

# COS 126

## General Computer Science Fall 2010

Robert Sedgewick

What is COS 126? Broad, but technical, introduction to **computer science**.

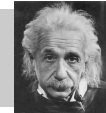
Goals.

- Demystify computer systems.
- Empower you to exploit available technology.
- Build awareness of substantial intellectual underpinnings.

Topics.

- **Programming** in Java.
- Machine architecture.
- Theory of computation.
- **Applications** to science, engineering, and commercial computing.

“Computers are incredibly fast, accurate, and stupid; humans are incredibly slow, inaccurate, and brilliant; together they are powerful beyond imagination.” – Albert Einstein



### The Basics

■ Lectures. [Sedgewick]

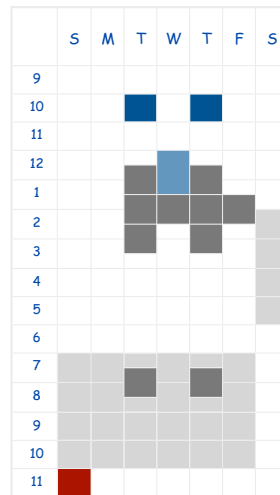
■ RS office hours. ← everyone needs to meet me!

■ Precepts. [Gabai · Ginsburg · Vertanen · Lee · Lee · Nagorna · Steiglitz · Jones · Stewart · Zhu ]

- Tips on assignments / worked examples
- Questions on lecture material.
- Informal and interactive.

■ Friend 016/017 lab. [Ugrad assistants]

- Help with systems/debugging.
- No help with course material.



Reading period. No lectures; precepts Jan 4-5.

See [www.princeton.edu/~cos126](http://www.princeton.edu/~cos126) for full details and preceptor office hours.

### Grades

Course grades. No preset curve or quota.

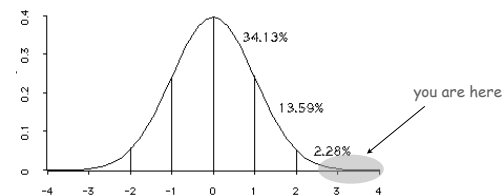
9 programming assignments. 40%.

2 exams (in class, 10/21-22 and 12/16-17). 50%.

Final programming project (due Dean's date - 1). 10%.

Extra credit / staff discretion. Adjust borderline cases.

participation helps, frequent absence hurts



Due dates

	Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3	4
SEP	5	6	7	8	9	10	11
	12	13	14	15	16	17	18
	19	20	21	22	23	24	25
	26	27	28	29	30		
						1	2
OCT	3	4	5	6	7	8	9
	10	11	12	13	14	15	16
	17	18	19	20	21	22	23
	24	25	26	27	28	29	30
	31						
		1	2	3	4	5	6
NOV	7	8	9	10	11	12	13
	14	15	16	17	18	19	20
	21	22	23	24	25	26	27
	28	29	30				
				1	2	3	4
DEC	5	6	7	8	9	10	11
	12	13	14	15	16	17	18
	19	20	21	22	23	24	25
	26	27	28	29	30	31	
							1
JAN	2	3	4	5	6	7	8
	9	10	11	12	13	14	15
	16	17	18	19	20	21	22
	23	24	25	26	27	28	29
	30	31					

## Course Materials

Course website. [[www.princeton.edu/~cos126](http://www.princeton.edu/~cos126)]

- Submit assignments, check grades.
- Programming assignments.
- Lecture slides.

← print before lecture;  
annotate during lecture

← skim before lecture;  
read thoroughly afterwards

Required readings. Sedgewick and Wayne. Intro to Programming in Java: An Interdisciplinary Approach.



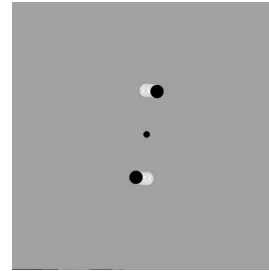
Recommended readings. Harel. What computers can't do.

