

Assignment #1

*Due: Friday, Oct 1**Sanjeev Arora*

Suggested reading (for lectures 1,2,3): Sipser Chapter 1.

A hint for this assignment: keep in mind the properties of regular languages. For instance, if you are trying to show that a language L is regular, it suffices to show that $\bar{L}$ is accepted by a nondeterministic automaton.

Collaboration Policy

You are allowed to collaborate with other people enrolled in this class. If you solved a particular problem in collaboration with somebody else, please mention the collaborator(s) name.

It is a violation of class rules to look at solutions to any of the problems from any other person or source, including online ones.

Problems (from lectures 1, 2, 3):

1. Problem 1.31
2. Let L be a regular language. Show that the language L' is also regular, where

$$L' = \{x : \text{no } w \in L \text{ is a substring of } x\}.$$

3. Problem 1.60
4. Problem 1.61
5. Suppose we augment the DFA model by allowing each state to have one epsilon arrow. Call such an automaton an ϵ -DFA. Such an automaton computes as an NFA does, but formally the transition function is of the form $\delta : Q \times \Sigma_\epsilon \rightarrow Q$, rather than to $\mathcal{P}(Q)$ as in an NFA.

Show that any NFA on n states can be converted to an ϵ -DFA with $\mathbf{O}(n^2)$ states.

6. The pumping lemma (Theorem 1.70) states that, if A is a regular language, then there exists a number p so that, for each string $s \in A$ of length at least p , there exist strings x, y, z so that
 - (a) $s = xyz$
 - (b) $|y| > 0$

(c) $|xy| \leq p$

(d) For each $i \geq 0$, $xy^iz \in A$.

Is the converse true? Prove, or give a counter example.