

Bits, bytes, and representation of information

- **digital representation means that everything is represented by numbers only**
- **the usual sequence:**
 - something (sound, pictures, text, instructions, ...) is converted into numbers by some mechanism
 - the numbers can be stored, retrieved, processed, transmitted
 - the numbers might be reconstituted into a version of the original
- **for sound, pictures, other real-world values**
 - make accurate measurements
 - convert them to numeric values

Encoding sound

- **need to measure intensity/loudness often enough and accurately enough that we can reconstruct it well enough**
- **higher frequency = higher pitch**
- **human ear can hear ~ 20 Hz to 20 KHz**
 - taking samples at twice the highest frequency is good enough (Nyquist)
- **CD audio usually uses**
 - 44,100 samples / second
 - accuracy of 1 in 65,536 ($= 2^{16}$) distinct levels
 - two samples at each time for stereo
 - data rate is $44,100 \times 2 \times 16$ bits/sample
 $= 1,411,200$ bits/sec $= 176,400$ bytes/sec ~ 10.6 MB/minute
- **MP3 audio compresses by clever encoding and removal of sounds that won't really be heard**
 - data rate is ~ 1 MB/minute

Analog versus Digital

- **analog: "analogous" or "the analog of"**
 - smoothly or continuously varying values
 - volume control, dimmer, faucet, steering wheel
 - value varies smoothly with something else
 - no discrete steps or changes in values
 - small change in one implies small change in another
 - infinite number of possible values
 - the world we perceive is largely analog
- **digital: discrete values**
 - only a finite number of different values
 - a change in something results in sudden change from one discrete value to another
 - digital speedometer, digital watch, push-button radio tuner, ...
 - values are represented as numbers



Discrete values vs continuous values

- **another kind of conversion**
 - letters are converted into numbers when you type on a keyboard
 - the letters are stored (a Word document), retrieved (File/Open...), processed (paper is revised), transmitted (submitted by email)
 - printed on paper
- **letters and other symbols are inherently discrete**
- **encoding them as numbers is just assigning a numeric value to each one, without any intrinsic meaning**

Representing letters as numbers

- **what letters and other symbols are included?**
- **how many digits/letter?**
 - determined by how many symbols there are
 - how do we disambiguate if symbols have different lengths?
- **how do we decide whose encoding to use?**
- **the representation is arbitrary**
- **but everyone has to agree on it**
 - if they want to work together

Important ideas

- **number of items and number of digits are tightly related:**
 - one determines the other
 - maximum number of different items = base ^{number of digits}
 - e.g., 9-digit SSN: $10^9 = 1$ billion possible numbers
- **interpretation depends on context**
 - without knowing that, we can only guess what things mean
 - what's 81615 ?

What's a bit? What's a byte?

