Assignment #2

Suggested reading: Sipser Chapters 2,3.

Collaboration Policy

You are allowed to collaborate with other people enrolled in this class. If you solved a particular problem in collaboration with somebody else, please mention the collaborator(s) name.

It is a violation of class rules to look at solutions to any of the problems from any other person or source, including online ones.

Problems:

1. Prove that the grammar in Exercise 2.9 is inherently ambiguous.

2. Prove that $F = \{a^i b^j \mid i = k \cdot j$ for some $k \geq 1 \}$ is not context free.

3. Let $A = \{wtw^R | w, t \in \{0, 1\}^*$ and $|w| = |t|\}$. Prove that A is not a context free language.

4. Let C be context-free, R be regular. Prove that $C \cap R$ is context-free.

5. Prove that the set of CFL's is not closed under intersection.

6. Prove that the set of CFL's is not closed under complementation.

7. Say that a language is *prefix-closed* if any prefix of any string in the language is also in the language.

   Let C be an infinite, prefix-closed, context-free language. Prove that C contains an infinite regular subset.

8. Prove that the set of decidable languages is closed under each of the regular operations, i.e., union, concatenation, complementation, intersection, and star.

9. Prove that single-tape Turing Machines that cannot write on the portion of the tape containing the input string can recognize only regular languages.

10. Prove that a language is decidable if and only some enumerator enumerates the language in lexicographic order.