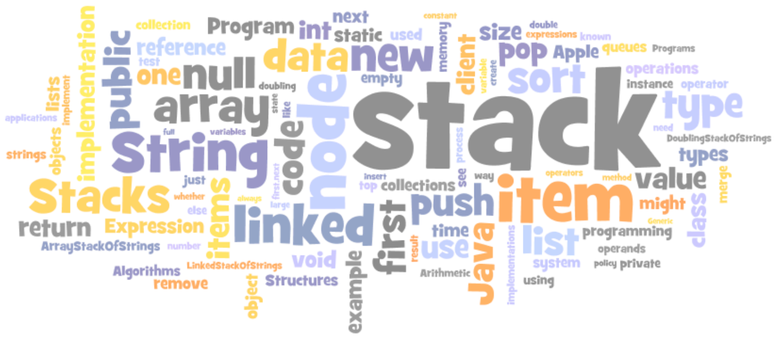


Data Structures and Data Types

Data types

- Set of values.
- Set of operations on those values.
- Some are built in to Java: `int, double, char, . . .`
- Most are not: `Complex, Picture, Charge, Stack, Queue, Graph, . . .`

↑ this lecture

Data structures.

- Represent data.
- Represent relationships among data.
- Some are built in to Java: arrays, `String, . . .`
- Most are not: linked list, circular list, tree, sparse array, graph, . . .

↑ this lecture ↑ TSP ↑ next lecture

Design challenge for every data type: What data structure to use?

- Requirement 1: Space usage.
- Requirement 2: Time usage for data-type methods

Collections

Fundamental data types.

- Set of operations (add, remove, test if empty) on generic data.
- Intent is clear when we insert.
- Which item do we remove?

LIFO = "last in first out"

Stack. (this lecture)

- Remove the item most recently added.
- Ex: cafeteria trays, Web surfing.

FIFO = "first in first out"

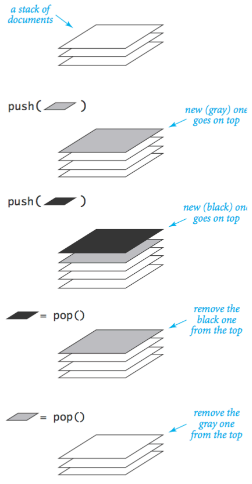
Queue. (see text)

- Remove the item least recently added.
- Ex: Registrar's line.

Symbol Table. (next lecture)

- Remove item with a given key.
- Ex: Phone book

Stacks

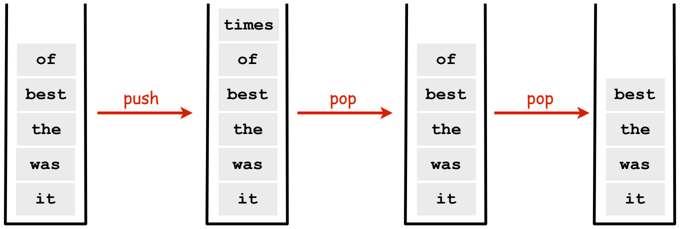


Operations on a pushdown stack

Stack API

public class *StackOfStrings

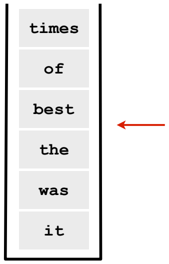
	*StackOfStrings()	*create an empty stack*
boolean	isEmpty()	*is the stack empty?*
void	push(String item)	*push a string onto the stack*
String	pop()	*pop the stack*



Stack Client Example 1: Reverse

```
public class Reverse
{
   public static void main(String[] args)
   {
      StackOfStrings stack = new StackOfStrings();
      while (!StdIn.isEmpty())
         stack.push(StdIn.readString());
      while (!stack.isEmpty())
         StdOut.print(stack.pop());
      StdOut.println();
   }
}
```

```
% more tiny.txt
it was the best of times

% java Reverse tiny.txt
times of best the was it
```