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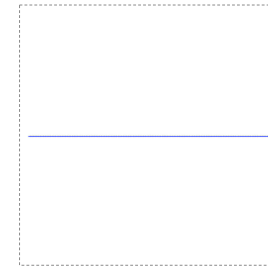
## Programming Assignments

Desiderata.

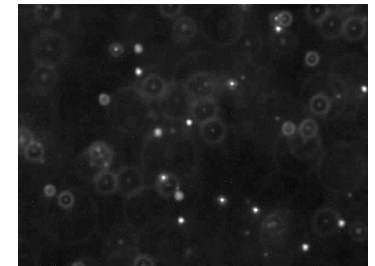
- Address an important scientific or commercial problem.
- Illustrate the importance of a fundamental CS concept.
- You solve problem from scratch!



N-body simulation



pluck a guitar string



estimate Avogadro's number

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## Programming Assignments

Desiderata.

- Address an important scientific or commercial problem.
- Illustrate the importance of a fundamental CS concept.
- You solve problem from scratch!

Due. Mondays 11pm via Web submission.

Computing equipment.

- Your laptop. [OS X, Windows, Linux, iPhone, ... ]
- OIT desktop. [Friend 016 and 017 labs]

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## What's Ahead?

Lecture 2. Intro to Java.

Precept 1. Meets today/tomorrow.

Not registered? Go to any precept now; officially register ASAP.

Change precepts? Use SCORE.

← see Colleen Kenny-McGinley in CS 210  
if the only precept you can attend is closed

Assignment 0. [[www.princeton.edu/~cos126/assignments.php](http://www.princeton.edu/~cos126/assignments.php)]

- Due Monday 11PM.
- Read Sections 1.1 and 1.2 in textbook.
- Install Java programming environment + a few exercises.
- Lots of help available, don't be bashful.

END OF ADMINISTRATIVE STUFF

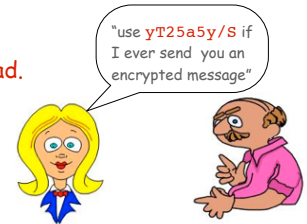
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# 0. Prologue: A Simple Machine

## Secure Chat with a One-Time Pad

Alice wants to send a secret message to Bob

- Sometime in the past, they exchange a **one-time pad**.
- Alice uses the pad to encrypt the message.



```
Secure Chat 1.0 [alice]
[alice]: Hey, Bob
[bob]: Hi, Alice!
[alice]: SENDMONEY

Encrypt SENDMONEY with yT25a5y/S
```

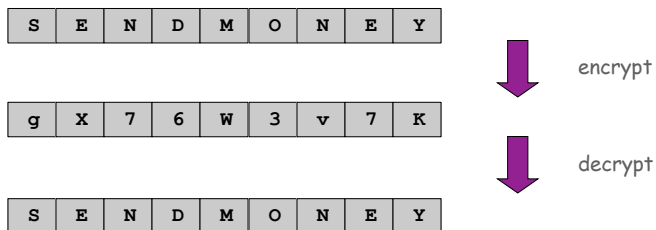
```
Secure Chat 1.0 [bob]
[alice]: Hey, Bob
[bob]: Hi, Alice!
[alice]: gX76W3v7K
```

Key point: Without the pad, Eve cannot understand the message.



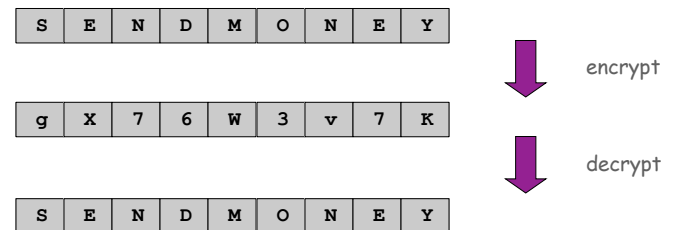
### Encryption Machine

Goal. Design a machine to encrypt and decrypt data.



### Encryption Machine

Goal. Design a machine to encrypt and decrypt data.



Enigma encryption machine.