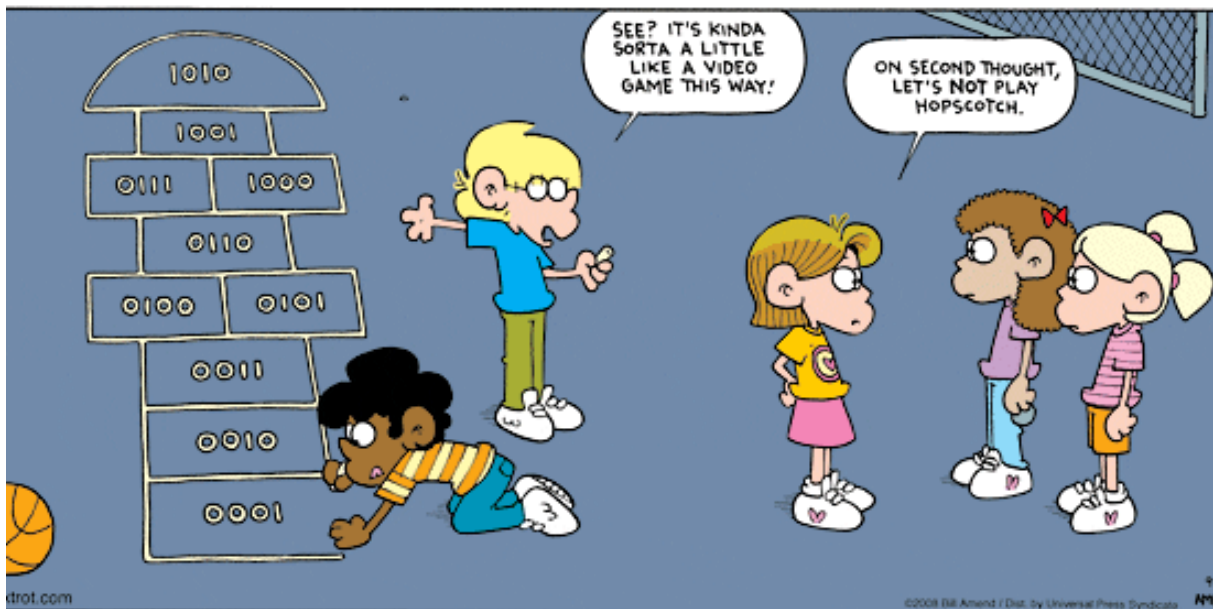
- **a bit is the smallest unit of information**
- **represents one 2-way decision or a choice out of two possibilities**
 - yes / no, true / false, on / off, M / F, ...
- **abstraction of all of these is represented as 0 or 1**
 - enough to tell which of TWO possibilities has been chosen
 - a single digit with one of two values
 - hence "binary digit"
 - hence bit
- **binary is used in computers because it's easy to make fast, reliable, small devices that have only two states**
 - high voltage/low voltage, current flowing/not flowing (chips)
 - electrical charge present/not present (RAM, flash)
 - magnetized this way or that (disks)
 - light bounces off/doesn't bounce off (cd-rom, dvd)
- **all information in a computer is stored and processed as bits**
- **a byte is 8 bits that are treated as a unit**

A review of how decimal numbers work

- **how many digits?**
 - we use 10 digits for counting: "decimal" numbers are natural for us
 - other schemes show up in some areas
 - clocks use 12, 24, 60; calendars use 7, 12
 - other cultures use other schemes (quatre-vingts)
- **what if we want to count to more than 10?**
 - 0 1 2 3 4 5 6 7 8 9
 - 1 decimal digit represents 1 choice from 10; counts 10 things; 10 distinct values
 - 00 01 02 ... 10 11 12 ... 20 21 22 ... 98 99
 - 2 decimal digits represents 1 choice from 100; 100 distinct values
 - we usually elide zeros at the front
 - 000 001 ... 099 100 101 ... 998 999
 - 3 decimal digits ...
- **decimal numbers are shorthands for sums of powers of 10**
 - $1492 = 1 \times 1000 + 4 \times 100 + 9 \times 10 + 2 \times 1$
 - $\quad = 1 \times 10^3 + 4 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$
- **counting in "base 10", using powers of 10**

Binary numbers: using bits to represent numbers

- just like decimal except there are only two digits: 0 and 1
- everything is based on powers of 2 (1, 2, 4, 8, 16, 32, ...)
 - instead of powers of 10 (1, 10, 100, 1000, ...)
- counting in binary or base 2:
 - 0 1
 - 1 binary digit represents 1 choice from 2; counts 2 things; 2 distinct values
 - 00 01 10 11
 - 2 binary digits represents 1 choice from 4; 4 distinct values
 - 000 001 010 011 100 101 110 111
 - 3 binary digits ...
- binary numbers are shorthands for sums of powers of 2
 - $11011 = 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1$
 $= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
- counting in "base 2", using powers of 2



Binary (base 2) arithmetic

- works like decimal (base 10) arithmetic, but simpler

- addition:

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

- subtraction, multiplication, division are analogous

Bytes

- **"byte" = group of 8 bits**
 - on modern machines, the fundamental unit of processing and memory addressing
 - can encode any of $2^8 = 256$ different values, e.g., numbers 0 .. 255 or a single letter like A or digit like 7 or punctuation like \$
ASCII character set defines values for letters, digits, punctuation, etc.
- **group 2 bytes together to hold larger entities**
 - two bytes (16 bits) holds $2^{16} = 65536$ values
 - a bigger integer, a character in a larger character set
Unicode character set defines values for almost all characters anywhere
- **group 4 bytes together to hold even larger entities**
 - four bytes (32 bits) holds $2^{32} = 4,294,967,296$ values
 - an even bigger integer, a number with a fractional part (floating point), a memory address
- **etc.**
 - recent machines use 64-bit integers and addresses (8 bytes)
 $2^{64} = 18,446,744,073,709,551,616$