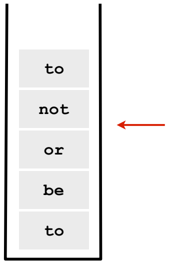


stack contents when StdIn is empty

Stack Client Example 2: Test Client

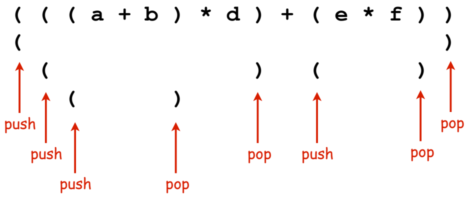
```
public static void main(String[] args)
{
   StackOfStrings stack = new StackOfStrings();
   while (!StdIn.isEmpty())
   {
      String item = StdIn.readString();
      if (item.compareTo("-") != 0)
         stack.push(item);
      else
         System.out.print(stack.pop());
   }
   System.out.println();
}
```

```
% more test.txt
to be or not to - be - - that - - - is

% java StackOfStrings  < test.txt
to be not that or be
```



stack contents just before first `pop()` operation

Stack Client Example 3: Balanced Parentheses



Stack Client Example 3: Balanced Parentheses

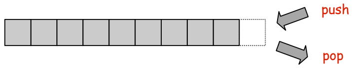
```
public class Balanced
{
   public static void main(String[] args)
   {
      StackOfStrings stack = new StackOfStrings();
      while (!StdIn.isEmpty())
      {
         String item = StdIn.readString();
         if (item.compareTo("(") == 0)
            stack.push(item);
         if (item.compareTo(")") == 0)
         {
            if (stack.isEmpty())
            {  StdOut.println(“Not balanced”); return; }
            stack.pop();
         }
      }
      if (!stack.isEmpty()) StdOut.println(“Not balanced”);
      else                  StdOut.println(“Balanced”);
   }
}
```

```
% java Balanced
( ( ( a + b ) * d ) + ( e * f ) )
Balanced

% java Balanced
( ( a + b ) * d ) + ( e * f ) )
Not balanced
```

Stack: Array Implementation

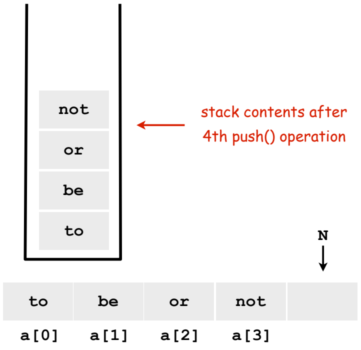
Array implementation of a stack.

- Use array `a[]` to store `N` items on stack. ← PROBLEM: How big to make array? (Stay tuned.)
- `push()` add new item at `a[N]`.
- `pop()` remove item from `a[N-1]`.



```
public class ArrayStackOfStrings
{
   private String[] a;
   private int N = 0;

   public ArrayStackOfStrings(int max)
   {  a = new String[max];  }

   public boolean isEmpty()
   {  return (N == 0);  }

   public void push(String item)
   {  a[N++] = item;  }

   public String pop()
   {  return a[--N];  }
}
```

Temporary solution: Make client provide capacity.



stack contents after 4th push() operation

Array Stack: Trace

	StdIn	StdOut	N	a[]				
				0	1	2	3	4
			0					
push	to		1	to				
	be		2	to	be			
	or		3	to	be	or		
	not		4	to	be	or	not	
	to		5	to	be	or	not	to
pop	-	to	4	to	be	or	not	to
	be		5	to	be	or	not	be
	-	be	4	to	be	or	not	be
	-	not	3	to	be	or	not	be
	that		4	to	be	or	that	be
	-	that	3	to	be	or	that	be
	-	or	2	to	be	or	that	be
	-	be	1	to	be	or	that	be
	is		2	to	is	or	not	to

Stack Challenge: Stack Sort?

Q. Can we always insert pop commands (-) to make strings come out sorted?

Ex 1: 6 5 4 3 2 1 - - - - - -

Ex 2: 1 - 2 - 3 - 4 - 5 - 6 -

Ex 3: 4 1 - 3 2 - - - 6 5 - -

Stack Challenge: Stack Sort?

Q. Can we always insert pop commands (-) to make strings come out sorted?

Ex 1: `6 5 4 3 2 1 - - - - - -`

Ex 2: `1 - 2 - 3 - 4 - 5 - 6 -`

Ex 3: `4 1 - 3 2 - - - 6 5 - -`

A. NO.

Ex. Cannot do `5 6 x x x x` because 6 must come out of stack before 5

Note: in a QUEUE, they always come out in the same order they went in.

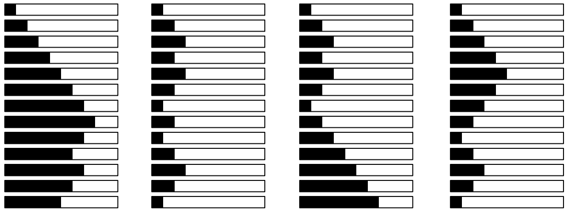
Array Stack: Performance

Running time. Push and pop take constant time. ✓

Memory. Proportional to client-supplied capacity, not number of items. ✗

What's the problem?