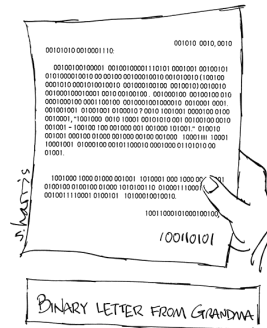
- "Unbreakable" German code during WWII.
- Broken by Turing bombe.
- One of first uses of computers.
- Helped win Battle of Atlantic by locating U-boats.



Data is a sequence of bits. [bit = 0 or 1]

- Text.
- Programs, executables.
- Documents, pictures, sounds, movies, ...

thousands of bits



Copyright 2004, Sidney Harris  
http://www.sciencecartoonsplus.com

billions of bits

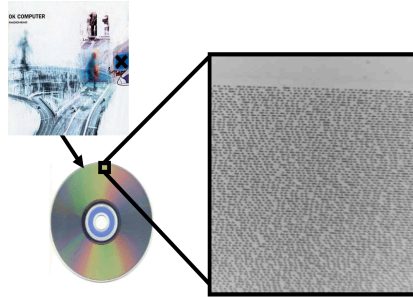


image courtesy of David August

Data is a sequence of bits. [bit = 0 or 1]

- Text.
- Programs, executables.
- Documents, pictures, sounds, movies, ...

Ex. Base64 encoding of text.

- Simple method for representing **A-Z, a-z, 0-9, +, /**
- 6 bits to represent each symbol (64 symbols)

000000	A	001000	I	010000	Q	011000	Y	100000	g	101000	o	110000	w	111000	4
000001	B	001001	J	010001	R	011001	Z	100001	h	101001	p	110001	x	111001	5
000010	C	001010	K	010010	S	011010	a	100010	i	101010	q	110010	y	111010	6
000011	D	001011	L	010011	T	011011	b	100011	j	101011	r	110011	z	111011	7
000100	E	001100	M	010100	U	011100	c	100100	k	101100	s	110100	0	111100	8
000101	F	001101	N	010101	V	011101	d	100101	l	101101	t	110101	1	111101	9
000110	G	001110	O	010110	W	011110	e	100110	m	101110	u	110110	2	111110	+
000111	H	001111	P	010111	X	011111	f	100111	n	101111	v	110111	3	111111	/

### Secure Chat with a One-Time Pad

First challenge: Create a one-time pad.

Good choice: A **random** sequence of bits (stay tuned).

Note: any sequence of bits can be encoded as characters

110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
y	T	2	5	a	5	y	/	s	encoded as characters

### One-Time Pad Encryption

Encryption.

- Convert text message to **N bits**.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
M	12	001100
...	...	...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64

## One-Time Pad Encryption

### Encryption.

- Convert text message to N bits.
- Use N random bits as one-time pad.

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
y	T	2	5	a	5	y	/	S	

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## One-Time Pad Encryption

### Encryption.

- Convert text message to N bits.
- Use N random bits as one-time pad.
- Take bitwise XOR of two bitstrings.

XOR Truth Table

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

sum corresponding pair of bits: 1 if sum is odd, 0 if even

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR

$0 \oplus 1 = 1$

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## One-Time Pad Encryption

### Encryption.

- Convert text message to N bits.
- Use N random bits as one-time pad.
- Take bitwise XOR of two bitstrings.
- Convert binary back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
w	22	010110
...	...	...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR
g	X	7	6	W	3	v	7	K	encrypted

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## Secure Chat with a One-Time Pad

### Alice wants to send a secret message to Bob

- Sometime in the past, they exchange a **one-time pad**.
- Alice uses the pad to encrypt the message.

