COS 597C: How to Solve it	**Fall 2009**

Assignment #1

Due: Tuesday, September 24 — *Sanjeev Arora*

Please be succinct, trying to give a proof sketch rather than gory details.

1. Suppose a gambler enters a casino with k dollars, and proceeds to make a sequence of fair bets (i.e, win \$ 1 or lose \$ 1 with equal probability). What is the chance that he goes broke before he doubles his money? What is the chance that he increases his money to $1.5k$ without going broke first? What is the chance that he increases his money to $3k$ before he goes broke? (Rough asymptotic expressions are OK if needed.)

2. Calculate the expected size of the largest clique in a random graph $G(n, 1/2)$. Try to use any of the concentration inequalities from the lecture for this problem and report your results.

3. Suppose we take n points $x_1, x_2, \ldots, x_n$ in the square of unit length in $\Re^2$. The *length* of edge $\{x_i, x_j\}$ is the Euclidean distance between x_i, x_j.

   (a) Show that there is constant c independent of n such that the optimum traveling salesman tour of the points has length no more than $c\sqrt{n}$.

   (b) Show that there is a constant c' such that the sum of *squares* of edge lengths in the optimum salesman tour is no more than c'.

   (c) Use these facts to derive a concentration bound for the length of the optimum salesman tour when $x_1, x_2, \ldots, x_n$ are randomly chosen in the unit square.