- API does not call for capacity (never good to change API)
- Client might have multiple stacks
- Client might not know what capacity to use (depends on its client)



Challenge. Stack implementation where space use is not fixed ahead of time.

Linked Lists

Sequential vs. Linked Data Structures

Sequential data structure. Put object one next to another.

- TOY: consecutive memory cells.
- Java: array of objects.

Linked data structure. Include in each object a link to the another one.

- TOY: link is memory address of next object.
- Java: link is reference to next object.

Key distinctions.

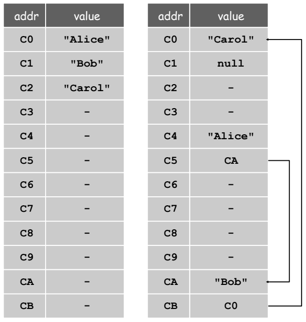
get ith element

- Array: arbitrary access, fixed size.
- Linked list: sequential access, variable size.

get next element

Linked structures.

- Not intuitive, overlooked by naive programmers
- Flexible, widely used method for organizing data



addr	value
C0	"Alice"
C1	"Bob"
C2	"Carol"
C3	-
C4	-
C5	-
C6	-
C7	-
C8	-
C9	-
CA	-
CB	-

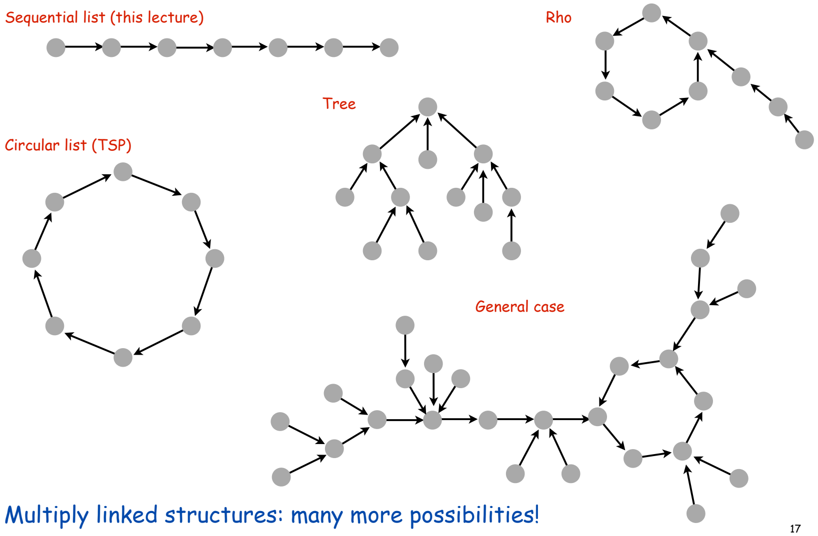
array

addr	value
C0	"Carol"
C1	null
C2	-
C3	-
C4	"Alice"
C5	CA
C6	-
C7	-
C8	-
C9	-
CA	"Bob"
CB	C0

linked list

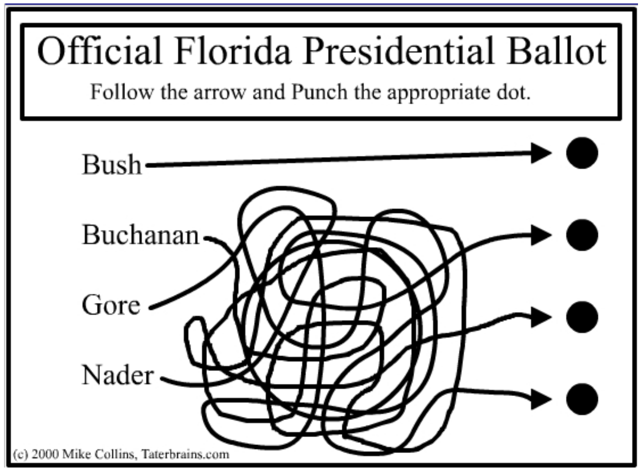
Singly-linked data structures

From the point of view of a particular object, all of these structures look the same: ●→



Multiply linked structures: many more possibilities!

Linked structures can become intricate



Linked Lists

Linked list.

- Simplest linked structure.
- A recursive data structure.
- A item plus a pointer to another linked list (or empty list).
- Unwind recursion: linked list is a sequence of items.

