COS 487: Theory of Computation **Fall 2010**

Assignment #5

Due: Monday, Dec 6 6PM *Sanjeev Arora*

Suggested reading: Arora Barak Chapter 3,4.

Additional reading: Sipser Chapter 8.

Collaboration Policy

You are allowed to collaborate with other people enrolled in this class. If you solved a particular problem in collaboration with somebody else, please mention the collaborator(s) name.

It is a violation of class rules to look at solutions to any of the problems from any other person or source, including online ones.

Problems:

1. Arora Barak 3.1

   Show the following language is undecidable.

   $$\{\langle M\rangle : M \text{ is a machine that runs in } 100n^2 + 200 \text{ time}\}\ .$$

2. Arora Barak 3.2

   Show that $\mathbf{SPACE(n)} \neq \mathbf{NP}$. (Note that we do not know if either class is contained in the other).

3. Arora Barak 4.2

   Prove that SPACETM is **PSPACE**-complete.

   $$SPACETM = \{\langle M, w, 1^n\rangle : \text{DTM } M \text{ accepts } w \text{ in space } n\}\ .$$

4. Arora Barak 4.5

   Show that $2SAT \in \mathbf{NL}$.

5. Sipser 8.13

6. Sipser 8.23

7. Consider the following game.

   A farmer wants to move n different animals across a river in a boat. However, some of these animals cannot coexist peacefully on a boat. A graph is given with an edge between two animals if they are incompatible. The farmer wants to ship as many of his animals as is possible in one shipment.

The captain of the ship is very concerned about having too many animals on the boat, even if they are peaceful – he does not want them to damage the boat or make a mess.

The farmer and the captain agree to negotiate which animals will be shipped in the following way. First, the animals are divided into an even number of groups, say by a neutral third party. The groups are also given a fixed ordering. Then, the negotiations begin, with the farmer going first. On the farmer's turn, the farmer considers the first group and the second group, and chooses a subset of one of the groups, which the captain will agree to ship. On the captain's turn, the captain considers groups three and four, and chooses one of them, which he vetoes and will not ship. The farmer then chooses a subset of the other group, which the captain will agree to ship. They continue counting off the groups in pairs this way and taking turns until all the groups have been considered.

It is against the rules of the game for the farmer to attempt to ship two incompatible animals.

Show that the following problem is PSPACE-complete:

Given the compatability graph for the animals, a fixed bundling of the animals into groups, a fixed ordering of the groups, and a quota k, determine if the farmer can now negotiate in such a way as to guarantee that at least k animals are shipped.

8. Show the above problem remains PSPACE-complete even if no group is of size more than 2.