"use yT25a5y/S if I ever send you an encrypted message"



Secure Chat 1.0 [alice]  
 [alice]: Hey, Bob  
 [bob]: Hi, Alice!  
 [alice]: SENDMONEY  
 Encrypt SENDMONEY with yT25a5y/S

Secure Chat 1.0 [bob]  
 [alice]: Hey, Bob  
 [bob]: Hi, Alice!  
 [alice]: gx76W3v7K

**Key point:** Without the pad, Eve cannot understand the message. But how can **Bob** understand the message?

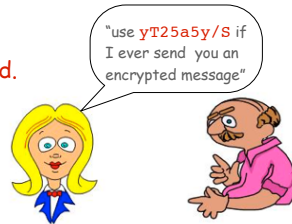


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## Secure Chat with a One-Time Pad

Alice wants to send a secret message to Bob

- Sometime in the past, they exchange a **one-time pad**.
- Alice uses the pad to encrypt the message.
- Bob uses the same pad to decrypt the message.



```
Secure Chat 1.0 [alice]
[alice]: Hey, Bob
[bob]: Hi, Alice!
[alice]: SENDMONEY

Encrypt SENDMONEY with yT25a5y/S
```

```
Secure Chat 1.0 [bob]
[alice]: Hey, Bob
[bob]: Hi, Alice!
[alice]: gX76W3v7K
SENDMONEY

Decrypt with yT25a5y/S
```

Key point: Without the pad, Eve cannot understand the message.



## One-Time Pad Decryption

Decryption.

- Convert encrypted message to binary.

g X 7 6 W 3 v 7 K encrypted

## One-Time Pad Decryption

Decryption.

- Convert encrypted message to binary.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
W	22	010110
...	...	...

g X 7 6 W 3 v 7 K encrypted  
 100000 010111 111011 111010 010110 110111 101111 111011 001010 base64

## One-Time Pad Decryption

Decryption.

- Convert encrypted message to binary.
- Use **same** N "random" bits (one-time pad).

g X 7 6 W 3 v 7 K encrypted  
 100000 010111 111011 111010 010110 110111 101111 111011 001010 base64  
 110010 010011 110110 111001 011010 111001 100010 111111 010010 one-time pad  
 y T 2 5 a 5 y / s

## One-Time Pad Decryption

### Decryption.

- Convert encrypted message to binary.
- Use same N random bits (one-time pad).
- Take bitwise XOR of two bitstrings.

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

g	X	7	6	W	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted
110010	010011	110110	111001	011010	111001	100010	111111	010010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR

↑  
1 ^ 1 = 0

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## One-Time Pad Decryption

### Decryption.

- Convert encrypted message to binary.
- Use same N random bits (one-time pad).
- Take bitwise XOR of two bitstrings.
- Convert back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
M	12	001100
...	...	...

g	X	7	6	W	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted
110010	010011	110110	111001	011010	111001	100010	111111	010010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	one-time pad
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR
S	E	N	D	M	O	N	E	Y	message

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## Why Does It Work?

Crucial property. Decrypted message = original message.

Notation	Meaning
a	original message bit
b	one-time pad bit
^	XOR operator
a ^ b	encrypted message bit
(a ^ b) ^ b	decrypted message bit

### Why is crucial property true?

- Use properties of XOR.
- $(a \wedge b) \wedge b = a \wedge (b \wedge b) = a \wedge 0 = a$

↑ associativity of ^  
↑ always 0  
↑ identity

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

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## One-Time Pad Decryption (with the wrong pad)

### Decryption.

- Convert encrypted message to binary.

g	X	7	6	W	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted

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## One-Time Pad Decryption (with the wrong pad)

### Decryption.

- Convert encrypted message to binary.

g	X	7	6	W	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted
									base64

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## One-Time Pad Decryption (with the wrong pad)

### Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).

g	X	7	6	W	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted
									base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits

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## One-Time Pad Decryption (with the wrong pad)

### Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).
- Take bitwise XOR of two bitstrings.

g	X	7	6	W	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted
									base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits
001000	001011	001110	010101	000100	001110	001010	010001	000000	XOR

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## One-Time Pad Decryption (with the wrong pad)

### Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).
- Take bitwise XOR of two bitstrings.
- Convert back into text: **Oops.**

g	X	7	6	W	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	encrypted
									base64
101010	110000	000011	100000	011011	000011	101110	011010	101111	wrong bits
001010	100111	111000	011010	001101	110100	000001	100001	100101	XOR
K	n	4	a	N	0	B	h	l	wrong message [usually gibberish]

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## Eve's problem

**Key point:** Without the pad, Eve cannot understand the message.



But Eve has a computer. Why not try all possible pads?