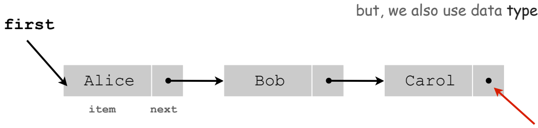
Node data type.

- A reference to a `String`.
- A reference to another `Node`.

```
public class Node
{
   private String item;
   private Node next;
}
```

Confusing point:

Purpose of data **structure** is to represent data in a data **type** but, we also use data **types** to implement data **structures**



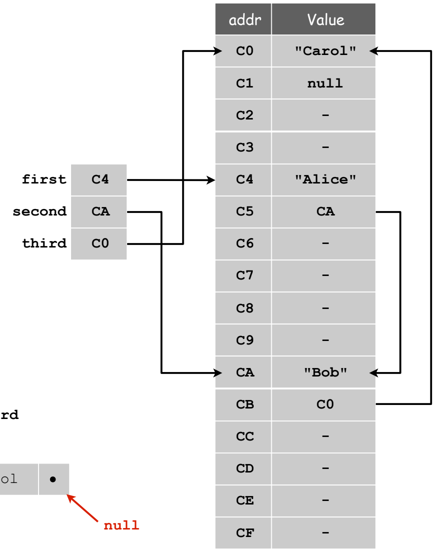
special pointer value **null** terminates list

Building a Linked List

```
Node third  = new Node();
third.item  = "Carol";
third.next  = null;
```

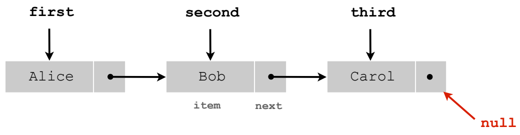
```
Node second = new Node();
second.item = "Bob";
second.next = third;
```

```
Node first  = new Node();
first.item  = "Alice";
first.next  = second;
```

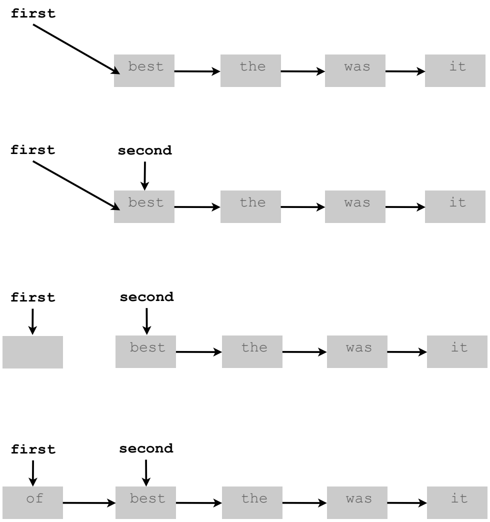


addr	Value
C0	"Carol"
C1	null
C2	-
C3	-
C4	"Alice"
C5	CA
C6	-
C7	-
C8	-
C9	-
CA	"Bob"
CB	C0
CC	-
CD	-
CE	-
CF	-

main memory



Stack Push: Linked List Implementation

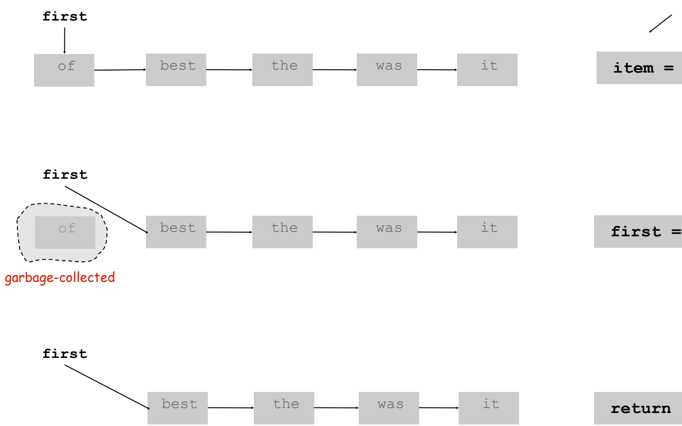


`second = first;`

`first = new Node();`

```
first.item = item;
first.next = second;
```

Stack Pop: Linked List Implementation



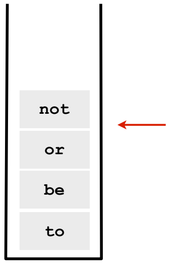
`item = first.item;`

`first = first.next;`

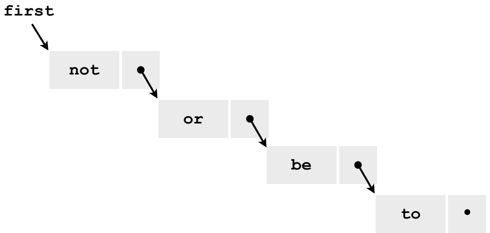
`return item;`

Stack: Linked List Implementation

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      private String item;
      private Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```