Interpretation of bits depends on context

- **meaning of a group of bits depends on how they are interpreted**
- **1 byte could be**
 - 1 bit in use, 7 wasted bits (e.g., M/F in a database)
 - 8 bits storing a number between 0 and 255
 - an alphabetic character like W or + or 7
 - part of a character in another alphabet or writing system (2 bytes)
 - part of a larger number (2 or 4 or 8 bytes, usually)
 - part of a picture or sound
 - part of an instruction for a computer to execute
 - instructions are just bits, stored in the same memory as data
 - different kinds of computers use different bit patterns for their instructions
 - laptop, cellphone, game machine, etc., all potentially different
 - part of the location or address of something in memory
 - ...
- **one program's instructions are another program's data**
 - when you download a new program from the net, it's data
 - when you run it, it's instructions

Powers of two, powers of ten

1 bit = 2 possibilities

2 bits = 4 possibilities

3 bits = 8 possibilities

...

n bits = 2^n

$2^{10} = 1,024$ is about 1,000 or 1K or 10^3

$2^{20} = 1,048,576$ is about 1,000,000 or 1M or 10^6

$2^{30} = 1,073,741,824$ is about 1,000,000,000 or 1G or 10^9

the approximation is becoming less good
but it's still good enough for estimation

- **terminology is often imprecise:**
 - " 1K " might mean 1000 or 1024 (10^3 or 2^{10})
 - " 1M " might mean 1000000 or 1048576 (10^6 or 2^{20})

Converting between binary and decimal (version 1)

- **binary to decimal:**

$$\begin{aligned}1101 &= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 13\end{aligned}$$

- **decimal to binary:**

- start with largest power of 2 smaller than the number
- for each power of 2 down to 2^0
- if you can subtract that power of 2, do so and write "1"
- otherwise write "0"

- start with 13, subtract 8, write "1"
- with 5, subtract 4, write "1"
- with 1, can't subtract 2, write "0"
- with 1, subtract 1, write "1"
- answer is 1101

Converting between binary and decimal (version 2)

- **decimal to binary (from right to left):**

- repeat while the number is > 0 :
- divide the number by 2
- write the remainder (0 or 1)
- use the quotient as the number and repeat
- answer is the resulting sequence in reverse (right to left) order

- divide 13 by 2, write "1", number is 6
- divide 6 by 2, write "0", number is 3
- divide 3 by 2, write "1", number is 1
- divide 1 by 2, write "1", number is 0
- answer is 1101

Hexadecimal notation

- binary numbers are bulky
- hexadecimal notation is a shorthand
- it combines 4 bits into a single digit, written in base 16
 - a more compact representation of the same information
- hex uses the symbols **A B C D E F** for the digits 10 .. 15

0 1 2 3 4 5 6 7 8 9 A B C D E F

0	0000	1	0001	2	0010	3	0011
4	0100	5	0101	6	0110	7	0111
8	1000	9	1001	A	1010	B	1011
C	1100	D	1101	E	1110	F	1111

ASCII (better representation: uses hex)

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SPC	!	"	#	\$	%	&	'	()	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[\]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{		}	~	DEL

Color

- **TV, computers, etc., use Red-Green-Blue (RGB) model**
- **each color is a combination of red, green, blue components**
 - $R+G = \text{yellow}$, $R+B = \text{magenta}$, $B+G = \text{cyan}$, $R+G+B = \text{white}$
- **for computers, color of a pixel is usually specified by three numbers giving amount of each color, on a scale of 0 to 255**
- **this is often expressed in hexadecimal so the three components can be specified separately (in effect, as bit patterns)**

Things to remember

- **digital devices represent everything as numbers**
 - discrete values, not continuous or infinitely precise
- **all modern digital devices use binary numbers (base 2)**
 - instead of decimal (base 10)
- **it's all bits at the bottom**
 - a bit is a "binary digit", that is, a number that is either 0 or 1
 - computers ultimately represent and process everything as bits
- **groups of bits represent larger things**
 - numbers, letters, words, names, pictures, sounds, instructions, ...
 - the interpretation of a group of bits depends on their context
 - the representation is arbitrary; standards (often) define what it is
- **the number of digits used in the representation determines how many different things can be represented**
 - number of values = base ^{number of digits}
 - e.g., 10^2 , 2^{10}