**There are too many possibilities!**

- 54 bits implies  $2^{54}$  possibilities.
- If Eve could check 1 million per second, it would take **571** years to try them all!
- If we add another bit, it would take her another 571 years.

1000 bits:  $2^{1000}$  possibilities.

Age of the universe in microseconds:  $2^{70}$

AAAAAAAA	gX76W3v7K
AAAAAAAAB	gX76W3v7L
AAAAAAAAC	gX76W3v7I
...	
qwDgbDuav	Kn4aN0Bh1
...	
yT25a5y/S	SENDMONEY
...	
////////+	f0EFpIQE0
/////////	f0EFpIQE1

Exponential growth dwarfs technological improvements.

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## Goods and Bads of One-Time Pads

**Good.**

- Easily computed by hand.
- Very simple encryption/decryption processes.
- Provably unbreakable if bits are truly random. [Shannon, 1940s]

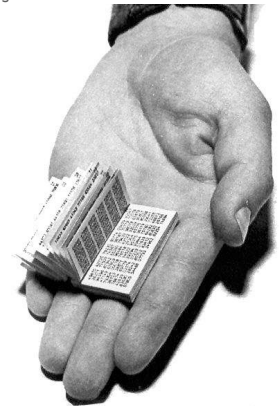
eavesdropper Eve sees only random bits

**Bad.**

- Easily breakable if pad is re-used.
- **Pad must be as long as the message.**
- Truly random bits are very hard to come by.
- Pad must be distributed securely.

"one time" means one time only

impractical for Web commerce



a Russian one-time pad 34

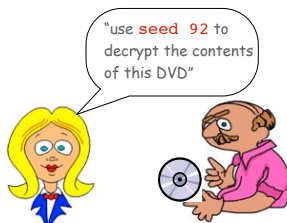
## Pseudo-Random Bit Generator

**Practical middle-ground.**

- Make a "random" bit generator gadget.
- Alice and Bob each get identical small gadgets [same gadget works for both]
- Alice and Bob also each get identical books of small **seeds**.

instead of identical large one-time pads

**Goal.** Small gadget that produces a long sequence of bits.



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## Pseudo-Random Bit Generator

**Goal:** Small gadget that produces a long sequence of **pseudo-random bits**.

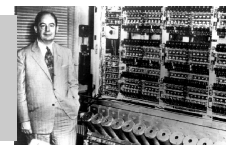
- Enigma
- **Linear feedback shift register.**
- Linear congruential generator.
- Blum-Blum-Shub generator.
- [many others have been invented]

**Pseudo-random? Bits are not really random:**

- Bob's and Alice's gadgets must produce the same bits from the same seed.
- Bits must have as many properties of random bits as possible (to foil Eve).

Ex 1. approx 1/2 0s and 1/2 1s  
Ex 2. approx 1/4 each of 00, 01, 10 11

"Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin."  
– Jon von Neumann (left)  
– ENIAC (right)

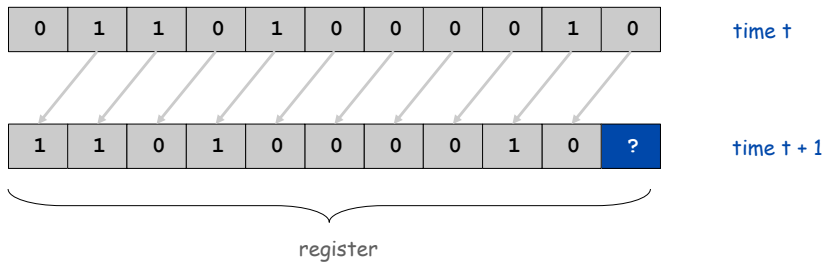


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## Shift Register

### Shift register terminology.

- Bit: 0 or 1.
- Cell: storage element that holds one bit.
- Register: sequence of cells.
- Seed: initial sequence of bits.
- Shift register: when clock ticks, bits propagate one position to left.