stack contents after
4th push() operation
in test client

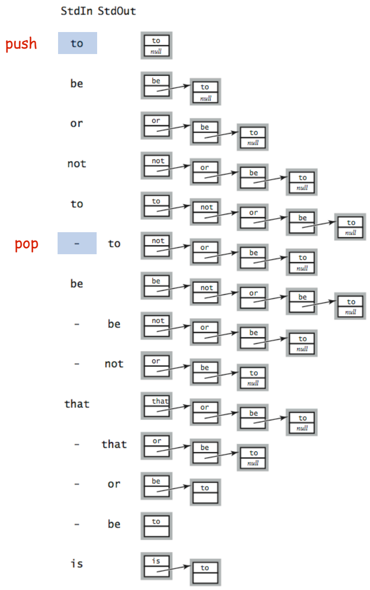


Note difference between `first` and `second`:

`first`: an instance variable that retains state

`second`: a local variable that goes out of scope

Linked List Stack: Trace



StdIn	StdOut
to	
be	
or	
not	
to	
-	to
be	
-	be
-	not
that	
-	that
-	or
-	be
is	

Linked-List Stack: Performance

Running time. Push and pop take constant time. ✓

Memory. Always proportional to number of items in stack. ✓

Stack Data Structures: Tradeoffs

Two data structures to implement the Stack data type.

Array.
- Every push/pop operation take constant time.
- But… must fix maximum capacity of stack ahead of time.

Linked list.
- Every push/pop operation takes constant time.
- But… uses extra space and time to deal with references.

Client can evaluate performance tradeoffs to choose among implementations
(implicitly choosing among underlying data structures)

List-Processing Challenge 1

What does the following code do?

```
...
Node list = null;
while (!StdIn.isEmpty())
{
   Node old = list;
   list = new Node();
   list.item = StdIn.readString();
   list.next = old;
}
for (Node t = list; t != null; t = t.next)
   StdOut.println(t.item);
...
```

List-Processing Challenge 1

What does the following code do?

```
...
Node list = null;
while (!StdIn.isEmpty())
{
   Node old = list;
   list = new Node();
   list.item = StdIn.readString();
   list.next = old;
}
for (Node t = list; t != null; t = t.next)
   StdOut.println(t.item);
...
```

A. Reverses the strings in StdIn.

Note: Better to use a Stack, represented as a linked list.
In this course, we always do list processing in data-type implementations.

List-Processing Challenge 2

What does the following code do?

```
...
Node list = new Node();
list.item = StdIn.readString();
Node last = list;
while (!StdIn.isEmpty())
{
   last.next = new Node();
   last = last.next;;
   last.item = StdIn.readString();
}
...
```

List-Processing Challenge 2

What does the following code do?

```
...
Node list = new Node();
list.item = StdIn.readString();
Node last = list;
while (!StdIn.isEmpty())
{
   last.next = new Node();
   last = last.next;;
   last.item = StdIn.readString();
}
...
```

A. Puts the strings on StdIn in a linked list, in the order they are read (assuming at least 1 string on StdIn).

Note: Could use a Queue, represented as a linked list (see text).
In this course, we always do list processing in data-type implementations.
This code might be the basis for an initialization method in a data type.

Parameterized Data Types

Parameterized Data Types

We implemented: `StackOfStrings`.

We also want: `StackOfMemoryBlocks`, `StackOfURLs`, `StackOfInts`, …

Strawman. Implement a separate stack class for each type.

- Rewriting code is tedious and error-prone.
- Maintaining cut-and-pasted code is tedious and error-prone.

Generics

Generics. Parameterize stack by a single type.

"Stack of Apples"

parameterized type

```
Stack<Apple> stack = new Stack<Apple>();
Apple  a = new Apple();
Orange b = new Orange();
stack.push(a);
stack.push(b);    // compile-time error
a = stack.pop();
```

sample client

Can't push an "Orange" onto a "Stack of Apples"

Generic Stack: Linked List Implementation

String stack (for reference)

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      private String item;
      private Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

```
public class Stack<Item>
{
   private Node first = null;

   private class Node
   {
      private Item item;
      private Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(Item item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

parameterized type name chosen by programmer

Autoboxing

Generic stack implementation. Only permits reference types.

Wrapper type.
- Each primitive type has a wrapper reference type.
- Ex: `Integer` is wrapper type for `int`.
- Wrapper type has larger set of operations than primitive type.
- Values of wrapper type are objects.

Autoboxing. Automatic cast from primitive type to wrapper type.
Autounboxing. Automatic cast from wrapper type to primitive type.

