COS 487: Theory of Computation **Fall 2008**

Assignment #2

Due: Tuesday, October 14 *Sanjeev Arora*

1. Let C be a context-free language and R be a regular language. Show that $C \cap R$ is context-free.

2. Let B be the language of palindromes over $\{0, 1\}$ containing an equal number of 0's and 1's. Show that B is not context-free.

3. Let $C = \{x\#y \mid x, y \in \{0, 1\}^* \text{ and } x \neq y\}$. Show that C is context-free. (Construct either a CFG or a PDA that recognizes C)

4. Show that a language is decidable iff some enumerator enumerates the language in *lexicographical order*. (shorter strings appear first in lexicographic ordering)

5. Let $C_{CFG} = \{\langle G, k\rangle \mid L(G) \text{ contains exactly } k \text{ strings where } k \geq 0 \text{ or } k = \infty\}$. Show that C_{CFG} is decidable.

6. Let A and B be two disjoint languages. Say that language C *separates* A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

7. Given a natural number n, define $f(n)$ to be $n/2$ if n is even and $3n+1$ if n is odd. The $(3n+1)$ conjecture states that for any natural number n, the sequence $f(n), f(f(n)), \ldots$ reaches 1 in finitely many steps (note that then it remains 1).

The conjecture is well-studied in mathematics but it still remains open. Show that if A_{TM} is decidable, then there exists a Turing machine that can determine if the $(3n + 1)$ conjecture is true.

(This is one reason to expect A_{TM} to be undecidable, because if not, a Turing machine can solve a major open problem).

8. Show that there exists a language $L \subseteq \{1\}^*$ which is undecidable.