PRINCETON UNIVERSITY F'08	COS 521: ADVANCED ALGORITHMS
Homework 4	
Out: *Dec 17*	Due: *Jan 6*

You can collaborate with your classmates, but be sure to list your collaborators with your answer. If you get help from a published source (book, paper etc.), cite that. Also, limit your answers to one page or less —you just need to give enough detail to convince me. If you suspect a problem is open, just say so and give reasons for your suspicion.

§1 Show that no deterministic algorithm can solve the *byzantine generals problem* when the number of players is three and one of them is malicious.

§2 The algorithm for the *byzantine generals problem* given in class works with high probability but we did not describe how the players can detect when it works. Modify the algorithm so they can.

§3 In class we analyzed an estimator for the second frequency moment $F_2 = \sum_i m_i^2$ where m_i is the number of copies of element i in the data stream. Here the basic form of the estimator consisted of a counter $C = \sum_i \epsilon_i m_i$ (where $\epsilon_i \in \{-1, +1\}$) and the estimator value was C^2. A number of independent copies of this estimator were required to obtain a good estimate with high probability. Consider the following variant to estimate $F_3 = \sum_i m_i^3$. Maintain a counter $C = \sum_i \epsilon_i x_i$ where x_i is picked uniformly and randomly from $\left\{1, \omega, \omega^2\right\}$ (where ω is a complex cube root of unity). For purposes of this question ignore issues of independence in the choices of ϵ_i and assume they are picked independently. Show that the expected value of C^3 is F_3. Describe further how you can develop this into an estimator for F_3.

§4 Let $d : X \times X \to \Re^+ \cup \{0\}$ be a metric on a point set X. We say that *embeds with distortion* C into another metric d_2 on point set X_2 if there is a mapping $f : X \to X_2$ such that

$$\forall x, y \in X \qquad d_2(f(x), f(y)) \leq d(x, y) \leq C d_2(f(x), f(y)).$$

Find the smallest C such that the n-point metric defined by the n-cycle embeds with distortion C into $\Re^n$ with Euclidean norm.