```
Stack<Integer> stack = new Stack<Integer>();
stack.push(17);        // Autobox     (int -> Integer)
int a = stack.pop();   // Auto-unbox (Integer -> int)
```

Stack Applications

Real world applications.
- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.

Stack Client 4: Arithmetic Expression Evaluation

value stack
operator stack

Goal. Evaluate infix expressions.

`( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )`

operand

operator

Two stack algorithm. [E. W. Dijkstra]

- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parens: ignore.
- Right parens: pop operator and two values; push the result of applying that operator to those values onto the operand stack.

Arithmetic Expression Evaluation

```
public class Evaluate
{
   public static void main(String[] args)
   {
      Stack<String> ops  = new Stack<String>();
      Stack<Double> vals = new Stack<Double>();
      while (!StdIn.isEmpty())
      {
         String s = StdIn.readString();
         if      (s.equals("("))               ;
         else if (s.equals("+"))    ops.push(s);
         else if (s.equals("*"))    ops.push(s);
         else if (s.equals(")"))
         {
            String op = ops.pop();
            if      (op.equals("+"))
               vals.push(vals.pop() + vals.pop());
            else if (op.equals("*"))
               vals.push(vals.pop() * vals.pop());
         }
         else vals.push(Double.parseDouble(s));
      }
      StdOut.println(vals.pop());
   }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

Correctness

Why correct? When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

So it's as if the original input were:

```
( 1 + ( 5 * ( 4 * 5 ) ) )
```

Repeating the argument:

```
( 1 + ( 5 * 20 ) )
( 1 + 100 )
101
```

Extensions. More ops, precedence order, associativity, whitespace.

```
1 + (2 - 3 - 4) * 5 * sqrt(6*6 + 7*7)
```

Postfix

Observation 1. Remarkably, the 2-stack algorithm computes the same value if the operator occurs after the two values.

```
( 1 ( ( 2 3 + ) ( 4 5 * ) * ) + )
```

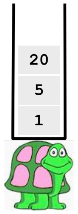
Observation 2. Now all of the parentheses are redundant!

```
1 2 3 + 4 5 * * +
```

Jan Lukasiewicz

Bottom line. Postfix or "reverse Polish" notation.

Real-World Stack Application: PostScript



PostScript (Warnock-Geschke, 1980s). A turtle with a stack.

- postfix program code
- add commands to drive virtual graphics machine
- add loops, conditionals, functions, types

PostScript code

```
100 100 moveto
100 300 lineto
300 300 lineto
300 100 lineto
stroke
```

units are points (72 per inch)

define a path

draw the path

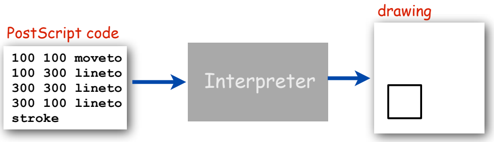
Simple virtual machine, but not a toy.

- Easy to specify published page.
- Easy to implement on various specific printers
- Revolutionized world of publishing.
- Virtually all printed material is PostScript.

Context/Definitions/Summary

Interpreter.

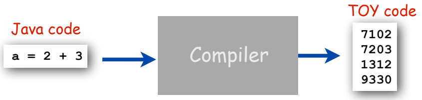
- Takes a program as input
- Does what that program would do.
- Simulates a virtual machine.


PostScript code

```
100 100 moveto
100 300 lineto
300 300 lineto
300 100 lineto
stroke
```

Compiler.

- Takes a program as input
- Produces a program as output.
- Produces code for a (real) machine.


Java code

```
a = 2 + 3
```

TOY code

```
7102
7203
1312
9330
```

TOY is our proxy for a real machine

Data Type and Virtual Machine are the same thing!

- Set of values = machine state.
- Operations on values = machine operations.

Virtual machines you have used
- LFSR
- Stack
- TOY
- PostScript
- Java Virtual Machine (another stack machine)

Data Structure.

- Represent data and relationships among data in a data type.
- array, linked list, compound, multiple links per node