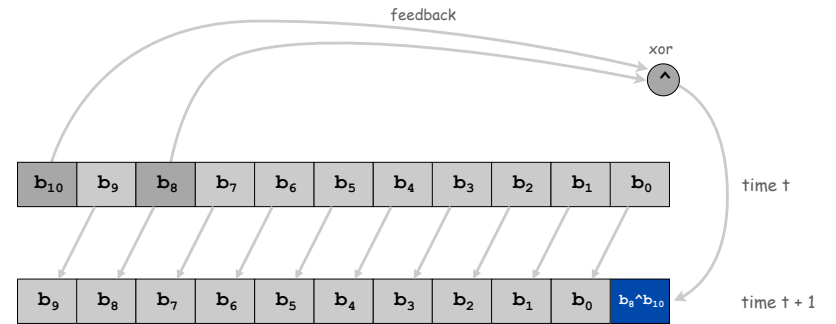


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## Linear Feedback Shift Register (LFSR)

### {8, 10} linear feedback shift register.

- Shift register with 11 cells.
- Bit  $b_0$  is XOR of previous bits  $b_8$  and  $b_{10}$ .
- Pseudo-random bit =  $b_0$ .



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## Random Numbers

Q. Are these 2000 numbers random? If not, what is the pattern?

```

1100100100111011011100101101011100110001011111010010000100110100101111001100100111111011100
00010101100010000111010100110100001110010011001110111110101000001000010001010010100011000
001011110001001001101011011100011010011011100111010111001000100111010101101000001010010001
0001101010101100000001011000001001110001011010101001010110011000011111100110000011111000110
00011011100110100111101001110010011101101101010101000000001000000001010000001000100001
0101010010000001010000011100100011011101011010100010100010100010101101010100001100001
0011100101100110010111011100100101011011000010101100100001011101001001010010101000111011
110110010101011100000010011000010111100100100011011010101010001100011001101101010010110
00011001110011111011100001010011001000111110101100001000111001010101100001101011001100011
1110101100010101101110100110101001110000111001001010111111101000000010010000010110100010011
0010101111100001000011001010011110001110001101101101101010101010000011011000011101011
0110100011010010111001010100111000001101100011010110110110011011000000110010000111
1101001100010011110101110001000101010101001100000011110000110001100110111100010100001110
001001010101110110001001011010110010100011100010100110011111001110000111011001100101
11111100100000111010000101000100110011011011101010100010000010101000010000100101000101
1000101001101000111010010110011001111111110000000011000000011100000110011000111111
10110000001011000010010100101100111001110001110011011001111011110001010001010000
1011100101001011000100010111011100110100011110010100011100110110110110110101010010001001101
011111100010000010101000011000010110110010011011101101010010100100010111101101101000101
010010100000110001000111010101010010000011101000110010010111101101001000101110101001001000011
01101001101001110101111010001000100101010101000000011000000110110000110111001101010111
1000010001100010101111010
    
```

A. No. This is output of {8, 10} LFSR with seed 01101000010!

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## LFSR Encryption

### Encryption.

- Convert text message to N bits.
- Initialize LFSR with given seed
- Generate N bits **with LFSR**.
- Take bitwise XOR of two bitstrings.
- Convert binary back into text.

### Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
w	22	010110
...	...	...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	LFSR bits
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR
g	X	7	6	W	3	v	7	K	encrypted

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## LFSR Decryption

### Decryption.

- Convert encrypted message to binary.
- Initialize identical LFSR with **same seed**
- Generate N bits **with LFSR**.
- Take bitwise XOR of two bitstrings.
- Convert back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
M	12	001100
...	...	...

g	X	7	6	W	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	LFSR bits
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR
S	E	N	D	M	O	N	E	Y	message

