be the set of all first halves of strings in L. In particular

$$L_{\frac{1}{2}-} = \{x \mid xy \in L \text{ for some } y \text{ with } |x| = |y|\}$$

(i) (5 points) Suppose L is decidable. Is $L_{\frac{1}{2}-}$ decidable? Explain briefly.

(ii) (5 points) Suppose L is enumerable. Is $L_{\frac{1}{2}-}$ enumerable? Explain briefly.

(iii) (15 points) Is the following language decidable? Prove your answer.

$$\{\langle M\rangle \ : \ M \text{ is a TM and } A_{\frac{1}{2}-} = \Sigma^*, \text{ where } A \text{ is } L(M)\}$$

THAT'S ALL FOLKS. GOOD LUCK. NEXT QUESTION IS OPTIONAL.

Question 5 (Optional) (25 points; you can work on this question after the alloted 4 hours)

This question refers to the notion of oracle reducibility. If A is a language, an *oracle* for A is a device that whenever it is given a string x replies YES if $x \in A$ and NO otherwise. If M is a Turing machine then M^A is the same machine that is allowed to use an oracle for A during its computation.

Suppose A is a language such that some Turing machine M needs just one query to oracle A to do the following: Given two strings x_1, x_2, decide for each whether or not it is in A.

Then show that A is decidable.