COS 487: Theory of Computation **Fall 2010**

Assignment #3

Due: Friday, Oct 29 6PM *Sanjeev Arora*

Suggested reading: Sipser Chapters 4, 5, 6.

Collaboration Policy

You are allowed to collaborate with other people enrolled in this class. If you solved a particular problem in collaboration with somebody else, please mention the collaborator(s) name.

It is a violation of class rules to look at solutions to any of the problems from any other person or source, including online ones.

Problems:

1. Show that the Post Correspondence Problem (see section 5.2), over a unary alphabet, is decidable.

2. Let $J = \{w \in \Sigma^* |$ either $w = 0x$ for some $x \in A_{TM}$, or $w = 1y$ for some $y \in \overline{A_{TM}}\ \}$. Show that neither J nor $\overline{J}$ is Turing recognizable.

3. Suppose the Church Turing Thesis is false, and some advanced civilization has figured out how to solve the halting problem for TMs. They wish to send a brief message to the rest of the universe allowing living beings everywhere to solve the halting problem. Imagine that the set of valid inputs for the halting problem are numbered $1, 2, 3, \ldots$ and everybody agrees about this numbering. Show that the length of the message needed to decide the halting problem on the first i inputs is only $\lceil \log i \rceil$ (for large enough i).

4. Prove that a language L is recursively enumerable if and only if it can be expressed as

$$L = \{x| \text{ there exists } y \text{ such that } \langle x, y\rangle \in R\ \}$$

   where R is a decidable language. In other words, you must prove that every language of this form is RE, and that every RE language has a related decidable language R that allows it to be expressed exactly as above.

5. (Rice's Theorem)

   Let P be a nontrivial property of the language of a Turing Machine. Prove that the property of determining whether a TM's language has property P is undecidable.

   More formally, let P be a language consisting of TM descriptions, where P fulfills two conditions. First, P is nontrivial – it contains some, but not all TM descriptions.

Second, P is a property of the $l$anguage – if $L(M_1) = L_(M_2)$ then $\langle M_1\rangle \in P$ iff $\langle M_2\rangle \in P$.

Prove that P is undecidable.

6. Using Rice's Theorem, prove the undecidability of the following languages:

   (a) $INFINITE_{TM} = \{\langle M\rangle \mid M$ is a TM and $L(M)$ is an infinite language $\}$.

   (b) $\{\langle M\rangle \mid M$ is a TM and $1011 \in L(M)$ $\}$.

   (c) $ALL_{TM} = \{\langle M\rangle \mid M$ is a TM and $L(M) = \Sigma^*$ $\}$.

7. Describe how you could write a Java program which, in the spirit of the Recursion Theorem (6.3), prints out its own source code.

8. Show that the function $K(x)$ (see definition 6.28) is uncomputable.