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## Goods and Bads of LFSRs

### Good.

- Easily computed with simple machine.
- Very simple encryption/decryption processes.
- Bits have many of the same properties as random bits.
- Scalable: 20 cells for 1 million bits; 30 cells for 1 billion bits.  
[ but need theory of finite groups to know where to put taps ]



a commercially available LFSR

### Bad.

- Still need secure, independent way to distribute LFSR seed.
- The bits are not truly random.  
[ bits in our 11-bit LFSR cycle after  $2^{11} - 1 = 2047$  steps ]
- Experts have cracked LFSR encryption.  
[ need more complicated machines ]

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## Other LFSR Applications

### What else can we do with a LFSR?

- DVD encryption with CSS.
- DVD decryption with DeCSS!
- Subroutine in military cryptosystems.

```

/*  efdtt.c  Author: Charles M. Hannum <root@ihack.net>  */
/*  Usage is: cat title-key scrambled.vob | efdtt >clear.vob  */

#define m(i) (x[i]^s[i+84])<<

    unsigned char x[5]      ,y,s[2048];main(
n){for( read(0,x,5      );read(0,s,n=2048
); write(1      ,s,n) )if(s
[y=s      [13]%8+20] /16%4 ==1 )int
i=m(      1)17 ^256 +m(0) 8,k =m(2)
0,j=      m(4) 17^ m(3) 9^k* 2-k%8
^8,a      =0,c =26;for (s[y] -=16;
--c;j      *=2)a= a*2^i% 1,i=i /2^j%1
<<24;for(j=      127; ++j<n;c>
y)
c
+=y^i^i/8^i>>4^i>>12,
i=i>>8^y<<17,a^=a>>14,y=a^a*8^a<<6,a=a
>>8^y<<9,k=s[j],k =7Wo~G_\216"[k
67]+2^"cr3sfw6v:*k+>/n."[k>>4]*2^k*257/
8,s[j]=k^(k&k*2&34)*6^c+~y
;})
    
```

<http://www.cs.cmu.edu/~dst/DeCSS/Gallery>

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## Typical Exam Question (TEQ) on LFSRs 1

Give first 10 steps of {3, 4} LFSR with initial fill 00001.

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**Goal.** Decrypt/encrypt 300 characters (1800 bits).

**Challenge.** Can we use an 11-bit LFSR?

A. Yes, no problem.

B. No, the bits it produces are not truly random.

C. No, need a longer LFSR.

D. No, experts have cracked LFSRs

**Important properties.**

- Built from simple components.
- Scales to handle huge problems.
- Requires a deep understanding to use effectively.

Basic Component	LFSR	Computer
control	start, stop, load	same
clock	regular pulse	2.8 GHz pulse
memory	11 bits	1 GB
input	seed	sequence of bits
computation	shift, XOR	logic, arithmetic, ...
output	pseudo-random bits	Sequence of bits

**Critical difference.** General purpose machine can be programmed to simulate ANY abstract machine.

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## A Profound Idea

**Programming.** Can write a Java program to simulate the operations of **any** abstract machine.

- Basis for theoretical understanding of computation. [stay tuned]
- Basis for bootstrapping real machines into existence. [stay tuned]

**Stay tuned.** See Assignment 5.

```
public class LFSR
{
    private int seed[];
    private int tap;
    private int N;

    public LFSR(String seed, int tap) { ... }

    public int step() { ... }

    public static void main(String[] args)
    {
        LFSR lfsr = new LFSR("01101000010", 8);
        for (int i = 0; i < 2000; i++)
            StdOut.println(lfsr.step());
    }
}
```

```
% java LFSR
1100100100111101101110010110101
1100110001011111101001000010011
0100101111001100100111...
```

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## A Profound Question

Q. What is a random number?

**LFSR does not produce random numbers.**

- It is a very simple deterministic machine.
- Not obvious how to distinguish the bits it produces from random.
- Experts have figured out how to do so.

Q. Are random processes found in nature?

- Motion of cosmic rays or subatomic particles?
- Mutations in DNA?

Q. Is the natural world a (not-so-simple) deterministic machine?

"God does not play dice."  
— Albert Einstein



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